The University of Zambia
School of Natural Sciences
Department of Physics
PHY 1010
Lecture 2B
Equations of Uniform Accelerated Motion

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1 Introduction

In this lecture, we will derive the equations of motion that are used to describe uniformly accelerated motion in a straight line.

2 Learning outcomes

By the end of this lecture, the student should be able to:

1. state the five equations of motion that are used to represent uniformly accelerated motion in a straight line;
2. use the equations of motion used to describe uniformly accelerated motion to solve problems.

3 Equations of Uniformly Accelerated Motion In A Straight Line

The velocity-time $(v-t)$ graph is particularly important for deriving the equations of uniform accelerated motion because its gradient is equal to the acceleration and the area under the graph is equal to the displacement.

$$\text{acceleration} = \text{slope of velocity time graph}$$

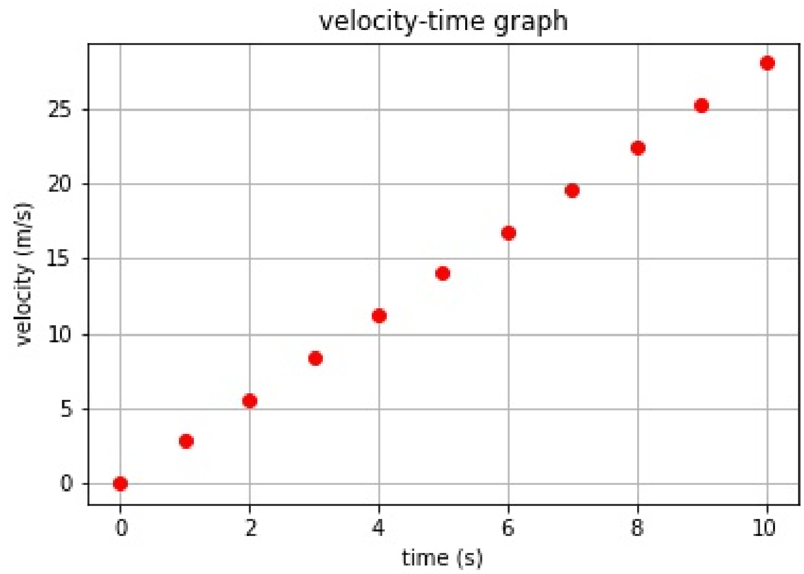

Figure 1: The velocity-time graph of a car accelerating at $a = 2.8$ m/s^2 for 10 s.

To obtain the gradient of a $v - t$ graph, we need to define the following physical quantities:

s - displacement (m)

t - time (s)

u - initial velocity (m/s) at $t = 0$

v - final velocity (m/s) after t

a - acceleration (m/s^2)

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$a = \frac{\Delta v}{t}$$

where

a is the average acceleration in m/s^2;

Δv is the change in velocity in m/s

t is the time taken in s

$$a = \frac{v-u}{t} \tag{1}$$

where

a is the average acceleration in m/s^2;

v is the final velocity in m/s;

u is the final velocity in m/s;

t is the time taken in s.

This is the first equation of motion used to describe uniform accelerated motion in a straight line.

Multiplying both side of the equation (1) by t, we obtain:

$$at = v - u$$

Moving u to the other side of the equation, we obtain the second equation of motion used to describe uniform accelerated motion in a straight line.

$$v = u + at \tag{2}$$

The area under the $v-t$ graph is equal to the total displacement s distance d moved. Recall, that we defined average velocity denoted $\bar{v}$ as follows

$$\bar{v} = \frac{1}{2}(u+v) \tag{3}$$

Next, we obtain the total displacement s in terms of average velocity $\bar{v}$ and time t as follows:

$$s = \bar{v}t \tag{4}$$

Substituting for the average velocity $\bar{v}$, we obtain

$$s = \frac{1}{2}\left(v+u\right)t \tag{5}$$

This is the third equation of motion used to describe uniformly accelerated motion in a straight line.

$$s = \frac{vt+ut}{2}$$

Substituting for v from the first equation of motion, we obtain:

$$s = \frac{(u+at)t+ut}{2}$$

$$s = \frac{ut + at^2 + ut}{2}$$

$$s = \frac{2ut + at^2}{2}$$

$$s = ut + \frac{1}{2}at^2 \tag{6}$$

This is the fourth equation of motion used to describe uniformly accelerated motion in a straight line.

We can re-write equation (1) as:

$$t = \frac{v - u}{a}$$

and substitute it into equation (6), we get.

$$s = u(\frac{v - u}{a}) + \frac{1}{2}a(\frac{v - u}{a})^2$$

This gives us

$$s = \frac{uv - u^2}{a} + \frac{1}{2}a\frac{(v - u)^2}{a^2}$$

$$s = \frac{uv - u^2}{a} + \frac{(v - u)(v - u)}{2a}$$

$$s = \frac{uv - u^2}{a} + \frac{(v^2 - uv - uv + u^2)}{2a}$$

$$s = \frac{2uv - 2u^2 + v^2 - 2uv + u^2}{2a}$$

$$s = \frac{v^2 - u^2}{2a}$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as \tag{7}$$

This is the fifth equation of motion used to describe uniform linearly accelerated motion in a straight line.

4 Uniform Accelerated Motion Due To Gravity

Uniform accelerated motion due to gravity is divided into upward motion from the surface of the Earth and downward motion towards the surface of the Earth. When an object is launched or thrown upwards, its velocity reduces at a steady 9.8 m/s every second until it reaches its maximum height, where its velocity is 0 m/s for a very short time. Every body thrown vertically upwards therefore accelerates at -9.8/m s^2.

On the other hand, if an object is dropped from a height, it accelerates towards the surface of the Earth at a steady 9.8 m/s every second. At the time, of being droped the object has an initial velocity of 0 m/s and once released will increase its velocity at a rate of 9.8/m s every second. This means that that the body will accelerate at 9.8 m/s^2.

In both cases (i.e, upward and downward motion), the five equations of motion for uniformly accelerated motion still apply because the acceleration is constant i.e. 9.8 m/s^2. Since our acceleration $a = g = 9.8\text{m/s}^2$, our equations of motion become:

$$g = \frac{v-u}{t} \tag{8}$$

$$v = u + gt \tag{9}$$

$$s = \frac{1}{2}(u+v)t \tag{10}$$

$$s = ut + \frac{1}{2}gt^2 \tag{11}$$

$$v^2 = u^2 + 2gs \tag{12}$$

During the upward motion of a body, the acceleration due to gravity has a value of $g = -9.8$ m/s^2. On the other hand, the acceleration due of gravity has a value of $g = 9.8$ m/s^2 in the downward motion of a body.

Example 3

A bullet from a rifle shot upwards has a muzzle velocity of 550 m/s. Calculate:

(a) the time it takes the bullet to reach its maximum height.

(b) the maximum height reached by the bullet.

(c) the total time of flight of the bullet

(d) the velocity with which the bullet strikes the ground.

Solutions

(a) The bullet exits the rifle with a muzzle velocity (i.e. initial velocity) $u =$ 550 m /s. When the bullet reaches its maximum height, its final velocity $v = 0$ m/s. Since the bullet is undergoing upward motion, acceleration due to gravity g becomes -9.8 m/s^2.

We can obtain, the time of flight t from the equation $v = u + gt$.

$$v = u + gt$$

$$0 \text{ m/s} = 550 \text{ m/s} + (-9.8 \text{ m/s}^2)t_{\text{rise}}$$

Dropping off units, we obtain

$$0 = 550 + (-9.8)t_{\text{rise}}$$

$$0 = 550 - 9.8t_{\text{rise}}$$

$$9.8t_{\text{rise}} = 550$$

Dividing both sides by 9.8, we get

$$t_{\text{rise}} = \frac{550}{9.8}$$

$$t_{\text{rise}} = 56.1 \text{ s}$$

(b) the maximum height reached by the bullet, s_{max}

Now that we know the rise time of the bullet, $t_{\text{rise}} = 56.1$ s and the initial velocity, $u = 550$ m/s, we can use the equation of motion

$$s = ut + \frac{1}{2}gt^2$$

to get the maximum height denoted s_{max}

$$s_{\text{max}} = ut + \frac{1}{2}gt^2$$

$$s_{\text{max}} = (550 \text{ m/s})(56.1 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(56.1 \text{ s})^2$$

Dropping off units, we get

$$s_{\text{max}} = (550)(56.1) - \frac{1}{2}(9.8)(56.1)^2$$

$$s_{\text{max}} \approx 15\ 434 \text{ m} \approx 15.434 \text{ km}$$

The bullet rises to a maximun height of $s_{\text{max}} \approx 15.434$ km

(c) the total time of flight of the bullet.

At maximum height, the bullet has an initial velocity $u = 0$ m/s. As the bullet returns to Earth it gathers velocity as its accelerates at $g = 9.8 \text{ m/s}^2$. The bullet on its downward motion covers a distance of $s = 15\ 434$ m. Therefore, we can work out the time of fall on its downward journey using

$$s = ut + \frac{1}{2}gt^2$$

. Substituting the values, we have

$$s_{\text{max}} = ut + \frac{1}{2}gt^2$$

$$15\ 434 \text{ m} = (0 \text{ m/s})t_{\text{fall}} + \frac{1}{2}(9.8 \text{ m/s}^2)t_{\text{fall}}^2$$

Dropping off units, we get

$$15\ 434 = (0)t_{\text{fall}} + \frac{1}{2}(9.8)t_{\text{fall}}^2$$

$$15\ 434 = 0 + 0.5(9.8)t_{\text{fall}}^2$$

$$15\ 434 = (4.9)t_{\text{fall}}^2$$

$$4.9t_{\text{fall}}^2 = 15\ 434$$

Dividing both sides by 4.9, we get

$$t_{\text{fall}}^2 = \frac{15\ 434}{4.9} = 3\ 149.8$$

Getting the square root on both sides, we obtain

$$t_{\text{fall}} = \sqrt{3\ 149.8}$$

$$t_{\text{fall}} = 56.1 \text{ s}$$

the fall time on the downward motion is again $t_{\text{fall}} = 56.1$ s.

Therefore, the total time of flight for both the upward and downward motion of the bullet is

$$t_{\text{flight}} = t_{\text{rise}} + t_{\text{fall}}$$

$$t_{\text{f}} = 56.1 \text{ s} + 56.1 \text{ s}$$

$$t_{\text{f}} = 112.2 \text{ s}$$

(d) the velocity with which the bullet strikes the ground.

Now that we know the fall time of the bullet on its downward motion, $t_{\text{fall}} = 56.1$ s, and its initial velocity at maximum height $u = 0$ m/s, we can calculate the final velocity with which the bullet strikes the ground, v.

$$v = u + gt$$
$$v = 0 \text{ m/s} + 9.8 \text{ m/s}^2 \times 56.1 \text{ s}$$

Dropping off units, we get

$$v = 0 + 9.8(56.1)$$

$$v = 549.78$$

$$v \sim 550 \text{ m/s}$$

The bullet strikes the ground with a velocity of 550 m/s. This is the same as the muzzle velocity with which the bullet is fired upwards from the gun.