Unit 5 Work, energy and power

Introduction

Energy is present in many forms, including mechanical energy, electro-magnetic energy, heat energy, and nuclear energy. The various forms are related to one another such that, when energy is transformed from one form to another, the total amount of energy remains the same. If an isolated system loses energy in some form, then, by the law of conservation of energy, the system must gain an equal amount of energy in other forms. For example, when an electric motor is connected to a battery, chemical energy is converted to electrical energy, which in turn is converted to mechanical energy. The transformation of energy from one form to another is an essential part of the study of physics, chemistry, biology, geology, and astronomy.

Objectives

After working through and completing this unit, you should be able to

- Define "work", "energy" and "power"
- Define work as the scalar product of the force, f applied to an object and the displacement s undergone by the object, i.e. $|f||s|\cos\theta$ where θ is the angle between the force vector and the displacement.
- State the work-energy theorem and use the same to calculate the increase or decrease in the kinetic energy of an object
- State the expressions $\frac{1}{2}mv^2$ and mgh as the kinetic and mechanical potential energies of an object respectively.
- State the law of conservation of mechanical energy and solve problems based on the same concept.
- Define power as the rate of energy expenditure.

The meaning of the term **work ,** symbol **W,** in physics is distinctly different from its meaning in our day-to-day affairs. It can be defined with the help of figure 5.3.1.

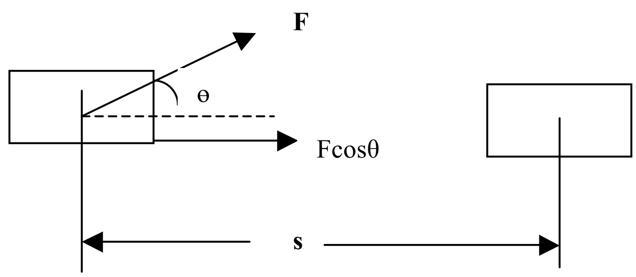


Figure 5.3.1

Here we see an object that undergoes a displacement **s** along a straight line under the action of a constant force ***F,*** which makes an angle **θ** with **s**.

The work, **W,** done by the constant force is defined as the product of the component of the force in the direction of the displacement and the magnitude of the displacement. Mathematically,

$$\mathbf{W = |F|\cos\theta)|s|} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.3.1)$$

According to this definition, a force does no work on an object if the object does not move, i.e. if **s = 0.** Note also that the work done by a force is zero when the force is perpendicular to the displacement, i.e. If $\theta = 90^o$ then **W = 0** since $\cos 90^o = 0$.

5.3.1 Positive and **negative work.**

The sign of the work done depends on the direction of ***F*** relative to **s**. The work done by an applied force is positive when the component **Fcosθ** is in the same direction as the displacement. For example when you lift a box, the work done by the force you exert on the box is positive because the lifting force is upward i.e. in the same direction as the displacement. Work is negative when the component of the applied force is in the direction opposite the displacement. A common example of a situation in which work is always negative is the work done by a frictional force when a body slides over a rough surface. This is so because the direction of the frictional force is opposite to the displacement. The negative sign comes from the fact that $\theta = 180^o$ and $\cos 180^o = -1$.

Finally, if an applied force acts along the direction of the displacement, then $\theta = 0^o$. Since $\cos 0 = 1$, the equation 5.3.1 becomes

$$W = |F||s| \quad \ldots\ldots\ldots\ldots\ldots\ldots (5.3.2)$$

Work is a scalar quantity, and its units are force multiplied by length so that the **S.I.** unit of work is the newton-meter (N.m). Another name for the newton-meter is the **joule (J).**

Example 5.3.1 Mr. Clean

A man cleaning his apartment pulls a vacuum cleaner with a force of 50N at an angle of $\mathbf{30^o}$ as shown in figure 5.3.2. A frictional force of $\mathbf{40^o}$ **N** retards the motion, and the vacuum cleaner is pulled a horizontal distance of **3 m**. Calculate

(a) the network done by the **50 N** pull;

(b) the work done by the frictional force and

(c) the net work done on the vacuum cleaner by all forces acting on it.

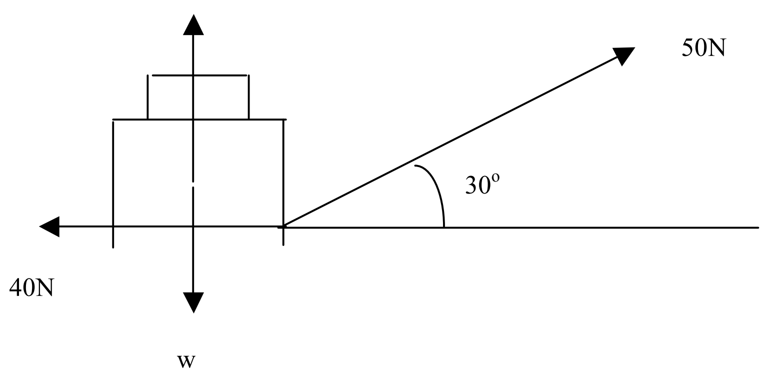


Figure 5.3.2

Solution: (a) We can make use of the relation

$$W_F = (F \cos\theta)s \text{ with } F = 50N$$

Given $\theta = 30^o$, and $s = 3$ m to get

$$W_F = (50N)(\cos 30^o)(3\ m) = 130\ J$$

(b) In this case, we take $f = 40N$ and $\theta = 180^o$ to get

$$W_f = (40N)(\cos 180^o)(3\ m) = -120\ J$$

(c) The normal force, **n**, the weight, **mg**, and the upward component of the applied force, $\mathbf{50\sin 30^o}$, do not do any work because they are perpendicular to the displacement. Thus, the net work done is

$$W_{net} = W_F + W_f = 10\ J.$$

Exercise. Find the net work done on the vacuum cleaner if the man pulls it horizontally with a force of **50 N**, assuming the frictional force is **40 N**. Answer: **30 J**

Work and kinetic energy

The figure below depicts an object of mass **m** moving to the right without friction under the action of a constant net force **F**. As the force is constant, we know from Newton's second law that the object will move with a constant acceleration **a**. If the object is displaced a distance **s**, the work done by the force is

$$W = Fs = mas \quad .... \quad (5.4.1)$$

since from Newton's second law F = ma.

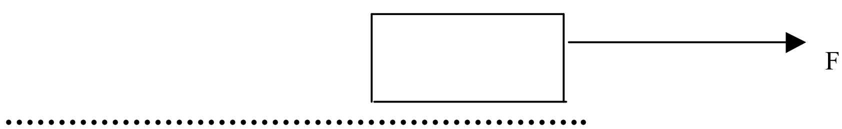


Figure 5.4.1

When an object undergoes constant acceleration, the following relation holds (unit 2)

$$V^2 = V_o^2 + 2as \text{ or } a\times s = \frac{v_f^i - v_i^2}{2} \quad .... \quad (5.4.2)$$

Using this in **(5.3.1),** obtains

$$W = m\left(\frac{v_f^2 - v_i^2}{2}\right)$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 \quad ...... \quad .. \quad (5.4.3)$$

The quantity $\frac{1}{2}mv^2$ is termed as the **Kinetic energy** of the object. It is a scalar quantity and has the same units as work. For example, a 1 kg mass moving with a speed of 4.0 m/s has a kinetic energy of 8.0 J. We can think of kinetic energy as the energy associated with the motion of an object.

5.4.1 The work - energy theorem

It is often more convenient to write equation 5.4.3 as

$$\mathbf{W = KE_f - KE_i} \quad ... \quad . \quad .... \quad (5.4.4)$$

According to this equation, the effect of the work done by the **net** force on an object is to change the kinetic energy of the object from some initial value $\mathbf{KE_i}$ to some final value $\mathbf{KE_f}$. Equation **5.4.4** is an important result known as the **work-energy theorem**. Thus, it is a foregone conclusion that the net work done on an object by the resultant force acting on it is equal to the change in the kinetic energy of the object.

This theorem also says that the speed of the object will increase if the work done on it is positive because the final kinetic will be greater than the initial kinetic energy. On the other hand, the speed will decrease if the net work is negative because the final kinetic energy will be less than the initial kinetic energy. Notice that the speed and hence kinetic energy of an object will change only if work is done on the object by some external force. Because of this connection between work and change in kinetic energy, we can also think of the kinetic energy of an object as the work the object can do in coming to rest.

Example 5.4.1 Towing a Car

A 1400 kg car has a net forward force of 4500N applied to it. The car starts from rest and travels down a horizontal highway. What are its kinetic energy and speed after it has traveled 100m? Ignore losses in energy due to friction, such as that caused by air resistance.

Solution: With the initial velocity given as zero, equation 5.4.3 reduces to

$$W = \frac{1}{2}mv^2$$

The work done by the net force on the car is therefore

$$W = F.s = (4500N)\ (100m) = 4.5 \times 10^5 \text{ J}$$

The work has entirely gone into changing the kinetic energy of the car. Thus the final value of the kinetic energy is also 4.50×10^5 J.

The speed of the car is found as follows:

$$4.5 \times 10^5 \text{ J} = \frac{1}{2}mv^2$$

$$\text{or} \quad v = 25.4 \text{ m/s}$$

5.4.2 Work done by a gas

Whenever a gas expands against some external force, it does work on the external agency. Conversely whenever a gas is compressed by the action of some outside force, work is done on the gas. To calculate the work done when a gas expands, consider an ideal gas enclosed in a cylinder, equipped with a movable piston as shown, figure 5.4.2.

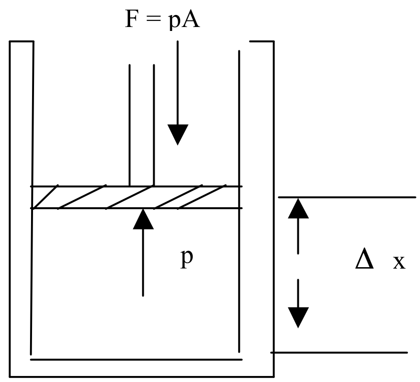


Figure 5.4.2

But $\Delta x = \Delta V$ where ΔV is the change in volume.

If the pressure exerted by the gas is **p** and the cross-sectional area of the cylinder is A, then the force exerted by the gas on the piston is pA. Suppose the piston moves out a small distance Δx the work ΔW done due to this force is given by

$$\Delta W = P.A\ \Delta x \quad \ldots\ldots\ \ldots (5.4.5)$$

$$\therefore \qquad \Delta W = P.\Delta V \quad \ldots\ldots\ldots \qquad (5.4.6)$$

If the pressure is constant i.e. the process is isobaric and the gas expands from its initial volume $\mathbf{V_i}$ to some final volume $\mathbf{V_f}$ the total work, W is equal to

$$\mathbf{W} = \sum \Delta \mathbf{W} = \sum_{V_1}^{V_2} P\Delta V \qquad (5.4.7)$$

according to convention, this work is taken to be positive, since the volume increases in the process.

Mechanical Potential energy

Potential energy (P.E) is sometimes referred as the energy possessed by an object as a consequence of its position in a given field, or a change in its shape. For example a mass lifted above the surface of the earth has stored in it gravitational potential energy (G.P.E.) as a consequence of its being in the Earth's gravitational field. It releases this energy when it falls back to the ground. A good example is water falling over a waterfall. The falling water is capable of running a turbine which, if hooked to a generator, is capable of producing electricity. Gravitational potential energy is mechanical energy just like kinetic energy. Mathematically, it is defined as the product of the mass of an object *m*, the gravitational acceleration *g* and the height through which the mass has been raised i.e.

$$P.E. = mgh. \;\ldots\ldots\ldots\ldots \quad (5.5.1)$$

If the initial and final heights (h_i, h_f) have been defined, the expression involves a change in potential energy i.e.

$$\Delta P.E. = mg(h_f - h_i) = mg\,\Delta h. \;\ldots\ldots\ldots\ldots \quad (5.5.2)$$

Conservation of mechanical energy

In the absence of dissipative forces such as friction, total mechanical energy in a system is conserved. For example if a mass m is lifted through a height h, the total mechanical energy stored in the mass is the potential energy given by

$$E_T = P.E. = mgh. \;\ldots\ldots \quad (5.6.1)$$

As the mass falls towards the ground, there is continuous conversion of some of the potential energy into kinetic energy until all the potential energy is converted into kinetic energy in which case the following equation obtains

$$E_T = P.E = mgh = K.E = \frac{1}{2}mv^2 \;.\;\;\ldots \quad (5.6.2)$$

The velocity of the object can then be easily calculated.

5.6.1 Conservative forces and the work-energy theorem

When the work done by a force depends only on the end points and not on the length of the path traversed by the object on which the force acts, then the force is a conservative force. An example is **gravity**.

Consider an object moving under the influence of a conservative force. If this is the only force acting on the object, then the work-energy theorem tells us that the work done by that force equals the change in the Kinetic energy of the object:

$$W_c = KE_f - KE_I \;\ldots\ldots\ldots \quad (5.6.3)$$

Since the force is conservative, W_c also equals the difference in the potential energies. Mathematically, this is

$$PE_I - PE_f = KE_f - KE_i \;\ldots\ldots \quad (5.6.4)$$

Re -arranging**,**

$$KE_I + PE_I = KE_f + PE_f \;\ldots\ldots \quad (5.6.5)$$

This is the **law of conservation of mechanical energy**. Alternatively, the law can be written

$$\Delta KE + \Delta PE = 0 \quad ... \quad . (5.6.6)$$

where $\Delta KE = KE_f - KE_i$ is the **change** in the kinetic energy, and Δ **PE= PE$_f$ - PE$_i$** is the **change** in the potential energy.

The law of conservation of mechanical energy states that the total mechanical energy of a system remains constant if the only force that does work is a conservative force.

This is equivalent to saying that, if the kinetic energy of a conservative system increases (or decreases) by some amount the potential energy must decrease (or increase) by the same amount. We have already encountered gravity as a conservative force. Thus, if the force of gravity is the only force doing work on an object, then the total mechanical energy of the object is conserved and the above law takes the form

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f \quad .... \quad ..... (5.6.7)$$

Example 5.6.1: The Atwood machine, figure 5.6.1.

Two masses in a frictionless Atwood machine have an initial speed of 0.20 ms^{-1}, with M_2 moving upward. How high does m_2 rise above its initial position before momentarily coming to rest? Take $M_1 = 3.7$kg and M_2=4.1 kg.

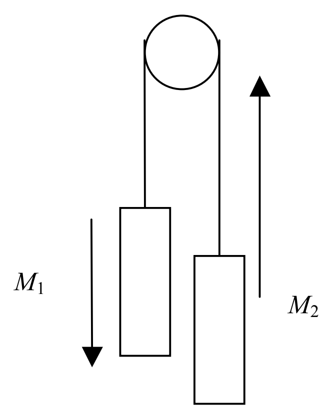


Figure 5.6.1

Solution

Initial speed $v_i = 0.2\,\text{ms}^{-1}$ for both masses.

Initial kinetic energy is given by

$$K.E_i = \frac{1}{2}M_2(0.2)^2 + \frac{1}{2}M_1(0.2)^2.$$

Let M_2 rise to a height h before coming to rest momentarily before changing direction. Thus potential energy change for the two masses is

$$\Delta PE_{M_2} = M_2 gh - 0 \text{ for } M_2$$

and potential energy change for M_1

$$\Delta PE_{M_1} = 0 - M_1 gh,$$

where the reference level for potential energies is the initial position level of the two masses. Total potential energy change

$$\Delta PE = (4.1)(9.8)h - (3.7)(9.8)h = 3.92h$$

Kinetic energy change for the two masses is

Mass $M_2 = 0 - \frac{1}{2}(4.1)(0.04) = -0.082\,\text{J}$; mass $M_1 = 0 - \frac{1}{2}(3.7)(0.04) = 0.074\,\text{J}$.

Total kinetic energy change $\Delta KE = -0.156\,\text{J}$

Conservation of mechanical energy requires that:

$$\Delta KE + \Delta PE = 0 \quad \ldots\ldots\ldots\ldots\ldots(5.6.6)$$

since gravity is a conservative force. Adding the two expressions as required by 5.6.6, we get

$$-0.156 + 3.92h = 0$$

$$h = 0.040\,\text{m} \text{ or } 4\text{ cm}$$

Note: You can solve the same problem using Newton's second law. The acceleration $a = 0.50\,\text{ms}^{-2}$. (Can you prove this result?). Using the relation

$$v_f^2 = v_i^2 + 2ah = 0 = (.2)^2 - 2(0.051)h$$

From which $h = 0.04\,\text{m}$ as obtained using the energy merhod.

Example 5.6.2 A sled and its rider together weigh 800N. They move down a **frictionless** hill through a vertical distance of 10 m. Use conservation of mechanical energy to find the speed of the sled – rider system at the bottom of the hill, assuming that the rider pushes off with an initial speed of 5 m/s.

Solution:

The forces acting on the sled and rider as they move down the hill are as shown in figure 5.6.3. In the absence of a frictional force, the only force acting are the normal force, **n**, and the gravitational force, W

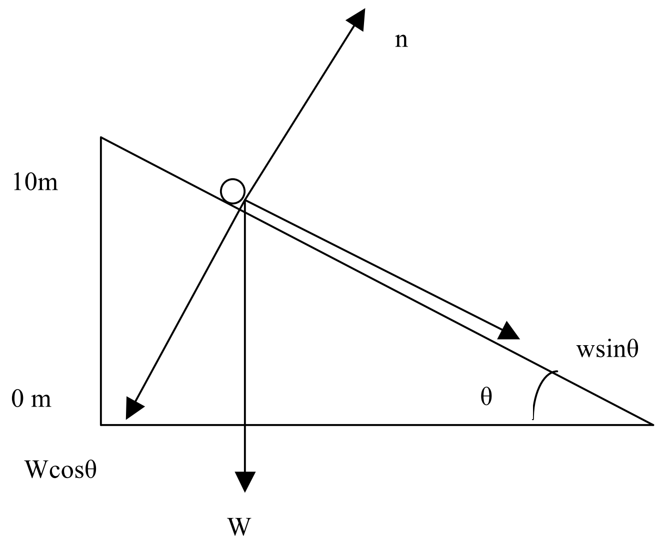


Figure 5.6.3

At all points along the path, **n** is perpendicular to the direction of travel and hence does no work. Likewise, the component of weight perpendicular to the incline (**ω cosθ)** does no work. The only force that does any work is the component of the gravitational force **F** along the slope of the hill. As a result, the only force doing any work is the gravitation force and we are justified in using the equation for conservation of mechanical energy. Not that in this case the initial energy includes Kinetic energy because of the initial speed.

$$\therefore \quad \tfrac{1}{2} mv_i^2 + mgy_i = \tfrac{1}{2} mv_f^2 + mgy_f \quad \text{.........}(5.6.8)$$

or

$$\tfrac{1}{2} v_i^2 + gy_i = \tfrac{1}{2} v_f^2 + gy_f \quad \text{.......}(5.6.9)$$

If we select the origin at the bottom of the incline, we get

$$\tfrac{1}{2}(5\ m/s)^2 + (9.8\ m/s^2)(10\ m) = \tfrac{1}{2} v_f^2 + (9.8\ m/s^2)(0)$$

or $v_f = 14.9$ m/s.

Student Activity 5.6.1

A child and a sled with a combined mass of 50.0 kg slide down a frictionless slope. If the sled starts from rest and has a speed of 3.0 ms^{-1} at the bottom, what is the height of the hill? Answer: 0.459 m.

5.6.2 **Non-conservative forces** and the **work-energy theorem**

In practical situations, non-conservative forces, such as friction are usually present. Therefore, the total mechanical energy in a system is not constant. The effect of non-conservative forces can be accounted for by using the **work-energy** theorem. If $\mathbf{W_{nc}}$ represents the work done on an object by all non-conservative forces and $\mathbf{W_c}$ the work done by all conservative forces, the work-energy theorem can be written as

$$W_{nc} + W_c = KE_f - KE_i \qquad \ldots\ldots \quad (5.6.1)$$

Where W_{nc} , W_c is the work done by non-conservative forces and conservative forces respectively and KE_f , KE_i is the final and initial kinetic energies of the system respectively.

Suppose the conservative force is the gravitational force, then

$$W_c = mgy_i - mgy_f = PE_i - PE_f \qquad \ldots\ldots (5.7.2)$$

and we have $\mathbf{W_{nc}} = (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2) + (mgy_f - mgy_i)$

$$W_{nc} = (KE_f - KE_i) + (PE_f - PE_i) \qquad .(5.6.3)$$

That is, **the work done by all non-conservative forces equals the change in kinetic energy plus the change in potential energy.**

Since the total mechanical energy is given by

$$\mathbf{E_T = KE + PE,}$$

we can also state the above equation as

$$W_{nc} = (KE_f + PE_f) - (KE_i + PE_i) = E_f - E_i \quad ..(5.6.4)$$

i.e. the work done by all non-conservative forces equals the change in the total mechanical energy of the system.

The work done by non-conservative forces can either add energy to an object or remove energy from the object. For example, a jet engine adds energy to an airplane whereas the drag force due to air-resistance removes energy from the airplane.

Example 5.6.1:

A 3 kg crate, slides down a rough ramp at a loading dock. The ramp is 1 m in length and is inclined at an angle of 30° as shown. The crate starts from rest at the top and experiences a constant frictional force of magnitude 5 N. Use the principle of conservation of energy to determine the speed of the crate when it reaches the bottom of the ramp.

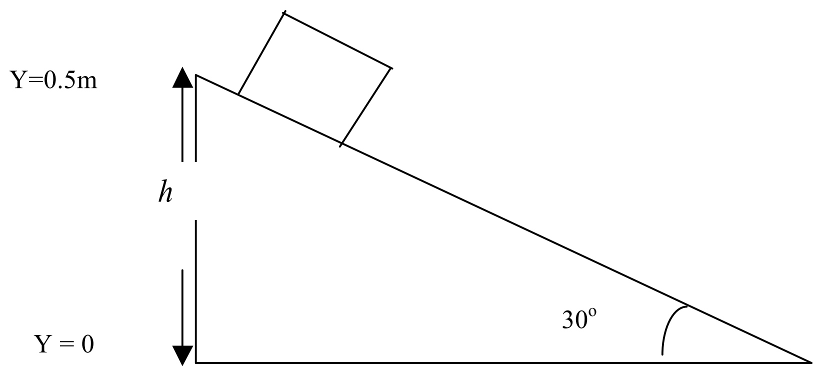


Figure 5.6.1

Solution.

Because v_i = 0, the initial kinetic energy is zero. If the **y** co-ordinate is measured from the bottom of the incline, then y_i = 0.5 m. Therefore the total mechanical energy of the crate at the top is all potential energy, given by

$$\mathbf{E_i} = \mathbf{PE_i} = (3\ \text{Kg})\ (9.80\ \text{ms}^{-2})\ (0.5\ \text{m}) = 14.7\ \text{J}.$$

Note that KE_i = 0.

When the crate reaches the bottom, its potential energy is zero since its elevation is y_f = 0. Therefore, the total mechanical energy at the bottom is all kinetic energy: $E_f = KE_f = ½\, mv_f^2$.

It cannot, however, be said that $\mathbf{E_i = E_f}$ as there is a non-conservative force that does work on the crate: the force of friction. In this case $\mathbf{W_{nc} = - fs}$ where **s** is the displacement along the ramp. With **f = 5 N** and **s = 1 m**, we have

$$W_{nc} = - fs = (-5\ \text{N})\ (1\ \text{m}) = -5\ \text{J}$$

This says that some mechanical energy is lost because of the presence of the retarding frictional force. Applying the **work-energy theorem** gives

$$W_{nc} = E_f - E_i$$

$$-fs = ½\, mv_f^2 - mgy_1 \quad \text{or} \quad \mathbf{v_f^2} = \frac{19.4J}{3Kg} = 6.47 m^2/s^2 \text{ from which } v_i = 2.54 \text{ m/s.}$$

Student Activity 5.6.2

The coefficient of sliding friction between a 900 kg car and the pavement is 0.80. If the car is moving at $25 ms^{-1}$ along level pavement when it begins to skid to a stop, how far will it go before coming to a stop? Answer: 40 m

Power

From a practical viewpoint, it is interesting to know not only the work done on an object but also the rate at which the work is being done. Power is defined as the rate of doing work i.e. power is work done per unit time.

Average power

$$\mathbf{P} \equiv \frac{\Delta W}{\Delta t} \quad \ldots\ldots\ldots\ldots \quad (5.7.1)$$

where ΔW is the work done and Δt is the time interval during which the work is done. As we have seen before

$$\Delta W = F\Delta s. \quad \ldots\ldots\ldots\ldots (5.3.2)$$

Therefore

$$P = \Delta W/\Delta t = F\Delta s/\Delta t = Fv \ldots\ldots \quad (5.7.2)$$

Note that $\frac{\Delta s}{\Delta t} = v$. Equation 5.8.2 shows that the average power delivered to an object is equal to the product of the force acting on the object during some time interval and its average velocity during this time period.

The S.I. unit of power is the joule per second (J/s) also called the watt.

$1W = 1\ J/s = 1\ kg\ m^2/s^3$

The watt is commonly used only in electrical applications, but it can be used in other scientific areas. For example, an automobile engine can be rated in watts as well as in horse power.

Example 5.7.1 Power delivered by an elevator motor

An elevator of 1000 kg is designed to carry a maximum load of 800 kg. A constant frictional force of 4000 N retards its motion upwards, as in figure 5.7.1

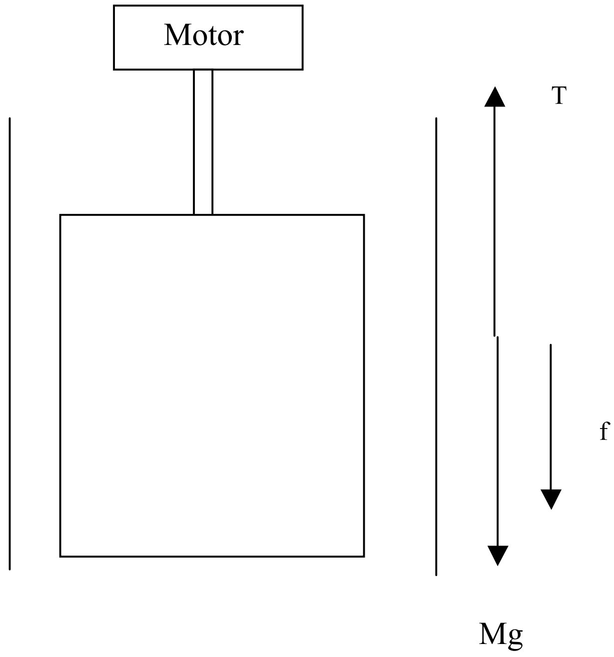


Figure 5.7.1

What minimum power must the motor deliver to lift the fully loaded elevator at a constant speed of 3 m/s?

Solution:

The motor must supply the force **T** that pulls the elevator upwards. From Newton's second law and from the fat that **a = 0** because **v** is constant, we get

$$T - f - mg = 0$$

where **M** is the total mass (elevator + load) equal to 1800 Kg. Therefore

$$T = f + mg$$

$$T = 4 \times 10^3 \text{ N} + (1.8 \times 10^3 \text{ Kg})(9.80 \text{ m/s}^2)$$

$$T = 2.16 \times 10^4 \text{ N}$$

From $P = Fv$ and the fact that **T** is in the same direction as **V**, we have

$$P = Tv = (2.16 \times 10^4 \text{ N})(3 \text{ m/s}) = 6.48 \times 10^4 \text{ W}$$

$$P = 64.8\text{kW}.$$

Self-assessment questions

Q1 A force of **40 N** acts on a box as shown

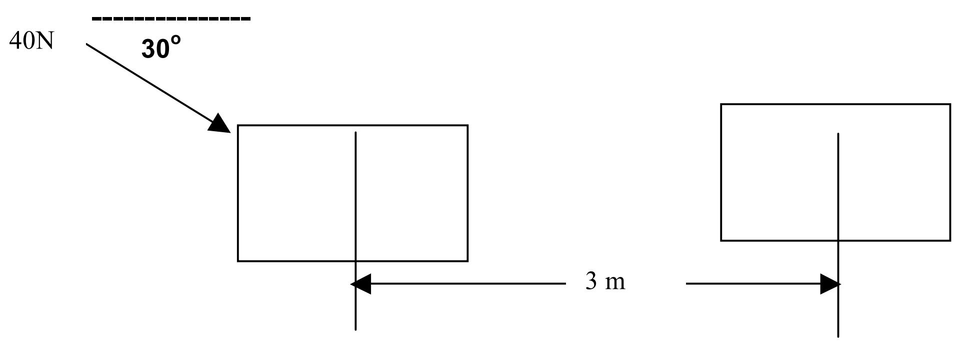


Figure 5.8.1

Calculate

(a) the work done by the force if it moves the box a total displacement of 3 m. Take the floor as frictionless.

(b) the total energy expenditure if the floor is found to have $\mu = 0.1$. Take mass of box as 10 kg

Q2 (a) Power can be calculated from the product of force applied and the velocity, $P = Fv$. Justify this expression, starting from the definition of power.

(b) A cyclist pedaling along a horizontal road provides 200W of useful power. The cyclist reaches a steady speed of 5.0 m/s. What is the value of the drag forces against which the cyclist is working?

(c) If the drag forces are proportional to the speed of the bicycle,

(i) show that the useful power the cyclist must produce at a speed v along the flat road is proportional to v^2;

(ii) predict what power the cyclist must produce to reach a speed of 6.0 m/s along the flat road.

(d) What useful power would the cyclist have to develop to maintain a speed of 5.0 m/s when climbing a hill of 1 in 30? Take the mass of the cyclist plus the bicycle to be 100 kg.

Q3 The intensity of solar radiation reaching the edge of the Earth's atmosphere is about 1400 Wm^{-2}. Use this information to estimate the total power output of the Sun. (Hint: assume no energy is absorbed between the Sun and the Earth, and that the radiation from the Sun spreads evenly over the surface of a sphere whose radius is equal to the distance from the Earth to the Sun, about 1.5×10^{11} m.).

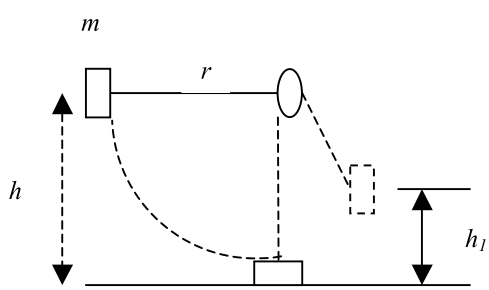
Q4 A team of dogs pulls a 100 kg sledge for a distance of 100 m at a constant velocity of 20m/s. If the coefficient of friction between the ice pavement and the sledge is 0.2, find the work done by the dogs.

Q5 Worked problem

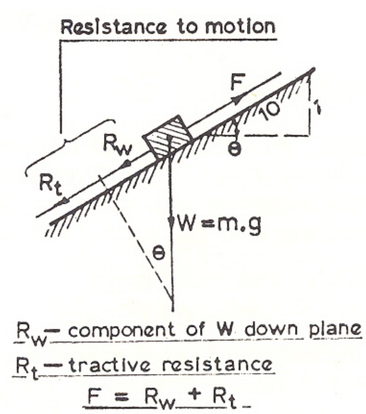
A hammer of mass 14 kg is attached to an arm carried on frictionless pivots and swings in a circle of one meter radius. The hammer is held in a horizontal position and on release strikes a specimen at the lowest point of the swing. If 60 J of energy is required to fracture the specimen, find

(i) the velocity at impact, and

(ii) the height to which the hammer will rise after fracturing the specimen.



5.8.2



Solution

(i) Just before impact, loss of potential energy = gain of kinetic energy or

$$mgh = \frac{1}{2}mv^2 \text{ .Or } v = \sqrt{2gh} = \sqrt{2 \times 9.807 \times 1} = 4.43 \text{ m/s}$$

(ii) By the principle of conservation of energy,

Kinetic energy before impact = initial potential energy

$= Wh = mgh = 14 \times 9.807 \times 1 = 137.3$ J.

After impact, kinetic energy = 137.3 – 60 = 77.3 J

Final potential energy = 14 ×9.807h_1 = 77.3 J

Therefore h_1 = 77.3/(14× 9.807) = 0.563 m = 563 mm.

Q6 Worked problem

Find the work done per minute in moving a trolley of mass 270 kg at a speed of 9 km/hr against a resistance of 90 N

(i) on level ground and

(ii) up an incline of 1 in 10.

Solution

(i) Resistance to motion = 90 N = applied force

Distance moved in one minute = 9 ×1000/60 = 150 m

Work done = 90 ×150 Nm = 13 500 J = 13.5 kJ

(ii) Resistance to motion = tractive resistance + component of weight down the incline

= 90 + $W\sin\theta$, and $W = mg$

= 90 + 270×9.807 ×(1/10) = 355. Work done = 355 ×150 = 53.25 kJ.