A PRIMER TO UNIVERSITY PHYSICS

(WITH EASY TO UNDERSTAND SOLVED EXAMPLES)

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Introduction

General objectives:

After working through the problems in each Chapter the student should be able to:

- understand the concepts,
- work out strategies of solving problems,
- approach challenging problems with confidence, and
- appreciate the application of physics in everyday situations.

Preface

The basis of solving any physics problem lies in the understanding of the basic concepts of physical processes involved in the problem. It should be noted that even the complicated problems in physics have the very simple concepts intertwined in them, however, the understanding of how these concepts are intertwined and how they affect the physical processes of the problem must be fully understood clearly before obtaining a reasonable solution to the problem. At advanced level, the number of constraints (i.e. factors that affect physical processes) are founded in the lower level physics. In such cases several approximations have and assumptions have to be made. One may not need to obtain actual numerical values as in lower level physics but the solution obtained may show a trend or prove theory etc.

The aim of this book is to show the reader how to tackle most first year undergraduate physics problems by grounding the reader in the basic concepts in each chapter. For example in mechanics and static electricity, the reader must fully understand the concept of vectors and how in particular forces affect physical processes in any dynamic system of course we deal with simple mechanical dynamic and static systems. The implications of Sir Isaac Newton's three laws to physical processes must be understood and not only their definitions which might appear like mere statements or postulations. Thus, it becomes imperative that simple constraints that affect static and dynamic systems be well understood.

In the Chapter on thermodynamics also, the physical processes which take place in a system must be understood before one can attempt to solve problems. It is necessary that the reader has an understanding of the first and second laws of thermodynamics, otherwise solutions in this field can appear to be difficult when in actual fact this need not be the case.

The physical processes in thermodynamics and heat normally follow certain specific patterns-these patterns must be remembered when solving problems, for example, when a block of ice is heated, it must melt first before changing into water at 0 °C. There is no other way that this process can take place, therefore, the reader must always understand that this process takes only one path. The book does not provide solutions to all problems that the student will encounter but gives a clear on some of the most common pitfalls that usually arise in first year physics courses encountered.

It is our belief that the explanations, examples and problems given in this book will make the learning process and solving of problems in physics to first year undergraduates a manageable task.

Chapter 1

1.0 Introduction to Vectors

1.1 Vectors

In our daily activities we encounter and use vector quantities unknowingly. For example when we give directions to someone, we normally refer to compass directions and distances involved. In such a situation we might use one or more vectors to give directions to a specific location. Generally our description in this case will involve use of some coordinate system and we will also refer to some reference points such as buildings or landmarks.

In physics we normally use some kind of coordinate system to locate objects in the x-y plane. If we refer to the intersection of the x and y coordinates (origin) as a point of reference, then we can draw a straight line path from it that leads to the point of interest. The straight line then becomes a vector quantity as it has a certain magnitude as well a definite direction. In figure 1, the object is located at 4 units long in the x direction and 3 units long in the y direction. The magnitude of the line segment then will be 5 units long from the origin. The pair of units in the x and y direction can be represented by (4, 3).

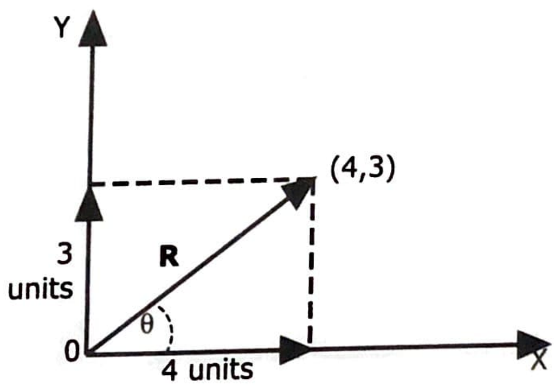


Figure 1 Coordinates

A vector quantity has both magnitute and direction hence our arrowed line segment in figure 1 is a vector as it has a magnitude and a direction with respect to the postive x-axis. In the case above, we could specify the angle between the positive x-axis and the arrowed line segment to give the direction of this vector. The coordinate system of specifying an object in the x-y plane is referred to as the Cartesian coordinate system and this can be extended by adding a z-axis perpendicular to the x-y plane.

Another way of specifying the position vector **R** is to give its magnitude and the direction by angle θ shown in figure 1, thus instead of (x,y), we can use (R,θ). It can be shown that:

$x = R\cos\theta$ and $y = R\sin\theta$

This is the Polar coordinate system of representation of vecrors. Hence we can easily see that R and θ can be expressed in terms of x and y if we consider the right angle triangle formed between **R** and the x-axis then:

$$R^2 = x^2 + y^2 \text{ and } \tan\theta = \frac{R_y}{R_x}$$

Note that the direction of a vector can be found from the tangent function.

Any quantity that has only magnitude and no direction attached to it is referred to as scalar quantity.

Vectors may also be referenced to compass directions, with the north-south axis being equivalent to the y-axis and the west-east axis equivalent to the x-axis. For example if we are told that a plane flew for 300 km north-east, then the displacement vector representing this movement can be drawn to scale in the direction 45° north of east which is the same as 45° with respect to the positive x-axis. This is illustrated in figure 2.

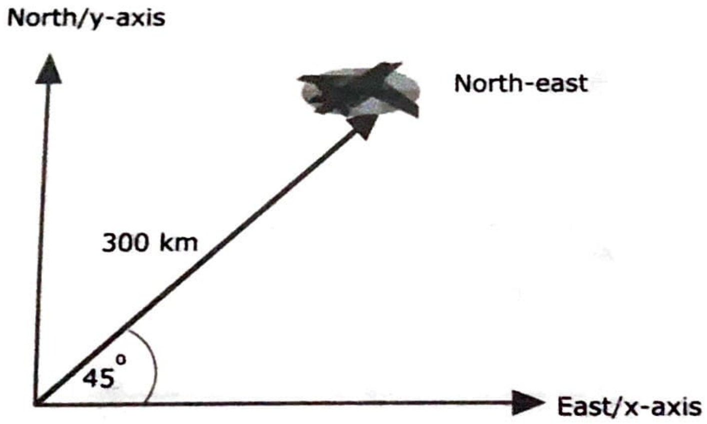


Figure 2 Position referencing using compass directions

1.2 Vector Addition

The addition of scalar quantities is straight forward, for example if one has 10 oranges and then 5 more are given to that person, the total number of oranges the person has is 15. Vector addition is not as simple as we have to take directions into consideration. This being the case various methods of addition of vectors have been devised which simplify the task.

Graphical addition of vectors is the simplest form especially if the vectors to be added lie in a straight line. If we want to add vector **A** lying along the x-axis to another vector **B** also lying along the x-axis then we simply take vector **B** and join its tail to the arrow head of vector **A**. Figure 2 shows this type of addition.

Figure 3 Graphical addition of vectors

Let vector **A** be 4 m long and vector **B**2 m long, the sum of vectors**A** + **B** = **R**, and the resultant vector **R** is 6 m along the positive x-axis. However, if vector **A**and **B** are not lying along the same line or parallel to each other, then we use the following method:

i. Move the second vector **B** parallel to it-self and shift it until its tail is at the head of vector **A**. In its new position vector **B** must maintain its length and direction;

ii. Then join the tail of vector **A** to the head of vector **B** in its new position. The resulting vector is the resultant **R**. See figure 3.

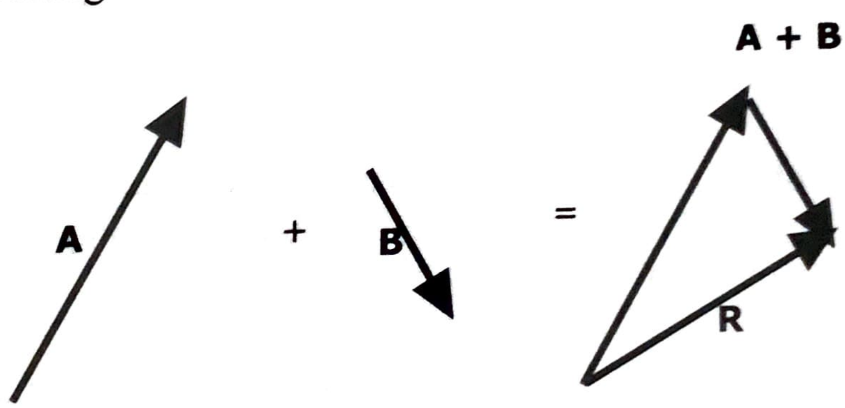


Figure 4 Graphical addition of vectors

The graphical method of adding vectors is cumbersome because one needs graph paper, a ruler and a protractor to carry out the measurements. In vector addition note that **A** + **B** = **B** + **A**.

1.3 Rectangular Components of a Vector

Any vector can be resolved into its x and y components. It is very clear from the Polar representation of vectors that the components are and as can be seen from figure 4.

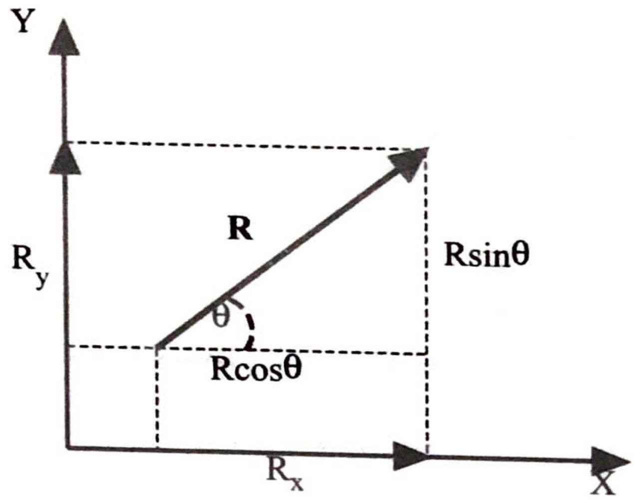


Figure 5 Vector in x-y plane

This being the case it follows that vector addition can then be performed easily by summing up the components of the vectors involved to get the components of the resultant vector.

Example 1.1

Add two velocity vectors, vector **A** is 4 ms^{-1} along the x-axis and vector **B** is 3 ms^{-1} at an angle of 60° with respect to the positive x-axis. Find the magnitude and the direction of the resultant vector.

Solution

To find the resultant of these velocity vectors we resolve the vectors into rectangular components on a table and then sum up x and y components in the respective columns to get components of the resultant vector.

Vector	x-components (ms^{-1})	y-components (ms^{-1})
A	4 cos 0° = 4.00	4 sin 0° = 0.00
B	3 cos 60° = 1.50	3 sin 60° = 2.60
R	R_x = 5.50	R_y = 2.60

Therefore, the magnitude of vector **R** is: $R = \sqrt{R_x^2 + R_y^2}$. At this stage we can now substitute the numerical values:

$$R = \sqrt{5.50^2 + 2.60^2} = 6.08 \text{ ms}^{-1}$$

The general direction of vector **R** can found by sketching the components of the vector before using the tangent function. See the illustration in figure

6. In sketching, draw the components proportionate to the magnitudes. Start with R_x and at the arrow head draw R_y then join the tail of R_x to the arrow head of R_y.

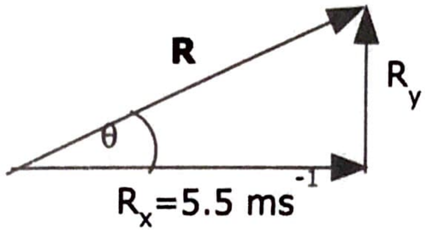


Figure 6 Components of a vector

As can be seen from the resultant is in the first quadrant where both components have positive values, hence, we can now proceed to get the actual direction with respect to the positive x-axis:

$tan\theta = \frac{R_y}{R_x}$, remembering to use the absolute values of these components.

tan = 2.60/5.50 = 0.4727, therefore, θ = 25.30° with respect to the positive x-axis.

The rectangular component method of addition of vectors is valid for any number of vectors and the order in which the vectors are added does not matter, for example:

$$\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{B} + \mathbf{A} + \mathbf{C} = \mathbf{A} + \mathbf{C} + \mathbf{B} = \mathbf{C} + \mathbf{A} + \mathbf{B} = \mathbf{C} + \mathbf{B} + \mathbf{A}$$

Example 1.2

Three displacement vectors **A** = 3 m, 20°above the x-axis, **B**= 10 m, 100° with respect to the positive y-axis and **C**= 5m 220° with respect to the positive x-axis lie in the same plane. Find the resultant displacement vector.

Solution

At first we have to draw a sketch of the three vectors on the x-y plane as in figure 7.

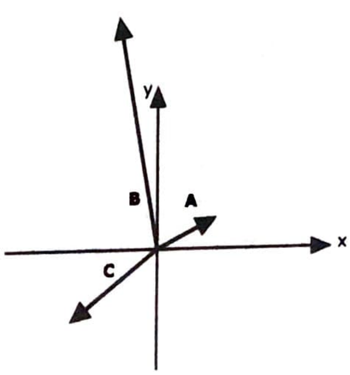


Figure 7

The next step is to tabulate the components.

Vector	x components (m)	Y components (m)
A	3 cos 20° = 2.81	3 sin 20°= 1.03
B	10 cos 100° = -1.74	10 sin 100° = 9.85
C	5 cos 220°= -3.83	5 sin 220°= -3.21
R	-2.76	7.67

Sketching components:

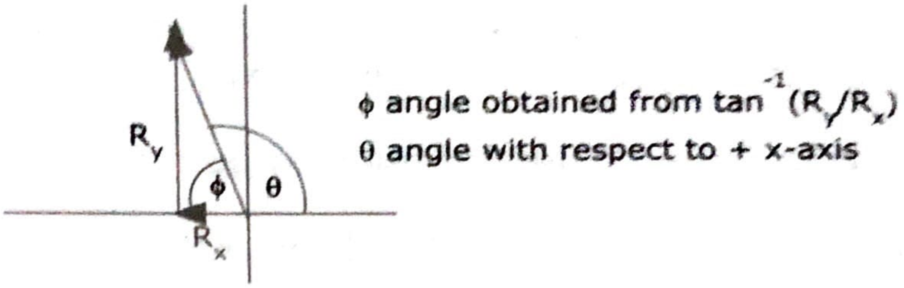


Figure 8

The sketch in figure 8 shows that the resultant is in the second quadrant and therefore, the angle calculated from the $\tan^{-1}(R_y/R_x)$ is ϕ. To get the angle θ with respect to the positive x-axis subtract ϕ from 180° i.e. $\theta = 180 - \phi$. We can now calculate angle ϕ from the components of the resultant.

$$\phi = \tan^{-1}(R_y/R_x) = \tan^{-1}(7.67 \div 2.76) = \tan^{-1} 2.779 = 70.2°$$

$$\therefore \theta = 180° - 70.2° = 109.8° \text{ wrt } + \text{x-axis}$$

Magnitude of the resultant: $R = \sqrt{2.76^2 + 7.67^2} = 8.15$ m.

Example 1.3

Two force vectors are acting at the same point in the x-y plane as shown in figure 9. Find the magnitude of the resultant.

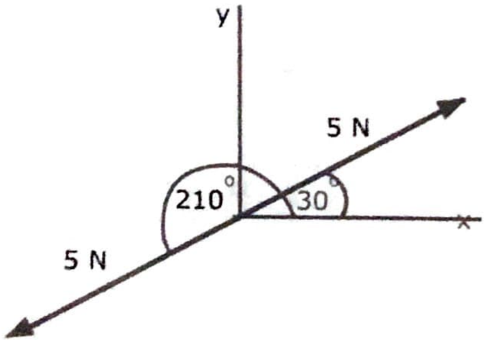


Figure 9

Solution

Using the component method:

Vector	x components (N)	y components (N)
1	5cos 30° = 4.33	5sin 30° = 2.50
2	5cos 210° = -4.33	5 sin 210° = -2.50
R	0	0

Here we see that the sum of components is equal to zero and hence the magnitude of the resultant vector is zero. This is a case where the two force vectors cancel out each other.

1.4 Vector Subtraction

The subtraction of vectors can be done easily by using the following simple rule: to subtract vector **B** from vector **A**, simply reverse the direction of vector **B** and then add it to **A**.

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

In reality if one is using the component method, then in the table of components the signs on the components of vector are reversed i.e. if we have $-\mathbf{B}_x$, then it will be changed to $+\mathbf{B}_x$ and in a similar manner reversal will be done on the component $\mathbf{B}_y$ or vice versa, that is if we have $+\mathbf{B}_x$ it will be changed to $-\mathbf{B}_x$ and $+\mathbf{B}_y$ to $-\mathbf{B}_y$. Vector subtraction is illustrated in figure 10.

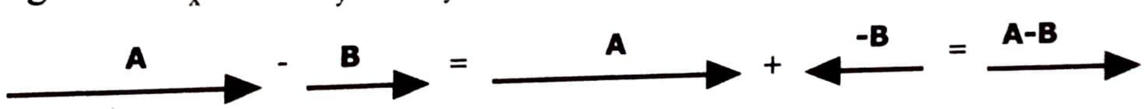


Figure 10 Graphical subtraction of vectors

Example 1.4

Two vectors **A** and **B** are shown on the drawing. Find the resultant of **A - B** and its direction.

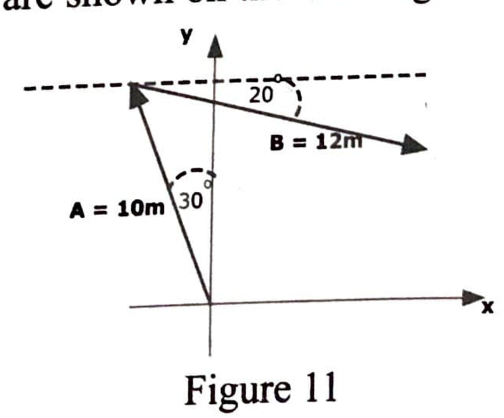


Figure 11

Solution

Since **A** - **B** = **A** + (-**B**), we will tabulate the components of vectors A and B, however we will reverse the signs on the components of vector **B** and then add the columns to get the resultant components see table below.

Vector	x-components (m)	y-components (m)
A	10 cos 120° = -5	10 sin 120° = 8.66
B	-(12 cos 340°) = -11.28	-(12 sin 340°) = +4.10
R	-16.28	12.76

Magnitude of resultant R = $\sqrt{-16.28^2 + 12.76^2}$ = 20.68 m and the direction is:

$\phi = \tan^{-1}(R_y/R_x) = \tan^{-1}(12.76/16.28) = \tan^{-1}(0.7838) = 38.1°$

$\therefore\ \theta = 180° - 38.1 = 141.9°$ wrt positive x-axis.

Chapter 2

2.0 Motion in One Dimension

2.1 Uniformly Accelerated Motion

An object is said to be in motion if its position changes in relation to its surroundings. The motion can be described by looking at how fast it is going, whether the speed is increasing or decreasing and the rate at which the speed increases or decreases. To simplify the equations describing the motion we will consider linear motion (straight line motion). The description of motion does involve properties such displacement, velocity, and acceleration, in terms of dimensions of time and length.

In the International System of Units popularly referred to as the S.I. system (Systeme Internationale), the unit of length is the meter (m) and time is in second (s).

2.2 Displacement and Distance

A change in position of an object without specifying the direction is defined only by the distance covered; it could be a straight line path or a meandering path. The distances covered are a scalar quantities see figure 1, where paths ACDEB, AFGHB are distances of unequal magnitudes but start and end at the same points.

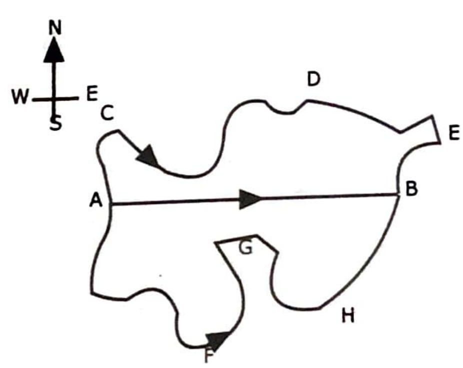


Figure 1 Displacement and distance

However, the distance **AB** eastwards is called a displacement because it has both magnitude and direction. **BA** westwards would be a displacement with the same magnitude as **AB** but opposite in direction.

$$AB = -BA.$$

Note that a displacement is the shortest distance between two points in a specified direction.

2.3 Speed and Velocity

If a person moves from point A to point B along path ACDEB in figure 10, in a given time interval then, his average speed can be defined as the actual distance travelled divided by the time taken.

$$\bar{v} = \frac{x}{t} \qquad 2.1$$

Equation 2.1 does not tell us anything about what happened during the journey. The cyclist could have stopped or could have slowed down or speeded up or slowed down. The quantity obtained from this equation is an average speed. For example if the cyclist took 2 hours and the distance of path ACDEB was 25 kilometers, then the average speed would be:

$$\bar{v} = \frac{25}{2} = 12.5\,\text{Km/h}$$

Speed or average speed is a scalar quantity as it has only magnitude.

Suppose the cyclist took the straight line path AB due east which is only 10 km and he took half an hour. Then his rate of change of distance would be called a velocity as the quantity calculated will have a direction.

$$v = \frac{\vec{s}}{t} = \frac{10}{0.5} = 20\,\text{Km/h, east}$$

Thus velocity can be defined as the displacement vector **AB** divided by time or change of displacement per unit time, it has both magnitude and direction. The S.I. unit of velocity and speed is metre per second (m/s). Other units for velocity and speed popularly used are mm/s, cm/s, and miles per hour (mph).

An object covering equal displacements in equal time intervals, no matter how short these are, is said to be moving with uniform velocity. Such objects can only move in a straight line, if the path is circular the direction will continuously be changing and so it cannot have uniform velocity although its speed maybe constant.

Example 2.1

A car travelling over a curved path covered a distance of 120 km in 3 hours. At the end of the journey it was found to be 60 km west of the starting point and the journey would have taken half an hour using the direct path.

a) What was its average speed?

b) What was its average velocity?

Solution

a) Average speed = distance travelled ÷ time taken

$$\bar{v} = \frac{120}{3} = 40 \text{ Km/h}$$

b) Average velocity = displacement ÷ time taken

$$\bar{v} = \frac{60}{0.5} = 120 \text{ Km/h}$$

Note the differences in the magnitudes of the average speed and average velocity.

2.4 Acceleration

Acceleration is the rate at which the speed or the velocity changes. If an object is gaining speed or velocity with time then it is said to be accelerating and its value is positive, whereas when the object is slowing down then its value is negative. Negative acceleration is generally referred to as deceleration.

Acceleration is said to be uniform if the velocity of an object changes by equal amounts in equal time intervals. Let the velocity of an object increase steadily from u to v in a time interval of t, then from the definition of acceleration, it can be said that the acceleration a, of the object is:

a = change in velocity ÷ time taken = (final velocity – initial velocity) ÷ time taken or

$$a = \frac{v-u}{t} \qquad 2.2$$

Equation 2.2 can be re-written in terms of final velocity $v$as:

$$v = u + at \qquad 2.3$$

The term at in equation 2.3 is a change in velocity as this term is added to the initial velocity of the object.

Example 2.2

A car starts from rest and accelerates uniformly to a velocity of 4 ms^{-1} in 8 s as it travels east. Find its acceleration.

Solution

Starting from rest means that: $u = 0$, and final velocity is: $v = 4$ ms^{-1} at $t = 8$ s

$$\therefore \quad a = \frac{v-u}{t} = \frac{4-0}{8} = 0.5 \text{ ms}^{-2}$$

Since the velocity is increasing steadily, the average velocity is the mean of the initial and final velocities.

Average velocity, $$\bar{v} = \frac{u+v}{2} \qquad 2.4$$

Example 2.3

Find the distance travelled by the car in Example 2.2.

Solution

To get the distance travelled we use equation (2.4) since we know u, v and t to find the average velocity and then use equation (2.1) to get the distance travelled.

$$\therefore \; \bar{v} = \frac{v-u}{2} = \frac{4-0}{2} = 2 \text{ ms}^{-2} \text{ and hence } x = \bar{v}t = 2+8 = 16 \text{ m}$$

If s is the displacement of the object covered in a time interval t, then we can calculate the average velocity of the object by dividing the displacement s by the time taken.

$$\bar{v} = \frac{s}{t} = \frac{u+v}{2} \qquad 2.5$$

Example 2.4

A car is travelling at 5 ms^{-1}is brought to rest in a time interval of 8 s and a distance of 20m. Calculate its average velocity.

Solution

$v = 0$ ms^{-1} and Δ..ms^{-1}, $\therefore \; \bar{v} = \frac{u+v}{2} = \frac{5+0}{2} = 2.5$ ms^{-1} or $\bar{v} = \frac{20m}{8s} = 2.5$ ms^{-1}.

Note that in both cases we get the same answer.

Multiplying the right-side terms of equation 2.5 by t on both sides:

$$s = \tfrac{1}{2}(u+v)t \qquad 2.6$$

Substituting for v in equation (2.6) with the right hand-side term from equation (2.3) yields:

$$s = \tfrac{1}{2}\left(u + u + v\right)t = \tfrac{1}{2}ut + at^2 \quad \text{or}$$

$$s = ut + \tfrac{1}{2}at^2 \qquad 2.7$$

Equation (2.7) is a derived one that can be used to calculate displacement if the initial velocity, the time of motion and acceleration are known, this equation supplements equation (2.1) from which one can calculate displacement if the average velocity and time are known.

Example 2.5

A car moving with a velocity of 15 ms^{-1} accelerates uniformly at 1.2 ms^{-2} for period of 30 s. Find the distance it will travel in this time.

Solution

Known quantities: $u = 15\ ms^{-1}$, $a = 1.2\ ms^{-2}$ and $t = 30$ s

We can use equation (2.7) to get the distance travelled:

$$s = ut + \tfrac{1}{2}at^2 = (15)(30) + \tfrac{1}{2}(1.2)\left(30^2\right) = 990 \text{ m.}$$

Re-arranging equation (2.2) in terms of time and substituting for t in equation (2.7), gives another useful equation:

$$t = \frac{v - u}{a}$$

$$s = u\left(\frac{v - u}{a}\right)t + \tfrac{1}{2}a\left(\frac{v - u}{a}\right)^2$$

Expanding the right hand-side term

$$s = \frac{uv}{a} - \frac{u^2}{a} + \frac{v^2}{2a} - \frac{uv}{a} + \frac{v^2}{2a} = \frac{v^2 - u^2}{a}$$

The result $s = \dfrac{v^2 - u^2}{a}$ is re-written as:

$$v^2 = u^2 + 2as \qquad 2.8$$

Equation (2.8) is another derived equation from the basic equations of uniform linear motion.

Example 2.6

A plane lands on a runway with a velocity of 200 km/h and decelerates to 30 km/h at the rate of 10 ms^{-2}. What distance does it cover on the runway?

Solution

First convert the velocities from km/h to ms^{-1}:

$$200 \text{ km/h} = \frac{200km/h \times 1000m}{3600s} = 55.56 \text{ ms}^{-1}$$

$$30 \text{ km/h} = 8.33 \text{ ms}^{-1}$$

Known quantities: $u = 55.56$ ms^{-1}, $v = 8.33$ ms^{-1} and $a = -10$ ms^{-2}

We can use equation (2.8) in which the only unknown is distance: $v^2 = u^2 + 2as$ from which we get $s = \frac{v^2 - u^2}{2a} = \frac{8.33^2 - 55.56^2}{2\times(-10)} = 150.9$ m

2.5 Summary of equations of motion in a straight line

i) $s = \bar{v}t$

ii) $\bar{v} = \frac{u+v}{2}$

iii) $v = u + at$

iv) $v^2 = u^2 + 2as$

v) $s = ut + \frac{1}{2}at^2$

2.6 Graphical Analysis of Motion

The motions of objects are sometimes best understood by making graphical representations. Plots made are displacement versus time, velocity versus time and acceleration versus time. Such plots help in visualizing the motion and analysis of the motion of the object can be analyzed from them.

2.7 Displacement-time Graph

A displacement-time graph is normally plotted with the displacement along the y axis and time along the x axis. The type of line or curve obtained can give us information about what is happening to the velocity with time, we can tell whether the object's velocity is increasing or decreasing, its direction and velocity. A straight line sloping upwards to the right indicates that the object is moving equal distances for equal time intervals (constant velocity) and moving in the positive direction as in figure 11.

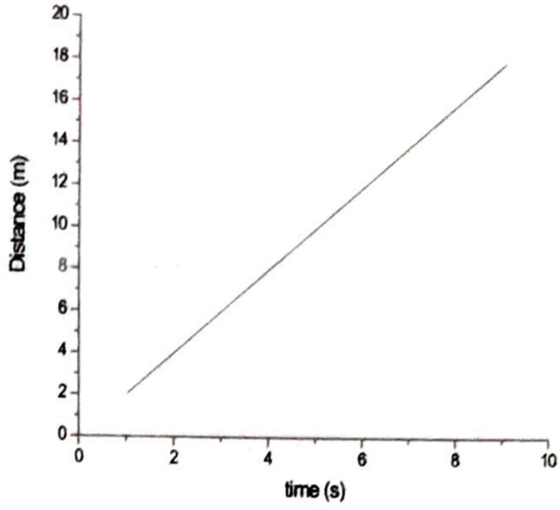


Figure 2 Displacement-time graph

The slope of a distance-time graph gives the average speed of the object for which the plot has been made. For example we can find the slope of the line in figure 2 by taking a distance interval and dividing it by its corresponding time interval. The result we get for the slope is the speed of the object. If we take the interval 2 m to 6 m then its corresponding time interval is 1 s to 3 s, therefore, the slope is:

$$\text{slope} = \frac{6-2}{3-1} = 2\,\text{m/s} = \text{speed of the object.}$$

The plot in figure 2 has a constant slope which implies that the object has a constant speed. If the plot was displacement-time graph then the slope would be the velocity of the object. The magnitude of the slope in this case is positive which means that the object is moving in the positive direction. A negative slope would imply that the object was moving in the opposite direction. In the latter case the line will have a slope from the top left sloping downwards to the right as in figure 3.

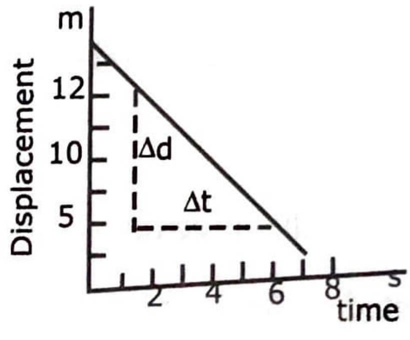


Figure 3 Displacement-time graph

In figure 12 the slope is: slope $= \frac{\Delta d}{\Delta t} = \frac{5-12}{6-1.5} = -1.56$ m/s, thus the velocity in this case is negative and is in the opposite direction to the positive direction. It follows that we can deduce the speed or velocity of an object from a distance-time graph.

In the case of non-constant velocity, the velocity can also be obtained from such graphs even though they involve curves. The velocity is obtained from the slope of a tangent drawn at the point of interest. Such a velocity refers only to that point and hence it is velocity at that instant in the motion of the object and is called instantaneous velocity. Figure 4 shows a tangent drawn at point A and its slope will give the instantaneous velocity at that point.

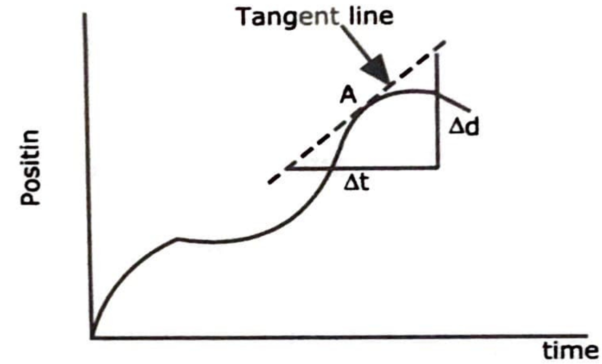


Figure 4: Position-time graph

Example 2.7

A cyclist's journey is plotted in figure 5. Find the direction and velocities represented by regions A, B and C.

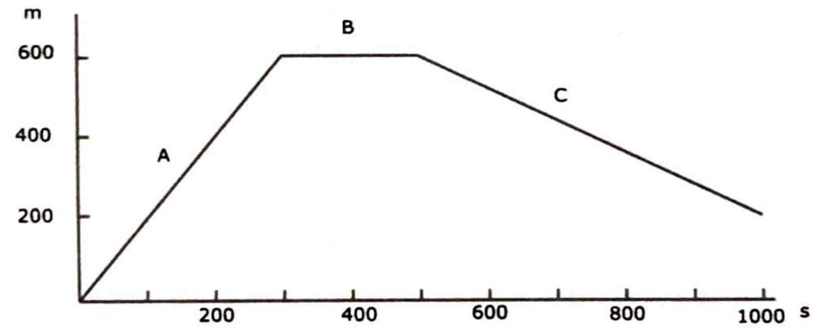


Figure 5: Displacement-Time Graph

Solution

First, we examine the slopes in the three regions of the motion and note that

Region A, the slope is slanted to the right upwards from the origin of the plot implying that the cyclist is moving in the positive direction.

Region B, has a zero slope implying that the cyclist is not moving.

Region C, the slope is from the top slanting downwards to the right implying that the cyclist is moving in the opposite direction to the original motion or negative direction.

Average velocities of the three regions are:

$$\text{Region A, } \frac{\Delta d}{\Delta t} = \frac{600-0}{300-0} = +2\,\text{m/s}$$

$$\text{Region B, } \frac{\Delta d}{\Delta t} = \frac{600-600}{500-300} = \frac{0}{200} = 0\ \text{m/s}$$

$$\text{Region C, } \frac{\Delta d}{\Delta t} = \frac{200-600}{1000-500} = \frac{-400}{500} = 0.8\,\text{m/s}$$

2.8 Free Fall Motion

Free fall motion is a special form of uniformly accelerated motion performed by bodies freely falling under the influence of gravity. It has been observed that objects fall vertically with the same acceleration. If we neglect air resistance, for example, a feather and a stone dropped from the same height in a vacuum will hit the bottom at the same time. This being the case, we can, therefore, use the equations of uniformly accelerated motion.

The acceleration of objects that are freely falling is called acceleration due to gravity. Acceleration due to gravity is always directed downwards towards the centre of the Earth, and has been assigned the symbol g, with its average value near the surface of the Earth being approximately 9.80 ms^{-1}. Note that the value of g decreases with increasing altitude and it also varies slightly with latitude as the Earth in not a perfect sphere.

Since free fall motion is in one direction only, we replace the variable s in the equations of uniformly accelerated motion with the variable y. Thus we have the following equations for this motion which involve the variables g, and y:

i) $v = u + gt$

ii) $v^2 = u^2 + 2gy$

iii) $y = ut + \frac{1}{2}gt^2$

iv) $y = \frac{1}{2}gt^2$, special case of a falling object starting from rest.

Acceleration due to gravity is always directed downwards and hence the value of g will have the value of -9.80 m/s. This is consistent with the choice of the upward direction as being positive. However, in certain instances we might choose the downward direction as positive to simplify calculations then, g will assume a positive value.

Example 2.8

A naughty boy drops a cat in well. He hears sound of the cat hitting the water after 2.5 seconds. How deep is the well into which the cat was dropped?

Solution

Firstly, note that the cat is falling freely and therefore, it is performing free fall motion. Hence we can use equation (*iv*).

Choosing the downward direction as positive g assumes a positive value.

$$\text{i.e. } y = \tfrac{1}{2} g t^2 = \frac{1}{2}(9.8)(2.5^2) = 30.6 \text{ m}$$

Example 2.9

A stone is shot vertically upwards with a velocity of 15 m/s. Find:

i) the maximum height reached, and

ii) the time taken to reach the maximum height.

Solution

i) Choosing the upward direction as positive, then the acceleration is $-g$ and the final velocity v at maximum height is zero.

$$v^2 = u^2 - 2gy$$

$$0 = 15^2 - 2(9.8)\,y$$

$$y = \frac{15^2}{2(9.8)} = 11.48 \text{ m}$$

ii) At maximum height the final velocity $v = 0$.

$$v = u - gt \text{ setting } v = 0$$

$$0 = 15 - 9.8t$$

$$t = \frac{15}{9.8} = 1.53 \text{ s}$$

Chapter 3

3.0 Motion in Two Dimensions

3.1 Projectile Motion

In the preceding Chapters we discussed motion of objects moving in a straight line (one dimension motion). However, we find that in everyday occurrences there are several instances where an object may have motion in two dimensions i.e. having components both in the x-axis and y-axis. Examples of such motion are plenty; objects such as balls, canon shots, spears etc. rarely follow straight lines. This type of motion which has aspects of motion in two dimensions is called projectile motion and the objects that perform it are called projectiles.

In projectile motion if we neglect the effect of air resistance, then a projectile moves with constant speed in the horizontal direction. In the vertical direction there is the effect of gravity, thus as the projectile rises or falls as it is under the influence of the acceleration due to gravity. These two motions are independent of each other except that they are linked by time. In time t, a projectile will move a certain distance in the horizontal direction as well as a certain distance in the vertical direction. The motion in the horizontal direction has zero acceleration whereas the vertical motion has a constant acceleration of 9.8 ms^{-2}. The motion follows a curved path called a trajectory.

3.2 Equations of Projectile Motion

Since the horizontal and vertical motions are independent of each other, we resolve the velocity into its horizontal and vertical components and treat them separately. In figure 1 the projectile is launched at an angle θ, the velocity of launch v_i can be resolved into components along the x-axis and y-axis.

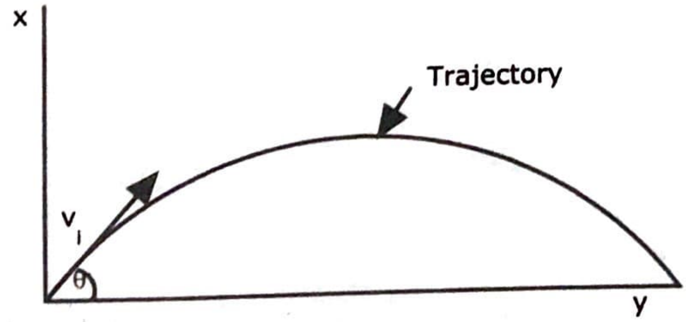


Figure 1 Trajectory of a projectile

3.3 Horizontal Component

It is linear motion with constant velocity and is described by the following equations:

$$v_{ix} = v_i \cos\theta = \bar{v} = v_{if}$$

The initial component of the velocity is equal to average horizontal velocity and also the final velocity. This latter fact is consistent with the assumption that the horizontal component of the velocity stays constant throughout the motion. This being the case, it becomes easy for us to calculate the horizontal distance travelled by the projectile using the equation:

$$x = v_{ix}.t = v_i \cos\theta.t$$

The horizontal distance x travelled by a projectile is called the range. The equation above is also valid for projection of projectiles from various heights. For example, the case in which the projectile is shot horizontally from the top of a building. The distance from the foot of the building to where the stone lands on level ground can be calculated using the same equation. This distance is called the range.

3.4 Vertical Component

In the vertical direction a projectile performs free fall motion and it follows that all the equations of uniformly accelerated motion apply. However, there is need to choose a reference point in assigning which direction is assigned as the positive direction. Choosing the direction upwards as positive means that the acceleration due gravity g will be assigned a negative value as it is always directed downwards. One may choose the downward direction as positive which will mean that the acceleration due gravity will be assigned a positive value, this usually done to simplify calculations in some instances. The following equations will apply to the first case of g being assigned a negative value:

i) $v_y = u_y - gt$ 3.1

ii) $v_y^2 = u_y^2 - 2gy$ 3.2

iii) $y = u_y t - \frac{1}{2} gt^2$ 3.3

iv) $y = -\frac{1}{2} gt^2$, is a special case where a projectile starts with zero vertical velocity. 3.4

The vertical component of the velocity is:

$v_y = v_{iy}\sin\theta$

Example 3.1

A stone is shot from a catapult horizontally with a velocity of17 ms^{-1} from a height of 1.5 m above the ground, assuming a level ground,

i) what is the horizontal distance it will travel before hitting the ground?

ii) what is its vertical velocity just before it hits the ground?

Solution

The initial velocity in the horizontal direction (x-axis) is: $v_x = 17$ ms^{-1}, and the initial velocity in the vertical direction $u_y = 0$ ms^{-1}. To calculate the horizontal distance travelled we use the equation $x = v_x t$. However, we do not have the time the stone travels, therefore, we calculate this first.

Choosing the point of release of the stone from the catapult as the origin i.e. $x = 0$, $y = 0$. The stone strikes the ground at $x = ?$, $y = -1.5$ m.

Set $y = -1.5$ m in equation (3.3) $y = u_y t - \frac{1}{2}gt^2$.

$$-1.5 = 0 - \left(\tfrac{1}{2}(9.8)t^2\right)$$

$$-1.5 = -4.9t^2$$

$t = \sqrt{\frac{1.5}{4.9}}$ s $= 0.55$ s and hence the horizontal distance $x = (17)(0.55) = 9.35$ m

Suppose we choose down as positive, then the equation of motion becomes,

$$y = u_y t + \tfrac{1}{2}gt^2$$

$$1.5 = 0 + \left(\tfrac{1}{2}(9.8)t^2\right)$$

$$1.5 = 4.9t^2$$

$$t = \sqrt{\frac{1.5}{4.9}}\ \text{s} = 0.55\ \text{s}$$

Note that we obtain the same result as before. In the vertical direction the stone is falling freely, neglecting air resistance:

Using the equation (3.2): $v_y^2 = u_y^2 - 2gy$, $g = -9.8$ ms^{-2}

$$v_y^2 = 0 - (2)(-9.8)(1.5)\,\text{m}^2\text{s}^2 = 29.4\ \text{m}^2\text{s}^2$$

$$= 5.4\ \text{ms}^{-1}$$

Similarly, taking down as positive i.e. $g = +9.8$ ms^{-2}

$$v_y^2 = u_y^2 + 2gy$$

$$v_y^2 = 0 + (2)(9.8)(1.5)\,\text{m}^2\text{s}^2 = 29.4\ \text{m}^2\text{s}^2$$

$$v_y = 5.4\ \text{ms}^{-1}$$

This example illustrates the fact that we can assign either up or down as positive but we have to remember to use the correct sign to assign to the acceleration due to gravity.

Example 3.2

A football player kicks a ball at an angle of 30° with a speed of 15 ms^{-1} on a level playing field. Find:

i) The maximum height the ball reaches.

ii) The time the ball takes to return to its original height.

iii) The horizontal distance travelled.

Solution

To solve this problem we will divide the motion into vertical and horizontal portions.

$v_{ix} = u_x = v\cos\theta = 15\cos 30° = 13.0$ ms^{-1} and $v_{iy} = u_y = v\sin\theta = 15\sin 30° = 7.5$ ms^{-1}

Taking up as positive.

At maximum height the ball has a vertical velocity $v_y = 0$

Using the equation (3.2): $v_y^2 = u_y^2 - 2gy$, we set $v_y^2 = 0$

$$0 = 7.5^2 - (2)(9.8)y$$

$$y = \frac{56.25}{19.6} = 2.87\text{ m}$$, this is the maximum height the ball rises to.

To get the total time of flight, we set the value of $y = 0$ after time t has elapsed which is the time the ball takes to return to the level of launch.

We use the equation (3.3): $y = u_y t - \frac{1}{2} g t^2$

$$0 = 7.5t - (\tfrac{1}{2})(9.8)t^2$$

$$0 = t(7.5\text{-}4.9t)$$

$$t = 0 \text{ or } t = \frac{7.5}{4.9} = 1.53\text{ s}$$

Therefore the horizontal distance travelled is obtained from the equation: $x = u_x t$

$$x = (2.87 \times 1.53)\text{ m} = 4.39\text{ m}$$

Example 3.3

A sloping roof of a building makes an angle of 30^0 with respect to the horizontal and the edge of the roof is 15 m above the ground. A ball rolling from the top of the roof attains a velocity of 6 ms^{-1} as it leaves the edge of the roof. See figure 2 below.

i) How long does it take to hit the ground?

ii) How far does it travel horizontally the bottom of the building to the point where it strikes the ground?

iii) What is its velocity just before it strikes the ground?

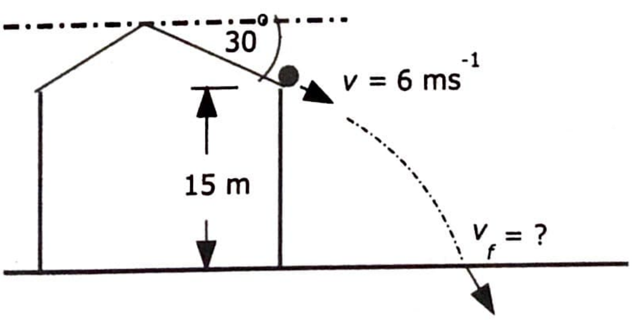


Figure 2

Solution

We place our coordinate axes at roof edge, hence upward is taken as positive and divide the motion into horizontal and vertical portions.

$v_x = v\cos30^\circ = (6)(0.866)\ \text{ms}^{-1} = 5.2\ \text{ms}^{-1}$ and $v_y = v\sin30^\circ = (6)(0.5)\ \text{ms}^{-1} = -3.0\ \text{ms}^{-1}$

Using the equation (3.3): $y = u_y t - \frac{1}{2}gt^2$, we set $y = -15$ m

$$-15 = -3t - 4.9t^2 \text{ or } 4.9t^2 + 3t - 15 = 0$$

Solving the resulting quadratic equation:

$t = \dfrac{-3 \pm \sqrt{9+294}}{9.8}$ from which we get $t = 1.47$s and $t = -2.08$s we discard the negative answer.

Hence the horizontal distance travelled is:

$$x = (v\cos30^\circ)(t) = (5.2)(1.47) = 7.6\ \text{m}$$

To find the final velocity just before the ball hits the ground we take into account that the ball is hitting the ground at an angle and therefore it will have horizontal and vertical components. We know the horizontal component and need to find the vertical component.

To get the vertical component we use the equation (3.1): $v_y = u_y - gt$,

$$v_y = -3.0 - (9.8)(1.47) = -17.41\ \text{ms}^{-1}$$

We can now use the Pythagorean theorem to get the final velocity,

$$v_f = \sqrt{v_x^2 + v_y^2} = \sqrt{5.2^2 + 17.41^2} = 18.17\ \text{ms}^{-1}$$

Note that the v_f is greater than v_i because the ball has been accelerated by gravity in the vertical direction.

Additional Equations

There are some points to note for projectiles fired at angle that start and end their motion at the same level in the horizontal direction:

i) The motion is symmetrical about the mid-point of the flight path.

ii) Gravity decelerates and accelerates a projectile by the same magnitude and hence the time taken to rise to maximum height is the same as the time for the projectile to fall to the same level it was launched from.

iii) The total flight time is twice the time the projectile takes to rise to maximum height.

It follows that, $v_y = u_y - gt = 0$

$$t_{up} = \frac{u_y}{g} = \frac{v\sin\theta}{g} \qquad 3.5$$

Hence the horizontal distance travelled by a projectile becomes,

$$\text{Range} = 2t_{up}v_x = 2v_x\cos\theta t_{up} \qquad 3.6$$

We now substitute the value of t_{up} from above into the range equation, giving us

$$\text{Range} = \frac{2(v\cos\theta)(v\sin\theta)}{g} = \frac{v^2\sin 2\theta}{g} \qquad 3.7$$

N.B. $2\cos\theta\sin\theta = \sin 2\theta$

The resulting equation tell us that the range is independent of the mass of a projectile and that it is maximum when $2\theta = 90°$ because $\sin 90° = 1$. Thus the angle of 45° gives the maximum range for any projectile. Since $\sin 90° = 1$, it follows that the equation for maximum range is,

$$\text{Range}_{max} = \frac{v^2}{g} \qquad 3.8$$

Example 3.4

A ball is kicked at 25 ms^{-1} on flat ground. Find:

i) Its maximum range,

ii) the range when the launch angle is 40°, and

iii) when the angle is 50°.

Solution

The maximum range is when the angle is 45°, hence,

$$R_{max} = \frac{v^2}{g} = \frac{25^2}{9.8} = 63.8\,\text{m}$$

Chapter 4

4.0 Forces and Newton's Laws of Motion

4.1 Force and Mass

In everyday life we associate a force with a pull or a push, for example an oxen applies a pulling force on a plough or a boy kicks a ball by applying a pushing force on the ball. These kinds of forces involve contact between two objects. There are forces that move objects without direct contact, for example a magnet lifts or moves objects without physical contact, such forces act at a distance. A force is a vector quantity since it has both magnitude as well direction and therefore, in diagrams involving forces we indicate the direction of a force by an arrow showing its direction. If drawn to scale, the length may be used to indicate the magnitude of a force vector.

The common forces we encounter on a daily basis are:

1. Tension- involves the pull on objects through cables, springs, bars etc.

2. Compression- these involve interaction of rigid objects which support weight, e.g. floors or may involve fluid pressure e.g. in brake systems or in the collision of objects.

3. Friction/viscosity- these forces cause resistance between sliding or moving objects.

4. Fundamental-these involve attractive or repulsive forces at a distance, i.e. gravity, magnetic and electrical forces.

Sir Isaac Newton in his work explained these interactions in a mathematical way and stated three laws that govern these interactions.

4.2 Inertia and Mass

To understand the term inertia, we first consider what happens to passengers in a moving vehicle when brakes are applied suddenly. What we observe in this situation is that the passengers are thrown forward. This happens because as the vehicle starts slowing down the passenger tend to maintain the same speed the vehicle was moving at. Equally a passenger in a sports car at high speed take-off experiences a push in the back as there is a tendency for the passenger to remain stationary. The tendency for objects to remain in motion or resist being moved from rest is called inertia. In general, we can define inertia as the tendency for objects at rest to remain at rest and those in motion to remain in motion with the original speed. The concept of inertia which is about the ability of objects to resist forces tending to change the state of motion is cardinal in Newton's first law of motion.

In order to quantify inertia we use mass to compare objects. In S.I. units mass is measured in kilograms. Thus an object having the same inertia as the standard one kilogram mass is said have a mass of one kilogram.

4.3 Newton's First Law of Motion

Newton stated that an object in motion continues that motion with constant velocity, unless acted upon by external forces that compel it to change. This law can be re-stated to say an object continues to move with constant velocity if the vector sum of external forces acting on it is zero. The law applies to moving as well as stationary objects.

4.4 Newton's Second Law of Motion

Newton's second law of motion gives us a quantitative definition of force. It states that:

The net force acting on an object equals the product of the mass and the acceleration the object experiences. The direction of the force is the same as that of the acceleration.

In equation form: $\mathbf{F}_{net} = ma$ or Net force = mass x acceleration 4.1

This equation can be extended to 3-dimension space along each coordinate axis,

$$\boldsymbol{F}_{x\text{-net}} = \sum F = ma_x$$

$$\boldsymbol{F}_{y\text{-net}} = \sum F = ma_y$$

$$\boldsymbol{F}_{z\text{-net}} = \sum F = ma_z$$

In S.I. units the unit of force is the Newton given the symbol N. A Newton is defined as that net force which applied to a one kilogram mass gives it an acceleration of 1 ms^{-2}. The Newton is not a basic unit, therefore, in calculations it is replaced by its equivalent in meter, second and kilogram.

$$F = ma$$

$$1 \text{ N} = (\text{kg})(\text{m/s}^2) = 1 \text{ kg.m/s}^2$$

In equation 4.1 it is clear that the mass dictates the acceleration that an object will undergo, especially if we re-write it in terms of acceleration:

$$a = \frac{F_{net}}{m} \qquad 4.2$$

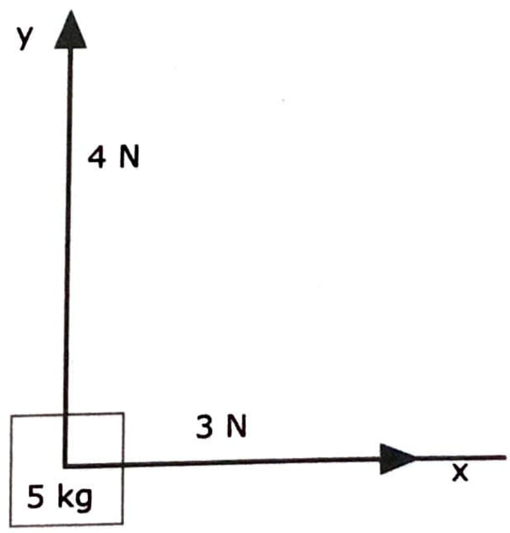
It can be easily shown that since there is an inverse relationship between acceleration a, and mass m, it follows that if the mass is large, the acceleration produced by a given force will be small.

Note that if the sum of forces acting on the object is equal to zero, then the object will not be accelerated. The sum referred to in equation (4.2) is a vector sum of forces acting on the object.

The Newton is an absolute unit of force since it does not refer to the pull of gravity of the Earth or any other planet.

Example 4.1

Two forces are acting on a 5 kg mass as shown in the diagram, 4 N along the x-axis and 3 N along the y-axis. What is the magnitude of the acceleration produced by these forces?



Solution

First we calculate the net force, noting that the forces are at right angles, we can get the resultant force by using the Pythagorean theorem:

$$\sum F = \sqrt{(3N^2) + (4N^2)} = 5 \text{ N}$$

Now we can use equation (4.1) to find the acceleration:

$$\sum F = ma \text{ or } a = \frac{\sum F}{m} = \frac{5}{5} = 1.0 \text{ ms}^{-2}$$

Example 4.2

A 1000 kg car accelerates from rest to 10 ms^{-1} in 7 s along a straight stretch of the road. How large is the force required to achieve this velocity?

From Newton's second law the net force required is: $F_{net} = ma$

If we assume constant acceleration then:

$$a = \frac{v_f - v_i}{t} = \frac{10 - 0}{7} = 1.43 \text{ ms}^{-2}$$

Thus $F_{net} = 1000 \times 1.43 = 1430$ N

Note that there are positive signs on velocity, acceleration and force which implies that all these vectors have the same direction.

4.5 Free Body Diagrams

In working with forces acting on an object, we may have several forces acting at the same time. In applying Newton's second law we need to resolve these forces acting directly on the object into rectangular components before summing them up. This is normally achieved by drawing a diagram which indicates all the forces at play. For example in figure 1, there is a force **F** applied to the wagon at an angle of 40° with respect to the horizontal. This force may be due to an individual pulling the wagon, our primary interest is on the force rather than the individual. The force may be through a rope attached to the wagon.

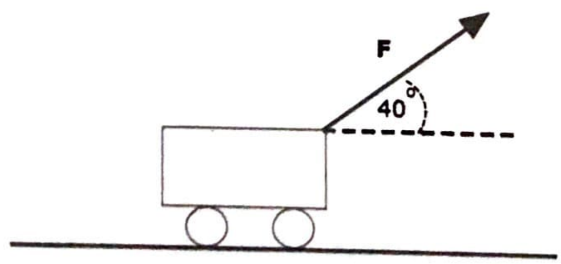


Figure 1 Wagon being pulled by force **F**

Our interest is only on forces acting on the wagon and we proceed to sketch on a free body diagram the forces and resolve them as in figure 2.

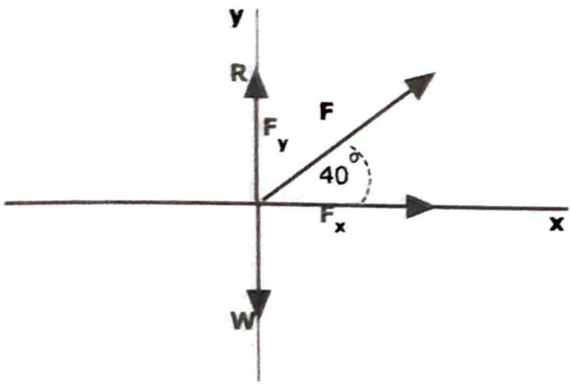


Figure 2 Resolution of forces acting on the wagon

Note that we have two forces the reaction force, **R** and the component of **F**, F_y acting in the upward direction, one downward force the weight, **W** and lastly the component of **F**, F_x acting to the right in the direction of motion. We can altenatively draw the diagram as in figure 3.

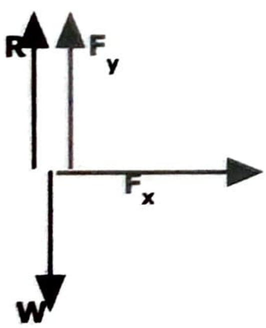


Figure 3Alternate drawing of the resolution of forces

The free body diagrams in figures 2 and 3 simplify the summing of forces acting on the wagon. In addition we can tabulate the components in a similar manner that is used in working with rectangular components in vector addition.

Table 1 Components of the vectors

Force	x-component	y-component
F	$F\cos 40^0$	$F\sin 40^0$
R	0	R
W	0	-W
Total	$F\cos 40^0$	$F\sin 40^0 + R - W$

Note that the magnitudes of the forces **R** and **W**are equal only if the wagon is stationary with no net force in the horizontal direction but W has a negative sign. However, if there is motion, the component of the vertical force F_y will reduce the reaction force.

4.6 Weight

In everyday spoken language we tend to confuse the concept of weight and mass of an object as being one and the same thing. In science these are very different in that the mass of an object is constant whereas the weight can vary depending on the location of that object, e.g. an object weighs one sixth of its weight on Earth when taken to the moon, however, its mass remains constant. Weight is a force exerted on an object by the force gravity. Even on the Earth the weight may vary from place to place as the acceleration due gravity varies from place to place on Earth.

The S.I. unit of weight is the Newton, abbreviated with capital N.

Example 4.3

In figure 1, the wagon is pulled with a force of 20 N which results in its acceleration horizontally. The wagon has a mass of 8 kg. Let friction be negligible, find:

i) The wagon's acceleration, and

ii) The reaction force **R** that the ground exerts on the wagon.

Solution

Examining the scenario reveals that there is no net vertical resultant force and therefore, it can be concluded that there is no acceleration in this direction.

i.e. $a_y = 0$, the equation for the vertical direction is:

$$R + 20N\sin 40^0 - 8kg \times 9.8ms^{-2} = 0$$

$$R + 12.86N - 78.4N = 0$$

$$R = 65.54N$$

Acceleration in the horizontal direction from equation 4.2 is

$a = \frac{\sum F_{xnet}}{m} = \frac{20\cos 40^0}{8} = 1.92\ ms^{-1}$, where $20\cos40^0$ is the net force.

4.7 Newton's Third Law

In studying the action of forces Sir Isaac Newton came to the conclusion that forces in nature act in pairs rather than in isolation. This law deals with the fundamental characteristic of forces. He came up with the following law which states that:

When an object exerts a force on another object, the second object exerts on the first a force of the same magnitude but in the opposite direction.

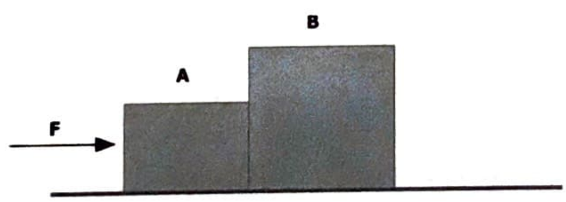
This law is sometimes referred to as "action-reaction" law, since it follows that: "for every action (force) there is an equal, opposite, reaction." In this action and reaction are referring to forces. For example if you push against a wall, the wall pushes against you with an equal but opposite force.

Example 4.4

Two blocks A and B are in contact on a frictionless horizontal surface and have masses of 2.0 kg and 4.0 kg respectively. A force of 12 N is applied to block A.

i) Find the force block A exerts on block B.

ii) If the same force is applied to block B, what is the force block A exerts on block B?



The blocks are in contact and therefore, they will undergo the same acceleration.

$$\text{i.e. } a = \frac{F}{m_A + m_B} = \frac{12N}{2kg + 4kg} = \frac{12N}{6kg} = 2\,\text{ms}^{-2}$$

To get this acceleration block A exerts a force on block B given by:

$$F_{AB} = m_B a = (4kg)(2ms^{-2}) = 8\,\text{N}$$

When the 12 N force is applied to block B, then block B exerts a force on A given by:

$$F_{BA} = m_A a = (2kg)(2ms^{-2}) = 4\,\text{N}$$

It should be noted that the reaction force of block A on B is the same but opposite in direction. The net force to the left of block B is:

$$12N - 4N = 8 N$$

The 8 N is the force needed to give block A an acceleration of 2 ms^{-2}.

4.8 Frictional Forces

Friction is the term used to describe the resistance that is experienced when two objects in contact slide against each other. For example, if one pushes a box on the floor, the bottom of the box will be sliding on the floor and the resistance offered to the sliding motion will be proportionate to the weight of the box. A heavy box will require much more effort than a lighter one. The resistance offered to the sliding motion is a frictional force i.e. it is the actual force that arises to oppose the relative motion between surfaces in contact. Friction is desirable in certain circumstances and undesirable in others. In everyday activities such as walking, braking systems, fixing nails, etc., it is desirable; whereas this is not the case in moving machinery parts.

4.9 Static Friction

We normally find that it takes more effort to set a heavy box sitting on the floor to start sliding and when in motion it is relatively easier to keep it sliding. The initial force applied increases before we reach a point at which motion just begins. This tells us that there is a force which prevents the box from sliding, it is this force which generates what we call static friction. The static frictional forces can have values ranging from zero to some maximum value based on the applied force, see figure 4.

4.10 Starting Friction

Starting friction is the maximum value given to static friction just before the box in our case starts moving relative to the floor. It is the peak value of the static friction beyond which motion begins.

4.11 Kinetic Friction

As earlier alluded to, once our box on the floor begins moving, it can be kept in motion with less effort than was required to set it in motion. Thus, the friction force generated is slightly less than the starting friction force; we call this the kinetic friction force.

The variation of friction forces is best visualized on a plot of frictional force versus applied force as shown in figure 4.

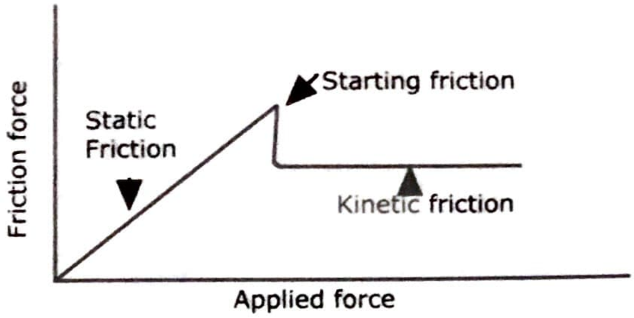


Figure 4 Plot of friction versus applied force

4.12 Coefficient of Friction

We can now look at factors that affect the frictional forces. It is well known that more effort is needed to start a laden box moving, which means that the more pressed together the contact surfaces are the larger the applied force needed for surfaces to slide. Examining figure 5 reveals that the force in this case will be the weight of the box and this force is perpendicular to the direction of motion. The perpendicular force with which either surface is pressed against each other affects the frictional force generated. We call the perpendicular force as the normal force.

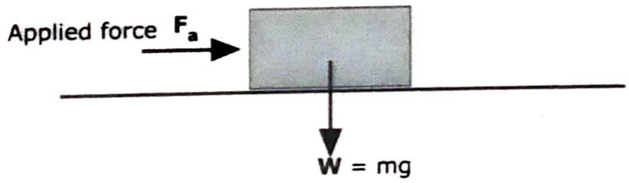


Figure 5 Box resting on a surface with a force $\mathbf{F_a}$ applied on it.

In addition we know that if the surfaces of the floor and the box are very smooth then it would easier to slide the box than if both surfaces are rough. It follows that the kinds of surfaces in contact do affect the frictional force. For example it is very difficult to walk over a slippery surface than a rough surface.

The normal force $\mathbf{F_N}$ is related approximately to the frictional force $\mathbf{f}$ by the following:

$$f_s \leq \mu_s F_N \quad \text{static friction}$$

$$f_k = \mu_k F_N \quad \text{kinetic friction}$$

The proportionality constant μ in the equations is called the coefficient of friction. The coefficient of static friction is larger than the coefficient of kinetic friction. The values of these coefficients can be determined experimentally. These values have been determined for combinations of surfaces in contact as can be seen in table 2.

Table 2

Coefficients of friction

Surfaces in contact	μ_s	μ_k
Rubber on dry concrete	0.9	0.7
Steel on steel	0.7	0.5
Wood on stone	0.5	0.4
Wood on wood	0.7	0.4
Steel on ice	0.02	0.01

Example 4.5

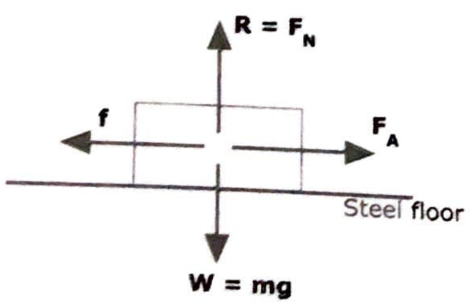
A 50 kg steel box is stationary on a level steel floor.

i) Calculate the minimum horizontal force needed to start the box moving.

ii) What is the minimum force needed to keep the box sliding at constant speed?

iii) What will be the effect of applying a 400 N to the box?

Solution



i) To start the box moving f must be equal to the starting frictional force.

$$f_s = \mu_s F_N = (0.7)(50)(9.8) = 343\,\text{N}$$

The box will only move if the applied force F_A is greater than 343 N.

ii) When it is set in motion, we can calculate the frictional force needed using the coefficient of kinetic friction.

$$f_K = \mu_K F_N = (0.5)(50)(9.8) = 254\,\text{N}$$

This is the force needed to keep the box in motion at constant speed.

iii) We note that the applied force of 400 N is greater than the starting frictional force of 343 N and it follows that a net force will arise that will cause the box to accelerate.

$$a = \frac{F_{net}}{m} = \frac{400-343}{50} = 0.8\,\text{ms}^{-2}$$

4.13 Weightlessness

Under normal circumstances the weight of an object is equal to the pull of gravity experienced by that object. However, there are situations in which the measured weight may vary depending on whether the object is being accelerated upwards or downwards. Since we normally use scales or balances to weigh, if the object being weighed undergoes acceleration the reading on the scale may not be the correct one. It may be more or less than the actual weight depending on the direction of acceleration i.e. upwards or downwards. The reading in this case is called an apparent weight. Consider a situation where an object is falling freely, the only force acting on it is gravity if we neglect air resistance, in this situation the object's apparent weight will be zero. We can then say that the object is weightless. This concept is best understood by considering a weight hanging on a balance in an elevator subjected to different accelerations.

Scenario 1

The case of an elevator that is not moving, the acceleration is equal to zero and the sum of forces in the vertical direction is equal to zero.

$$a_y = 0, \sum F_y = ma_y = 0$$

Hence, the balance will read the apparent weight which is equal to the pull of gravity (mg).

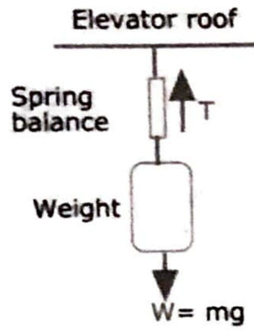


Figure 6 Weight hanging from a balance attached to an elevator roof.

No acceleration (velocity = 0)

The sum of forces: T+(-W) = 0 or T = W, i.e. the apparent weight is equal to the gravitational pull and therefore, the scale will read W, see figure 6.

Scenario 2

The elevator moving with constant velocity (either direction) implies that the acceleration is zero. In this case the spring balance will also read W since the sum of forces in the vertical direction will also be zero.

Scenario 3

Let the elevator be accelerating upwards and then we consider the sum of forces in the vertical direction. From Newton's second law taking up as the positive direction we have that:

$$\sum F_y = ma_y \text{ or } T + (-W) = ma_y$$

The spring balance reads the apparent weight given by T. We get from the above equation that:

$$T = W + ma_y$$

This result tells us that the apparent weight is much more than the actual weight found in scenarios 1 and 2.

Scenario 4

Let the elevator be accelerating downwards and then we consider the sum of forces in the vertical direction. From Newton's second law we get:

$$\sum F_y = ma_y \text{ or } T - W = m(-a_y)$$

Therefore, the apparent weight is: $T = W - ma_y$, or that the apparent weight is less than the actual weight by an amount equal to ma_y. Suppose the cable of the elevator snapped, and then the elevator will be in free fall motion together with its contents. We can re-write the latter equation as:

$$T = W - ma_y = mg - mg = 0, \text{ since } a_y = g$$

The tension will be equal to zero meaning that the spring balance will show a zero reading. The zero reading tells us that the object under these conditions is weightless or there is no tension exerted on the object attached to the balance.

The above discussion shows that the object's apparent weight (T) depends on its acceleration parallel to pull of gravity. In addition, objects undergoing free-fall motion are weightless (zero apparent weight) whenever the only force acting on them is the pull of gravity.

Example 4.6

A 25.0 kg girl sits on balance seat hanging in an elevator. What is her apparent weight when the elevator is:

i) accelerating upward with an acceleration of 1.75 ms^{-2},

ii) moving upward at a constant velocity, and

iii) accelerating downward with an acceleration of 0.75 ms^{-2}.

Solution

i) Accelerating upward is scenario 3 where, $T = W + ma_y$

T = 25 x 9.8 + (25)(1.75) = 245 +43.75 = 288.75 N i.e. apparent weight is greater than her actual weight.

ii) Elevator at constant velocity is scenario 2 where, $T = W$ meaning that the apparent weight is equal to the actual weight.

iii) Accelerating downward is scenario 4 where, $T = W - ma_y$.

T = 245 – 18.45 = 226.25 N

Note that the apparent weight is less than the actual weight.

Motion on an Inclined Plane

We now turn to resolution of motion on an inclined plane using Newton's second law of motion. Previously the forces acting on the object in question were resolved into its respective components in the x-y plane. We will confine the discussion to the object moving along the x-axis of the coordinate system taking the plane being parallel to this axis as in Figure 7.

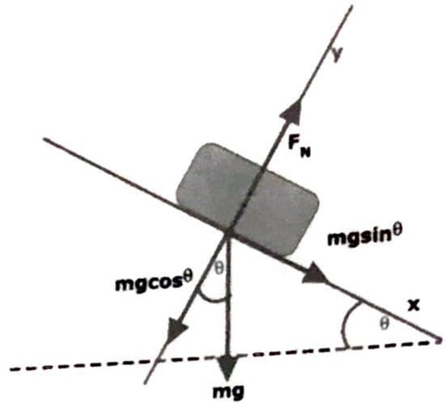


Figure 7 Object on an inclined plane.

The weight of the object in figure 7 is acting directly downwards labeled *mg*, and the normal F_N is perpendicular to the x-axis. We can resolve the forces acting on the object into *x* and *y* components along the respective axes. The weight of the object *mg* has components in both axes whereas the normal force F_N is the only acting along the positive y-axis.

Components:	x-axis	y-axis
	$mg\sin\theta$	$mg\cos\theta$
	0	F_N

If there is friction between the object and the plane its direction will be determined by the direction of motion on the plane e.g. if the object is sliding downwards in figure 7 then the frictional force would be acting in the opposite direction to the motion. There are two conditions that govern motion on an inclined plane:

i) $\sum F_y = 0$, this arises from Newton's first law of motion.

ii) $\sum F_{x,net} = ma_x$, from Newton's second law of motion.

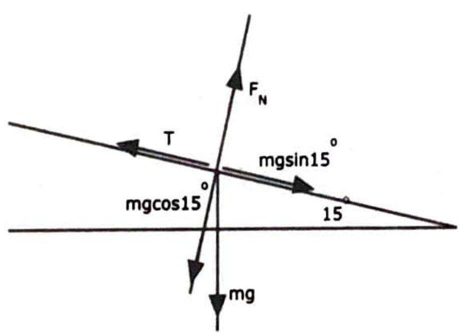
The two conditions allow us to apply these equations to motion on various configurations of motion on inclined planes. Remember to assign the axes with the correct signs along the respective axes of the coordinates superimposed on the plane.

Example 4.7

An electric motor on a stationary tow-truck is hooked to a boat mounted on a trailer resting on a slope of 15° with respect to the horizontal. The boat-trailer combination has a mass of 750 kg and the maximum working strength of the attached cable is 3000 N. Find the maximum acceleration the boat-trailer combination can be given.

Solution

First, we draw a free body diagram to show the forces acting on the boat-trailer combination.



We are interested in the motion along the slope, it follows that:

$$F_{net} = T - mg\sin 15^0, \text{ and}$$

from Newton's second law of motion $\sum F_{net} = ma$. We can, therefore write that:

$$T - mg\sin 15^0 = ma$$

$$a = \frac{T}{m} - g\sin 15^0$$

$$a = \frac{3000}{750} - (9.8)(0.2588) = 1.46 \text{ ms}^{-2}$$

Note that we did not use the equation relating to the perpendicular direction to the slope. If the problem involved friction then we could have used it.

Chapter 5

5.0 Work

5.1 Work and Energy

In physics the word "work" does not have the same meaning as everyday spoken language. Work is defined in terms of the application of a force and the displacement it causes an object to undergo. The physics version of work states that:

When a force acts on an object to produce a displacement, the work done by the force is defined as the product of the displacement and the component of the force in the direction of the displacement.

The latter part of the statement implies that the force need not be applied in the direction parallel to the direction of motion. If the applied constant force is at an angle θ with respect to the displacement vectors, then the work done is:

$$W = Fs\cos\theta \qquad 5.1$$

Figure 5.1 illustrates the situation in which a box is pulled through a displacement *s* by a constant force applied at an angle θ.

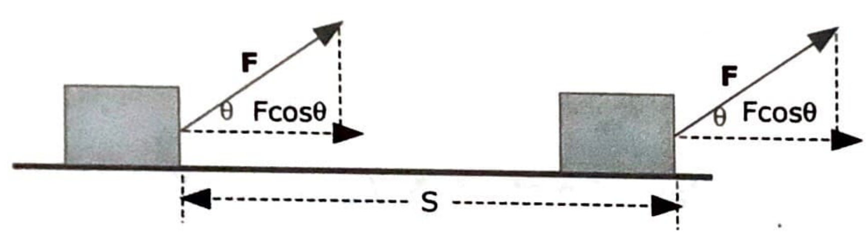


Figure 5.1 An object being moved by a force **F** applied at an angle θ.

Not that if the applied constant force is parallel to the displacement then, work from the definition is $W = Fs$ and that if the displacement is zero then there is no work done by the force. Although work is a product of two vector quantities, it is a scalar with no associated direction. The unit of work in the SI system is the Newton-metre (Nm) which is equivalent to one joule (1J).

One joule is the work done when a constant force of one Newton acts through a distance of one meter i.e. 1 J = 1 Nm. Note that there are other units of work depending on the units used for force and displacement.

Example 5.1

How many joules of work are done by a brick-layer who lifts a 1.5 kg cement block through 3.0 m at a construction site?

Solution

First, we find the force the brick-layer exerts on the block in lifting. The exerted force is equal to the weight of the block.

$$F = 1.5 \text{ kg x } 9.8 \text{ ms}^{-2} = 14.7 \text{ N}$$

$$W = F.s = 14.7 \text{ N x } 3 \text{ m} = 44.1 \text{ J}$$

5.2 Power

Power is the rate at which work is done by a force. Thus we can define power as the amount of work done in a given interval of time t.

$$\text{Power } P = \frac{Work}{time} = \frac{W}{t} \qquad 5.2$$

In the S.I. units, where work is in joules and time is in seconds, the unit of power is the watt.

$$1 \text{ watt (W)} = 1 \text{ J/s}$$

There is also another common unit of power which is predominantly used in North America called the horse-power (hp). The units of power are commonly used to specify the output of various machines such electric motors, engines etc. The relationship between the watt and the horse-power is that one horse-power is equivalent to 746 W.

Since a force is involved in doing work, it follows that power and work are related in some way. To get this relationship we consider a force F which moves an object through a distance s and takes time t.

Work done is $W = F.s$ and the power of the driving force $P = \frac{W}{t}$, if we substitute in this latter equation with the term for work, we get

$$P = \frac{F.s}{t} = F.\left(\frac{s}{t}\right) \qquad 5.3$$

The term in brackets from equations of linear motion is equal to the velocity of the object, therefore, we can write that

$$P = F.v \text{ or in general } P = Fv\cos\theta \qquad 5.4$$

Where θ is the angle between F and v. Usually equation 5.4 is re-written as:

$$W = P.t \qquad 5.5$$

Example 5.2

An electric motor mounted on Pick-up truck exerts a force of 2000 N on a cable fixed to a tree and pulls the Pick-up truck a distance of 15 m in a minute. What is the power supplied by the electric motor?

Solution

Using equation 5.2: $P = \frac{W}{t} = \frac{F.s}{t} = \frac{(2000)(15)}{60} = 500\ \text{W}$

Example 5.3

An electric motor rated at 20 kW output powers an elevator of a five storey building. The total mass of a loaded elevator is 1200 kg, what is the minimum time needed for it to rise to 25 m from the ground floor to the top-most floor?

Solution

This example is a case of work done against gravity, therefore, work is:

$$W = mgh = (1200)(9.8)(25) = 294{,}000\ \text{J}$$

From equation 5.2, we get $t = \frac{W}{P} = \frac{294{,}000}{20{,}000} = 14.7\ \text{s}$

5.3 Energy

Energy is defined as the property that gives something the ability or capacity to do work in the context of work as defined earlier. This being the case, it can be stated that work done on something is equivalent to the amount of energy used up in doing the work. There are three categories of energy of interest, namely; kinetic, potential and rest energies. Kinetic energy is the energy an object has by virtue of it being in motion, potential energy is the energy an object has by virtue of its position or state and rest energy is the energy an object has in its mass. This leads us to the conclusion that the unit energy is the same as that of work i.e. the joule.

5.4 Kinetic Energy

In order to find the kinetic energy an object in motion has, we consider the energy required to give it motion. It follows from above that when an object is brought to rest it must give up the energy it was given initially.

Consider a mass m initially at rest, if a force **F** is applied to it so that it undergoes a displacement **s** in the direction of the force. Then from Newton's second law we can say:

$$F = ma \text{, and}$$

$$W = ma.s \text{ at constant acceleration} \qquad 5.6$$

Final velocity squared is given by: $v_f^2 = v_i^2 + 2as$ but $v_i^2 = 0$ since the mass starts from rest. This equation reduces to:

$$v_f^2 = 2as \text{ or } as = \frac{v_f^2}{2} \qquad 5.7$$

Substituting the value of *as* from equation 5.7 into the work equation 5.6, we get:

$$W = m.\frac{v_f^2}{2} \text{ or work done appearing as kinetic energy (KE)}$$

Thus the work done appears as kinetic energy of the mass *m*, therefore, in general we can obtain the KE from the equation:

$$KE = \tfrac{1}{2}mv^2 \qquad 5.8$$

Example 5.4

What is the force required to stop a 20 g bullet whose velocity is 430 ms^{-1} from a pistol in distance of 25 cm in a block of wood?

Solution

$$\text{Work} = F.s = \Delta KE$$

$$F.s = KE_f - KE_i$$

$$= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \text{ , } v_f^2 = 0$$

$$F.(0.25) = 0 - \tfrac{1}{2}(0.02)(430^2)$$

$$F = -\frac{(.05)(0.02)(430^2)}{0.25} = -7{,}396\,\text{N}$$

The negative sign signifies the fact that the stopping force is in the opposite direction to the motion of the bullet.

5.5 The Work-Energy Theorem and Kinetic Energy

The work-energy theorem states that the work done on an object by net force, F_{net} is equal to the change in the object's kinetic energy. In other words the difference in the kinetic energies i.e. final kinetic energy minus the initial kinetic energy is equal to the work done on the object arising out of the applied net force.

i.e. Work done by F_{net} ,$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta KE$, v_f, v_i being final and initial velocities.

The net force in the direction of motion speeds up the motion and the net force in the opposite direction motion decreases it. Thus retarding net forces such as friction do negative work on the object as demonstrated in example 5.4.

Example 5.5

A 1000 kg car is travelling at 72 km/h when the engine is switched off. It slows down to a stop on level ground in a distance of 200 m. Calculate the average frictional force acting on the car.

Solution

The work done by a retarding force is in the opposite direction to the motion and the displacement of the car. The equation for this case is:

$$W = f.s\cos\theta = f.s\cos 180^0$$ where f is the frictional force.

But $W = \Delta KE$

$$f.s\cos\theta = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$f = \frac{\frac{1}{2}m\left(v_f^2 - v_i^2\right)}{s\cos\theta}$$

We have $v_i = 72 \text{ km/h} = \dfrac{72\times1000}{60\times60} = 20 \text{ m/s}$

$$\therefore f = \frac{(0.5)(1000)\left(0-20^2\right)}{(200)(-1)} = 1000 \text{ N}$$

5.6 Potential Energy

It has already been mentioned that potential energy is the energy possessed by an object by virtue of its position or rest energy stored in its mass. We will concentrate our discussion on the energy arising out of the position of the object. To calculate potential energy we consider an object like a stone which is raised to a certain height. It is a fact that when a stone is raised to a certain height, work is done and this work can be calculated since gravity opposes the raising. A stone with mass m is raised to a height h, the work done is:

$$W = mgh$$

If the stone is released from the height h, it will fall back to its original position and in so doing it will give up the energy it attained by virtue of position. The energy will appear as kinetic energy of the stone. The stone may deform the surface it lands on, thus the height it was raised to gives the stone an ability to do work. The work done by falling to the ground is called potential energy (PE), sometimes referred to as gravitational potential energy (GPE). It follows that the potential energy is equal to the work done on the stone.

$$W = PE = mgh \qquad 5.9$$

Example 5.6

Suppose your Physics textbook has a mass of 1 kg and is raised from the floor to a table-top 60 cm from the floor. What is the potential energy acquired by the text book?

Solution

In this instance we will calculate the potential energy with respect to the floor.

$$PE = mgh = (1)(9.8)(0.6) = 5.88\,\mathrm{N}$$

Note: It is always important to specify the reference when discussing potential energy of an object.

5.7 Conservation of Energy

The conservation of energy is one of the fundamental laws of nature. It is a law that addresses the constancy of energy in an isolated system. It states that "the total amount of energy in a system isolated from the rest of the universe remains constant, although energy transformations from one form of energy to another may take place within the system". The law does not allow the destruction of energy nor its creation; however, energy maybe transformed from one form to another with the total energy remaining the same as before. In some cases the original energy may be transformed into two or more forms e.g. the raised stone will have kinetic energy as it hits the ground and may result in the production of sound and heat as other forms of energy. For example the total mass of products of a chemical reaction should be equal to the total mass of the original substances. This law allows us to solve certain problems that would be very complex if we employed other methods.

Example 5.7

A child starts from rest on a frictionless slide. If the slide is 2 m high from its bottom end. Calculate the speed of the child at the bottom of the slide.

Solution

We will use the energy conservation law to find the speed of the child. At the top of the slide the child has PE and no KE and at the bottom the child has only KE and no PE. The potential energy at the top is all converted to kinetic energy at the bottom since there are no friction losses.

$$PE_{top} + KE_{top} = PE_{bottom} + KE_{bottom}$$

$$mgh + 0 = 0 + \tfrac{1}{2} mv^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{(2)(9.8)(2)} = 6.26\,\text{ms}^{-1}$$

Note that we did not know the child's mass in order to solve this problem.

5.8 Conservative and Non-conservative forces

The work done by conservative forces such work done against gravity can re-appear as kinetic energy. In this case the whole of the potential energy will manifest as kinetic energy. Common conservative forces are gravitational, magnetic and elastic forces. Work done against such forces depends only on the end point rather than the path taken to arrive at it.

The work done by non-conservative forces have a dependency on the path taken, i.e. work done may not give rise to potential energy which can be converted to an equal amount of kinetic energy. For example work done against friction cannot be recovered by re-tracing the path the object took. In such cases there is energy loss which is converted into heat, sound and light. Non-conservative forces do not conserve kinetic and potential energies of the object they act on.

Example 5.8

A 400 g toy car is shot up a 20° incline with a speed of 1.5 m/s as shown in figure 5.2.

i. How far up the incline does it go if the coefficient of friction between its wheels and the incline is 0.12?

ii. Compare the height reached with that it will attain if there was no friction.

iii.

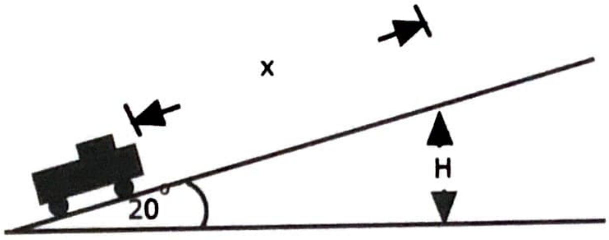


Figure 5.2 Toy car moving on an incline surface

Solution

i. We first find the frictional force the toy car experiences:

$$f = \mu F_N = \mu mg\cos 20^0 = 0.12 \times 0.4 \times 9.8 \times 0.940 = 0.442\,\text{N}$$

The toy car will slide up the incline a distance equal to $x\sin 20°$. The energy equation up the incline from the conservation of energy is:

$$\Delta KE = \Delta PE + W_f$$

$$\tfrac{1}{2}mv^2 = mgx\sin 20^0 + f.x$$

$$0.45 = 1.34x + 0.442x$$

$$x = 0.17\text{ m}$$

ii The height reached without friction is:

$$H = 0.17\sin 20^0 = 0.058\,\text{m}$$

The case of no friction the height reached is found from the conservation of energy by equating the kinetic energy to the potential energy:

$$\Delta KE = \Delta PE$$

$$\tfrac{1}{2} m v^2 = mgh$$

$$h = \frac{v^2}{2g} = 0.115\,\text{m}$$

We note that the toy car rises to a higher height if there is no loss of energy to a non-conservative force, namely, the frictional force.

Chapter 6

6.0 Momentum

6.1 Linear Momentum

It is observed in everyday experiences that it is easier to throw a light object than a heavy object and also that is easier to stop a lighter object than a heavy one. In trying to understand the phenomena, Sir Isaac Newton noted that the mass and the speed of the object played a part in these observations and he described these observations in terms of the product of the mass and its velocity. This product attempts to quantify the "quantity" motion of a mass in easily understood terms. The product of mass and velocity is called linear momentum for objects moving in straight lines.

The linear momentum is symbolized by the letter **p** and is a vector quantity. Thus the linear momentum of an object is the product of its mass *m* and velocity ***v***

$$\vec{p} = m\vec{v} \qquad 6.1$$

It has the same direction as that of the velocity of the object. The SI units of momentum are kg.m/s.

6.2 Impulse

A relationship exists between a net applied force to an object and its change in linear momentum. A net force causes the object to accelerate or decelerate as a result its velocity and momentum change. We know that force and mass are related through Newton's second law of motion.

$$F = ma \qquad 6.2$$

The acceleration or deceleration of an object is defined by the equation $a = \frac{v_f - v_i}{t}$. When we substitute this expression in equation 6.2 we get:

$$F = m.\frac{(v_f - v_i)}{t} = \frac{mv_f - mv_i}{t} \qquad 6.3$$

The right term in equation 6.3 becomes the change in momentum divided by time or the rate of change of momentum in time *t*, thus we can write that:

$$F = \frac{p_f - p_i}{t} = \frac{\Delta p}{t} \qquad 6.4$$

The latter equation 6.4 is a re-statement of Newton's second law in terms momentum. Usually to set an object such as a ball from rest, a force is applied for some finite time. If the force is great or time of application is long we expect a greater value of momentum. However, when we kick a ball or hit a golf ball, the contact time is usually very short. The product of the average force and the short time interval Δt is called the impulse.

$$\text{Impulse} = \overline{F}.\Delta t \qquad 6.6$$

Therefore, from equation 6.4 we can conclude that impulse is simply equal to the momentum change.

$$\overline{F}.t = \Delta p \qquad 6.7$$

Example 6.1

A golfer strikes a 45.7 g golf ball and his golf club is in contact with the golf ball for 0.45 milli-seconds. The golf ball flies off with a speed of 75 m/s. Find, the average force that the golf ball was subjected to during the contact period.

Solution

The golf ball starts from rest; the momentum change is given by equation 6.7.

$$\overline{F}.t = \Delta p = mv_f - mv_i$$

$$\text{From rest } \therefore mv_i = 0$$

$$\Delta p = 0.0457 \times 75 = 3.43 \text{ kg.m/s}$$

$$\overline{F} = \frac{\Delta p}{t} = \frac{3.43}{4.5 \times 10^{-4}} = 7.62 \times 10^3 \text{ N}$$

We see from the average force calculated that a tremendous force is applied on the golf ball to give the speed with which it flies off.

6.3 Conservation of Momentum

The conservation of momentum states that if two or more objects interact, the momentum before the interaction is equal to the momentum after interaction. We can say that the total linear momentum of an isolated system remains constant since it is a vector quantity. For example, suppose two billiard balls of masses m_1 and m_2 are moving in opposite directions with velocities v_1 and v_2 respectively collide, then the statement above can be stated in equation form as follows:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad 6.8$$

Where v_{1i}, v_{2i} are the initial velocities of ball 1 and 2 respectively, whereas v_{1f} and v_{2f} represent their final velocities.

In equation 6.8 the subscripts *i* and *f* denote initial and final velocities of the billiard balls. This equation is valid for a number of situations, e.g. the balls may be moving in the same direction before the collision or one of the balls maybe stationary. We may understand this concept better if we consider a special case of gun/bullet system that is cocked ready to fire and is suspended freely on ropes as in figure 6.1.

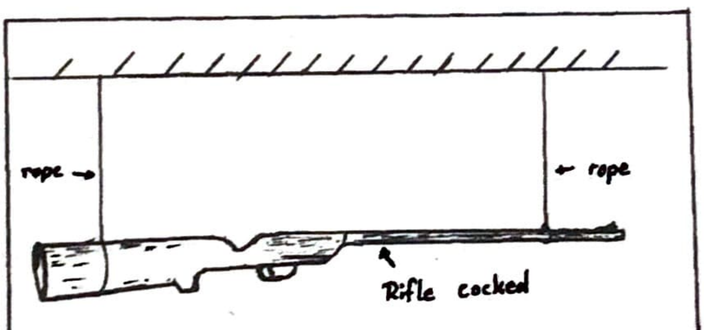


Figure 6.1 Riffle suspended on cords.

When the gun is fired, the exploding gasses in the bullet will expel the bullet and at the same time the escaping bullet will exert and an equal and opposite force on the gun. Let $\boldsymbol{F_b}$ be the force exerted on the bullet then by Newton's third law, the bullet exerts an equal and opposite force on the gun $\boldsymbol{F_g}$. We can write that:

$$F_b = -F_g \qquad 6.9$$

We can use the special form of Newton's second law in equation 6.4 to re-write the forces on the bullet and the gun.

$$F_b = \frac{m_b v_b}{t}, \text{ Similarly } F_g = \frac{m_g v_g}{t}$$

These forces act for exactly the same length of time on each other, thus the impulse of the bullet is equal to the impulse of the gun. It follows that if we set these terms against each other we get the result:

$$m_b v_b = -m_g v_g \qquad 6.10$$

The result in equation 6.10; states that the momentum of the bullet is equal to that of the bullet and since it is a vector quantity, the addition of these momenta give zero which is the situation that was there before the gun was fired. In other words, the momentum of the gun/bullet system has not changed and this proves that momentum was conserved.

Example 6.2

A 500 kg wheeled cannon rests on a frozen lake and fires horizontally a 10 kg cannon ball with a muzzle velocity of 400 m/s. Calculate the recoil velocity of the cannon carriage.

Solution

Let the direction the cannon ball takes be positive then the cannon carriage's recoil direction will be negative. We can write equation 6.8 as:

$$m_c v_{ci} + m_b v_{bi} = -m_c v_{cf} + m_b v_{bf}, \; v_{ci} = 0 \text{ and } v_{bi} = 0$$

$$\therefore 0 + 0 = -m_c v_{cf} + m_b v_{bf}$$

$$v_{cf} = \frac{m_b v_{bf}}{m_c} = \frac{10 \times 400}{500} = 8 \text{ m/s}$$

The cannon carriage will have a recoil velocity of -8 m/s.

6.4 Collision Phenomena

In everyday life we encounter collisions of objects, these collisions take several forms. Some collisions can be direct head-on collisions, collisions of moving objects into stationary ones or collisions at angle. In all these collisions the conservation of momentum and conservation of energy do apply. In some collisions part of the energy maybe transformed into non-mechanical energy such as heat and sound thus making it difficult to account for the energy after a collision.

Collisions are classified into two major types:

i) Elastic collisions

ii) Inelastic collisions

There are some intermediate collisions that we will not discuss.

6.5 Perfectly Elastic Collisions

A collision is said to be perfectly elastic if both kinetic energy and momentum are conserved. In general objects that retain the original shape after a collision are elastic. Collisions at a microscopic scale such as those that take place between atoms, atomic nuclei, electrons and molecules are perfectly elastic. This condition is never attained for collisions at a macroscopic scale such as billiard balls, a ball and the floor as some energy is lost during such collisions. However, it should be noted that in all collisions momentum is conserved.

If two objects undergo a perfectly elastic collision in a line connecting their centres, their momentum and kinetic energy will be conserved. We can write two equations that are valid for this collision:

i. $mv_i + MV_i = mv_f + MV_f$, from conservation of momentum. 6.11

ii. $\frac{1}{2}mv_i^2 + \frac{1}{2}MV_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}MV_f^2$, from conservation of energy. 6.12

Equations 6.11 and 6.12 can be re-written respectively as:

$$m(v_i - v_f) = M(V_f - V_i) \qquad 6.13$$

$$m(v_i^2 - v_f^2) = M(V_f^2 - V_i^2) \qquad 6.14$$

Dividing equation 6.14 by equation 6.13 we get:

$$v_i + v_f = V_f + V_i \qquad 6.15$$

Putting in initial and final velocities together we get:

$$v_i - V_i = V_f - v_f = -(v_f - V_f) \qquad 6.16$$

From equation 6.16 we note that $v_i - V_i$ is the velocity of approach and $v_f - V_f$ is the velocity of separation.

Example 6.3

A 20 g steel ball travelling along the positive x-axis with a velocity of 2.5 m/s collides with another 30 g steel ball travelling in the opposite direction at 1.5 m/s. The collision is elastic, find the final velocities after collision.

Let m represent the 20 g ball and M the 30 g ball. We use the conservation of momentum equation 6.11.

$$mv_i + MV_i = mv_f + MV_f$$

$$0.02 \times 2.5 + 0.03 \times (-1.5) = (0.02)v_f + (0.03)V_f$$

$$0.005 = 0.02v_f + 0.03V_f$$ Dividing both sides by 0.02 we get;

$$v_f + 1.5V_f = 1.5 \quad \text{(a)}$$

Using equation 6.16: $v_{i-} - V_i = V_f - v_f$

$$2.5 - (-1.5) = V_f - v_f$$

$$-v_f + V_f = 4 \quad \text{(b)}$$

Adding equations (a) and (b) gives:

$$2.5V_f = 5.5 \text{ or that } V_f = 2.2 \text{ m/s}$$

Substituting in equation (a):

$$v_f + (1.5 \times 2.2) = 1.5$$

$$v_f = -1.8 \text{ m/s}$$

The two separation velocities of the steel balls calculated show that the directions of the balls are reversed.

6.6 Perfectly Inelastic Collisions

A collision is said to be perfectly inelastic when the two colliding objects remain bound together after the collision. In such collisions the objects suffer permanent deformation; they do not separate but move with the same final velocity. For example a bullet that is fired into a block of wood and gets embedded in it, the two will move together after such a collision. Remember that momentum is always conserved in all collisions. In perfectly elastic collisions the conservation of momentum is sufficient for the computation of the final velocity if the masses and initial velocities are known using equation 6.17.

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f \qquad 6.17$$

Example 6.4

A 2.5 g bullet is shot into a 3 kg block hanging from a strong thin rope. The bullet gets lodged into the block and both move off with a speed of 0.8 m/s. Find the velocity of the bullet before it struck the block of wood and velocity of the bullet/block combination just after the collision in terms of *h*. This type of arrangement is called a ballistic pendulum.

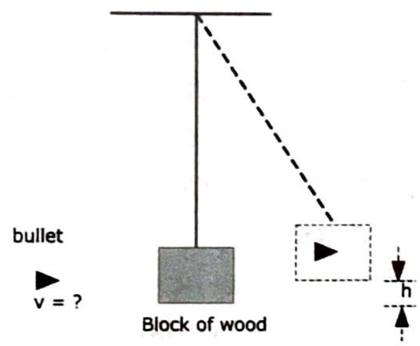


Figure 6.2 Wooden block suspended on a rope

Solution

To find the final combined velocity we use equation 6.17:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$0.0025 \times v_{ib} + 3 \times 0 = (0.0025 + 3) \times 0.8$$

$$v_{ib} = \frac{2.402}{0.0025} = 960.8\,\text{m/s}$$

The initial velocity of the bullet from this calculation is 960.8 m/s. For the latter part of the question we can employ energy consideration.

Let the shift in the centre of gravity of the block be *h* then from conservation of energy we can write:

Potential energy gained by bullet/block = kinetic energy of the bullet

Let the velocity of the bullet/block as they start moving together be v then:

$$(m_1 + m_2) gh = \frac{1}{2}(m_1 + m_2) v^2$$

$$v = \sqrt{\frac{2(m_1 + m_2) gh}{m_1 + m_2}}$$

$$v = \sqrt{19.6h} \text{ m/s}$$

6.7 Glancing Collisions

All collisions of given object take place in three dimension space (3-D). It follows that when considering the conservation of momentum we have to take into account the conservation of momentum in each axis, i.e. x, y and z axes. Most of the collisions we encounter can be confined to the two dimension space (2-D) e.g. pool balls. In such collisions we can formulate two component equations for the conservation of momentum. It means we have to properly label our masses and velocities with appropriate subscripts that identify them; remembering that momentum is a vector quantity and observe correct signs on the velocities. In glancing collisions the centres of mass do not lie in the same line. This being the case, the colliding objects deviate from original directions after collision as illustrated in figure 6.3.

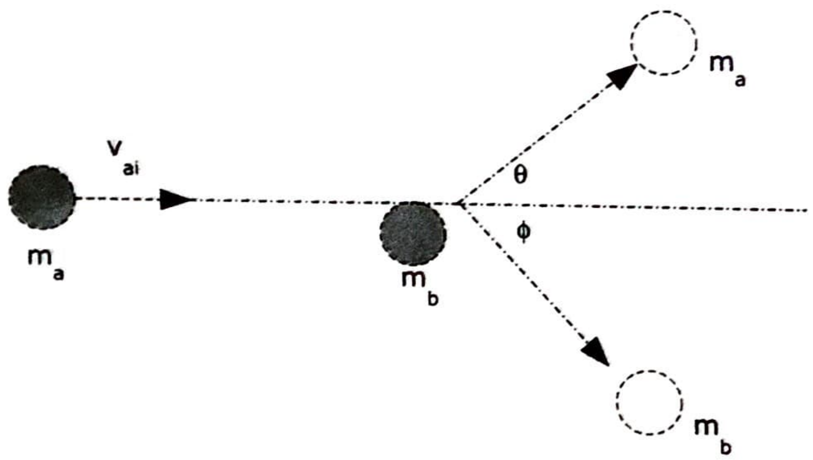


Figure 6.3 Spherical masses under-going a glancing collision

Suppose in figure 6.3, mass a has a velocity v_a and mass b is stationary. The collision of these two masses will result in mass a being deviated from its original direction at angle θ and mass b at angle □. We can write the following equations for the conservation of momentum in the x and y directions as follows:

Along the x-axis;

$$m_a v_{ai} + 0 = m_a v_{af} \cos\theta + m_b v_{bf} \cos\phi \qquad 6.18$$

and y-axis

$$0 + 0 = m_a v_{af} \sin\theta - m_b v_b \sin\phi \qquad 6.19$$

Note that both masses had no component of velocity along the y-axis and hence the right side of equation 6.19 is equal to zero. These equations will suffice in most cases, however, if the collision is elastic than we can generate a third equation 6.20 based on the conservation of energy.

$$\tfrac{1}{2} m_a v_{ai}^2 = \tfrac{1}{2} m_a v_{af}^2 + \tfrac{1}{2} m_b v_{bf}^2 \qquad 6.20$$

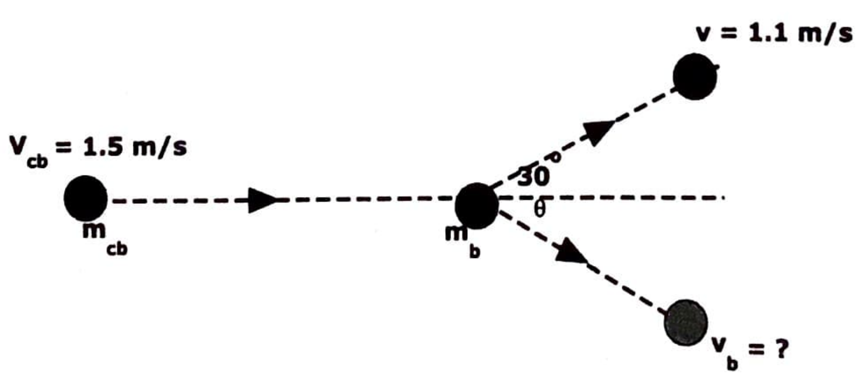
Kinetic energy of the system is not conserved if the collision is inelastic and equation 6.20 does not apply.

Example 6.5

A pool cue billiard ball of mass 0.170 kg moving with a speed of 1.5 m/s strikes another ball of mass 0.165 kg initially at rest. The collision causes the cue ball to deflect off at an angle of 30^0 with a speed of 1.1 m/s.

i) The direction of the cue ball is taken to be along the positive x-axis, write down the equations for the conservation of momentum in the x and y directions separately.

ii) Solve these equations to get the speed of the second ball and the angle it deflects at, assuming the collision is not elastic.



Solution

Conservation of momentum along the x-axis:

$$m_{cb}v_{cb}+0=m_{cb}v_{cbf}\cos 30^0+m_b v_b\cos\theta$$

$$0.170\times 1.5=0.170\times 1.10\times 0.866+0.165v_b\cos\theta$$

$$0.255=0.162+0.165v_b\cos\theta$$

$$0.165v_b\cos\theta=0.093 \qquad \text{(a)}$$

Conservation of momentum along the y-axis:

$$0+0=m_{cb}v_{cbf}\sin 30^0-m_b v_b\sin\theta$$

$$0=0.170\times 1.10\times 0.5-0.165v_b\sin\theta$$

$$0.165v_b\sin\theta=0.0935 \qquad \text{(b)}$$

Dividing equation (b) by equation (a):

$$\tan\theta=\frac{0.0935}{0.093}=1.0054$$

$$\theta=45.2^0$$

We use equation (a) to find the velocity of the second ball v_b:

$$v_b=\frac{0.093}{0.165\cos 45.2^0}=0.80\text{ m/s}$$

Chapter 7

7.0 Rotational Motion and Gravitation

7.1 Rotational Motion

So far we have discussed objects moving in linear fashion as well as those following curved paths such as projectiles. We now turn our attention to objects that follow circular paths and in particular we introduce the concepts of angular measure, displacement, velocity, and acceleration.

7.2 Angular Measure

When rigid objects follow a circular path or rotate about a fixed axis, the angular displacement can be stated in terms of the angles subtended between the starting point and the final point. Normally we measure angles in degrees. A complete rotation is assigned 360 degrees (360^{0}). It follows that 1^{0} is equivalent to $\frac{1}{360}$ of a full rotation. In SI units the preferred unit of angular displacement is the radian which is defined in terms of the arc length and the of the circular path.

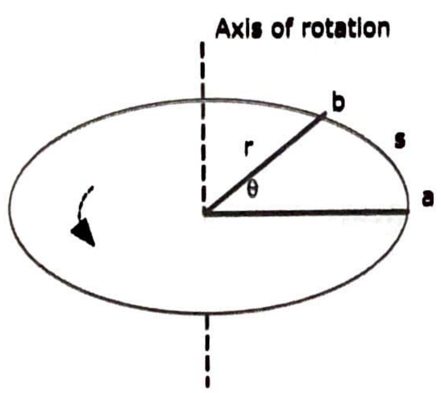


Figure 7.1 Circular path of an object.

If an object moving in a circular path of radius r moves from point a to $b$illustrated in figure 7.1 covering a distance s and angle ofθ then the radian (rad) measure is defined as:

$$\theta = \frac{s}{r} = \frac{arc\ length}{radius}\ \text{radians} \qquad 7.1$$

The ratio of the arc length s to the radius r gives the angle θ in radians abbreviated as rad. The stating of the angle in radians is a ratio of two lengths, therefore, in calculations it is treated as a number withot units. It has no bearing on other units in calculations that it is multiplied or divided with.

In order for us to convert between degrees and radians, we use fact that a full rotation of a circle of radius $r$has an arc length of $2\pi r$. It follows that the number of radians corresponding to one rotation or 360^0 is

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ rad} \qquad 7.2$$

Thus 2π rad corresponds to 360^0, hence the number of degrees in one radian is

$$1 \text{ rad} = \frac{360^0}{2\pi} = 57.3^0$$

We can in general convert from degrees to radians by using the following factor;

$$\frac{2\pi \text{ rad}}{360}$$

and also converting from radians to degerees

$$\frac{360^0}{2\pi \text{ rad}}$$

Example 7.1

A compact disc (CD) has a diameter of 12 cm, a point on the rim moves thorugh an angle of 150^0. How far does this point move in centimeters?

Solution

We first convert the angle θ from degrees to radians

$$\theta = \left(150^0\right)\left(\frac{2\pi \text{ rad}}{360^0}\right) = 2.62 \text{ rad}$$

Radius of CD is half the diameter that is 6 cm, and from equation 7.1

$$\theta = \frac{s}{r} \text{ or in terms of } s$$

$$s = r\theta = 6 \times 2.62 = 15.72 \text{ cm}$$

The above calculation tells us that the point on the rim moves an arc length of 15.72 cm on the rim of the CD.

Sometimes a complete rotation is referred to as one revolution, in which case one revolution (rev) is equivalent to 360^0. Hence

$$1 \text{ rev} = 360^0 = 2\pi \text{rad}$$

Angular Velocity

A rotating object that turns through an angle θ in time t has an average angular velocity given by the equation:

$$average\ angular\ velocity = \frac{angular\ displacement}{time\ taken}$$

The Greek letter ω is used as a symbol for angular velocity when θ is expressed in radians and time in seconds, then the equation above becomes:

$$\overline{\omega} = \frac{\theta}{t} \qquad 7.3$$

The unit for angular velocity in equation 7.3 is radians per second (rad/s). The other units used in connection with angular velocity are:

rev per sec (rps)

rev per min (rpm)

7.3 Relationship between Angular Velocity and Linear Speed

Consider a particle moving with uniform tangential velocity v in a circle of radius r as in figure 7.2.

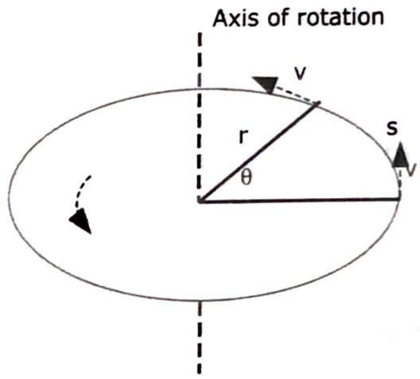


Figure 7.2 Path of an object performing circular motion.

In time t the particle covers a distance s given by $s = vt$ and the angular distance in the same time interval can be obtained from equation 7.1, $\theta = \frac{s}{r}$. We can substitute for the arc length s in the latter equation to get:

$$\theta = \frac{s}{r} = \frac{vt}{r} \qquad 7.4$$

We can then substitute for θ in equation 7.3 with the last expression in equation 7.4:

$$\omega = \frac{\theta}{r} = \frac{vt}{rt} = \frac{v}{r} \text{ or } \omega = \frac{v}{r} \qquad 7.5$$

Equation 7.5 give us the relationship between the angular velocity and the tangential velocity of a particle moving with uniform velocity in a circle. Note that the angular velocity must be expressed in rad/s. Equation 7.5 is generally written as:

$$v = \omega r \qquad 7.6$$

Example 7.2

Find the linear velocities of points 2.5 cm and 5.5 cm from the axis of rotating disc at 500 rpm.

Solution

First we have to convert the angular velocity from revolutions per minute to radians per second.

$$\omega = (500\text{rpm})\left(\frac{2\pi\text{rad/s}}{60\text{rpm}}\right) = 52.36 \text{ rad/s}$$

Therefore a point 2.5 cm from the axis of rotation has a tangential velocity of

$$v = \omega r = 52.36 \times 25 = 130.9 \text{ cm/s}$$

Similarly the point at 5.5 cm has tangential velocity of

$$v = 52.36 \times 5.5 = 288.0 \text{ cm/s}$$

7.4 Angular Acceleration

We know that rotating objects may speed up, slow down or rotate with a constant angular velocity. We are interested in knowing how they accelerate or decelerate. The rate of change of the angular velocity can be obtained in a similar manner to that of linear velocity. Thus we divide the change in average angular velocity by the time it takes for this to happen.

$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t} \qquad 7.6$$

Where $\bar{\alpha}$ is the average angular acceleration. The units of angular acceleration when ω is in radians are radians pers second (rad/s²).

Exampe 7.3

A CD is at a certain instant turning with an angular velocity of 15 rad/s then after an interval of 10 s it has an angular velocity of 60 rad/s. What is the angular acceleration of the CD?

Solution

We can use equation 7.6 to calculate the angular acceleration.

$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t\,s} \text{ rad/s}$$

$$\alpha = \frac{60 - 15}{10} = 4.5 \text{ rad/s}^2$$

7.5 Relationship between Angular Velocity and Tangential Velocity

Let us consider a particle performing uniform performing uniform circular motion, we know that its angular velocity will be constant and also that its tangential velocity also remains constant but its direction changes continuosly. In figure 7.3 the particle in moving from point A to B moves though angle θ. Let the tangential velocity vector at point A is v_o and point B is v_f, it follows from the definition of acceleration that $a = \frac{v_f - v_i}{t}$. This equation can be re-written in terms of the vector difference as $v_f - v_i = at$. The difference is shown on in figure 7.3 *at*.

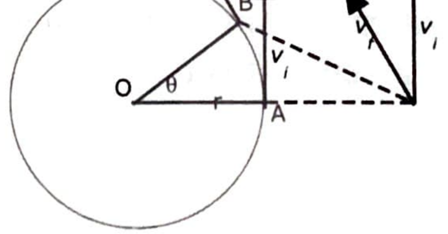


Figure 7.3 Diagram relating tangential velocity and the acceleration.

Note that the velocity vectors in figure 7.3 have been shifted away from the circular path maintaining the orientation and magnitude. From the diagram it can be seen that the acceleration is approximately directed towards the centre of the path, it follows that if the angle θapproaches zero the acceleration is towards the centre.

Let the particle pass over the arc AB in time t with a constant speed v. The distance AB is approximately equal to vt if the angle θ is very small. In figure 7.3 the triangle ABO is similar to the triangle formed by v_i, at and v_f.

$$\frac{vt}{r} = \frac{at}{v} \qquad 7.7$$

Simplifying equation 7.7 gives us the result:

$$a = \frac{v^2}{r} \qquad 7.8$$

We can conclude that for a particle describing uniform circular motion the acceleration is always directed towards the centre of the circle. The acceleration of a particle in uniform circular motion is called centripetal acceleration.

A particle moving in a circular path of radius r has an angular acceleration α and also a linear acceleration α_T in the direction of the tangential velocity v. For a constant radius r equation 7.5 says $\omega = \frac{v}{r}$ from which we say that:

$$\Delta\omega = \frac{\Delta v}{r} \text{ and } \alpha = \frac{\Delta\omega}{t}$$

Eliminating $\Delta\omega$ in the latter equation for angular yields:

$$\alpha = \frac{\Delta v}{rt} \qquad 7.9$$

The second term in equation 7.9 is a linear acceleration term and hence we can re-write the equation as:

$$\alpha = \frac{a_T}{r} \qquad 7.10$$

Equation 7.10 is usually written in terms of the tangential acceleration so that it reflects the fact that for a given α, it is proportional to the radius.

$$a_T = r\alpha\left(\alpha - \text{in rad/s}^2\right) \qquad 7.11$$

The tangential acceleration a_T represents the change in velocity whereas the centripetal acceleration represents the change in the direction of motion.

7.6 Centripetal Force

An object with acceleration toward the centre of the circular path has a net force in that direction. We can find the magnitude of the force according to

Newton's second law of motion as follows:

$$F_c = ma_c = m\frac{v^2}{r} \qquad 7.12$$

According to Newton's third law of motion the centirpetal force must have an equal and opposite force away from the circular path which we call the centrifugal force.

When a boy whirls a stone tied to string in a horizontal circlethe centripetal force needed to keep the stone in circular path is provided by the string. The string exerts a force on the stone towards the centre and the stone exerts an equal and opposite force away from the centre. The centripetal force is simply a net force acting radially towards the centre of the circular path. The net force maybe single force such as the tension in a string or in combination with other forces such as friction. For example a car moving around a curve without sliding on a level road, the static friction between the wheels and the road providesthe necessary centripetal force to keep it in a circular path. However, if the road is banked (i.e. slopping towards the centre of the circular path), the centripetal force maybe from banking plus friction or just from banking.

7.7 Road Banking

In road construction the curves in the road are suitably banked to reduce the reliance of cars on static friction to produce the necessary centripetal force for a given speed limit. The road is raised by a certain amount on the outer curve relative to the inner curve to produce suitable banking as in figure 7.4. Below the designed speed limit cars can go round the bend without the aid of friction to generate the necessary centripetal force to keep the car on the curved road.

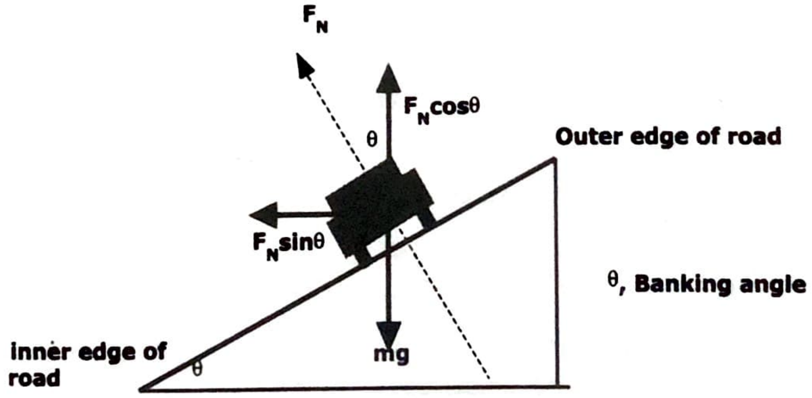


Figure 7.4 Resolution of forces on a banked road.

Figure 7.4 illustrates a car going round a curve of a banked road. In the diagram $\mathbf{F}_N$ is the normal force with respect to the road. Since the road makes an angle θ with respect to the horizontal, the normal force has a component $F_N \sin\theta$ that points toward the centre of circular path and it provides the needed centripetal force:

$$F_c = F_N \sin\theta = \frac{mv^2}{r} \qquad \text{(i)}$$

$$F_N \cos\theta = mg \qquad \text{(ii)}$$

The other component of the normal force in the vertical direction is $F_N\cos\theta$ which balances the weight *mg* of the car. Thus $F_N\cos\theta = mg$ (ii). We can now divide equation (i) by equation (ii) that gives a tangent function fronm which we can get the banking angle.

$$\frac{F_N \sin\theta}{F_N \cos\theta} = \frac{mv^2/r}{mg}$$

$$\tan\theta = \frac{v^2}{rg} \qquad 7.13$$

Equation 7.13 tells us that for a given speed *v*, the centripetal force required to negotiate a curve of radius *r* can be met by banking the road by an angle θ. Note that required angle is independent of the mass of the car.

Example 7.4

A high way is to be constructed in which there is a bend of 60 m radius and the speed limit on this portion of the road is 100 km/h. Find the banking angle if the vehicles have to depend on the banking angle to provide the required centripetal force.

Solution

From equation 7.13, we know that the required angle can be calculated from:

$$\tan\theta = \frac{v^2}{rg}$$

Given that maximum speed to negotiate this bend safely is 100 km/h, we convert this to the equivalent in m/s:

$$100 \text{ km/h} = 27.78 \text{ m/s}$$

$$\tan\theta = \frac{27.78^2}{60 \times 9.8} = 1.312$$

$$\theta = \tan^{-1} 1.312 = 52.7^{\circ}$$

If the road is banked at this angle, then vehicles travelling at this speed will have no tendency to skid even if the surface is slippery.

7.8 Equations of Angular Motion

There are similarities between linear and angular equations of motion. This being the case, one needs only to replace the symbols in equations of linear motions with those of its angular equivalents.

Linear motion		Angular motion
Displacement:	s	θ
Velocity:	v	ω
Acceleration:	a	α

It follows that the following are then the equations of angular motion:

i) $\theta = \omega . t$

ii) $\omega_f = \omega_i + \alpha t$

iii) $\bar{\omega} = \dfrac{\omega_f + \omega_i}{2}$

iv) $\omega_f^2 = \omega_i^2 + 2\alpha\theta$

v) $\theta = \omega_i t + \frac{1}{2}\alpha t^2$

These equations may be used in working out problems involving uniform circular motion in a similar manner done with equations of linear motion.

7.9 Newton's Law of Gravitation

Planetary motion approximates circular motion. This motion attracted a lot of attention and a number of scientists proposed various theories that characterise it. It is a well known fact that there is mutual attraction between planetary bodies. In our Solar system the Sun and each planet attract each other with forces equal in magnitude but opposite in direction. Sir Isaac Newton employed his third law to come up with the relationship between these forces and the masses involved. The forces are responsible for planets following a circular orbit are of gravitation origin. Gravity is a fundamental force which cannot be easily explained in terms of any other force. Based on the circular nature of planetary motion, we can conclude that gravitional forces are resiponsible for providing the necessary centripetal forces. Newton concluded that the force was of the form:

$$F_g \propto \frac{m_1 m_2}{r^2}$$

where m_1 and m_2 are masses of the two bodies separated by a distance r. Introducing a proportionality constant in the above relationship gives the following equation for a gravitational force:

$$F_g = G\frac{m_1 m_2}{r^2} \qquad 7.14$$

where G is the gravitational constant equal to 6.672×10^{-11} $N.m^2/kg^2$. Equation 7.14 is known as Newton's law of universal gravitation.The law states that every object in the iniverse attracts every other object with a force directly proportional to the product of the their masses and inversely proportional to the square of the distance between them. For example the Sun attracts the Earth with a gravitational force which is equal in magnitude to that the Earth exerts on the Sun but opposite in direction. The latter force can be calculated from equation 7.14.

Example 7.5

Two spherical steel balls of masses 5 kg and 3kg are suspended from strings and their centres are 2.5 m apart as in figure 7.5. Find the gravitational force of attracation between them.

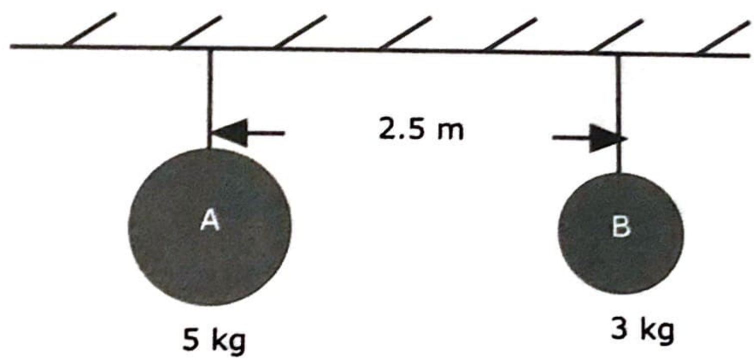


Figure 7.5 Masses suspended from thin strings.

Solution

According to Newton's law of gravitation, the gravitational force can be found by usinng equation 7.14.

$$F_g = G\frac{m_A m_B}{r^2} = \left(6.67\times10^{-11}\right)\left(\frac{5\times3}{2.5^2}\right) = 1.60\times10^{-10}\,\text{N}$$

The calculation tell us that the force the spheres exert on each other is very small, however, this force exists between them.

Example 7.6

Given information that an apple has a mass of 300 g and is placed on the ground. Use Newton's law of gravitation to calculate the mass of the Earth.

Solution

In this example we have the Earth-apple system which can be reperesented by two masses M and m for Earth and apple respectively separated by a distance equal to the radius r_e of the Earth essentially. It follows that the force the Earth exerts on the apple can be calculated from equation 7.14, and this force is equal to the apple's weight, $w = mg$. Hence:

$$F_g = G\frac{Mm}{r_e^2} = mg$$

$$G\frac{Mm}{r_e 2} = mg$$

$$\therefore M = \frac{gr_e^2}{G}\text{, where } r_e^2 = 6.4\times10^6 \text{ m}$$

$$M = \frac{(9.81)\left(6.4\times10^6\right)^2}{6.67\times10^{-11}} = 6.0\times10^{24}\text{ kg}$$

In this example we had the result $M = \frac{gr_e^2}{G}$, from which we can calculate the value of the acceleration due to gravity g by solving this equation for g:

$$g = \frac{GM}{r_e^2}\text{, where } M \text{ is mass of the Earth}$$

Note that this latter equation shows that the value of g varies withthe distance from the Earth's centre.

7.10 Orbital Motion

Man-made satellites which go round the Earth at various altitudes follow circular paths. The circular path described by satellites is called an orbit, hence, the motion is referred to as orbital motion as shown in figure 7.6.

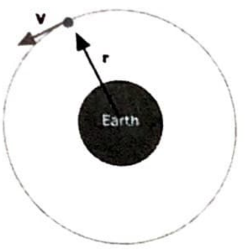


Figure 7.6 Satellite orbiting around the Earth.

The required centripetal force to hold a satellite in orbit is provided by the gravitational force. It follows that:

$F_g = F_c$, where these are gravitational force and centripetal force.

$G\frac{m_e m_s}{r^2} = \frac{m_s v^2}{r}$, m_e and m_s are Earth and satellite masses 7.15

Solving for the satellite's velocity, we get:

$$v = \sqrt{\frac{Gm_e}{r}} \qquad 7.16$$

For any satellite to maintain its orbit the centripetal froce must balance the gravitational force and must be moving at the speed given in equation 7.16.

Example 7.7

Find the velocity required to maintain a satellite in a stable circular orbit 650 km above the Earth's surface. Taking the radius of the Earth as 6.4×10^6 km and mass of 6.0×10^{24} kg.

Solution

For such a satellite the gravitational force provides the require centripetal force. Note that in equation 7.15 the satellite mass cancels out and we remain with equation 7.16 from which we can calculate the velocity. Note that satellite disatance from Earth's centre is 7,050 km.

$$v = \sqrt{\frac{Gm_e}{r}}$$

$$v = \sqrt{\frac{6.67\times10^{-11}\times6.0\times10^{24}}{7.05\times10^6}} = 7534 \text{ m/s}$$

Chapter 8

8.0 Rotational Work, Energy and Momentum

8.1 Rotational Work

We associate kinetic energy with moving objects and so far we have only discussed work in relation to objects moving in straight path. The question is - what about rotating objects such wheels, balls etc.? Suppose we have wheel mounted on axle where it can rotate freely as shown in figure 8.1 and wrap a string around it. If the string is pulled with a force ***F*** through a distance *s* it will set the wheel in motion.

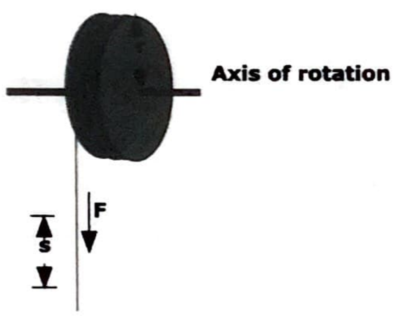


Figure 8.1 Wheel mounted on an axle with string wound around it.

Work is done in pulling the string through a distance *s* by the force ***F***. The work done by the definition of work is then:

$$W = F.s$$

Since the wheel turns through some angle θ as the string unwinds by a distance *s*. The distance *s* is related to θ through $s = r\boldsymbol{\theta}$ from equation 7.1. It follows that we replace the distance in the equation above for work done by a force ***F*** to obtain:

$$W = F.r\boldsymbol{\theta} \qquad 8.1$$

But the product $F.r$ is equal to the torque applied to the wheel through the string.

$$\tau = F.r \qquad 8.2$$

Hence, the work done through the angle θ is:

$$W = \tau\theta \qquad 8.3$$

According to the work-energy theorem, the work done on the wheel by a net force must manifest as kinetic energy. In order to calculate this kinetic energy we consider the fact that a wheel is composed of several small particles. If we examine a wheel that is rolling on level ground, then each particle in the wheel performs two motions, firstly each particle follows a circular path and secondly it is moving along the ground. Imagine that the mass of the wheel is concentrated at its geometric centre, then we have rotation and linear motions being executed by the particles. We will use this fact to calculate the total kinetic energy of the wheel which is the sum of kinetic energy of translation (linear motion) and kinetic energy of rotation. The kinetic energy of a wheel of *n* number of particles is:

$$KE_{\text{wheel}} = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 + \tfrac{1}{2}m_3v_3^2 + \ldots.. + \tfrac{1}{2}m_nv_n^2$$

Each small mass is such as m_1 is moving in a circle of radius r_1 and its tangential velocity $v_1 = \omega r_1$ and therefore, $\tfrac{1}{2}m_1v_1^2 = \tfrac{1}{2}\omega^2 r_1^2$

We can substitute the individual kinetic energies of the small masses in the equation of the combined kinetic energy above giving:

$$KE_{\text{wheel}} = \tfrac{1}{2}\omega^2 r_1^2 + \tfrac{1}{2}\omega^2 r_2^2 + \tfrac{1}{2}\omega^2 r_2^2 + \ldots. + \tfrac{1}{2}\omega^2 r_n^n$$

All parts of the wheel move with the same angular velocity ω, we can factor out $\tfrac{1}{2}\omega^2$ and we have the result:

$$KE_{\text{wheel}} = \tfrac{1}{2}\omega^2\left(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots.m_nr_n^2\right)$$

The product of mass and radius squared is a moment of inertia with the symbol I for a rotating mass. The total moment of inertia for the wheel the sum of $I = I_1 + I_2 + I_3 + \ldots. + I_n$.

$$KE_{\text{wheel}} = \tfrac{1}{2}\omega^2\left(I_1 + I_2 + I_3 + \ldots. + I_n\right) = \tfrac{1}{2}I\omega^2$$

$$KE_{\text{wheel}} = \tfrac{1}{2}I\omega^2 \qquad 8.4$$

Equation 8.4 gives the kinetic energy of rotation in terms of the moment of inertia.

The units of the moment of inertia I are kg.m^2. The moment of inertia depends on mass and the distribution of the matter forming the mass.

Rotational energy can easily be related to a work done on wheel by the applied torque. Let us take a case of a wheel rotating with an angular velocity ω_o

when a torque τ is suddenly applied to it, as a result the wheel turns through an angle θ. The work done by the torque is given by equation 8.3. If at that time the angular velocity becomes ω_f, then the Work-energy theorem states that: work done on the wheel = the change in the kinetic energy of the wheel.

$$\tau\theta = \tfrac{1}{2} I\omega_f^2 - \tfrac{1}{2} I\omega_o^2 = \tfrac{1}{2} I\left(\omega_f^2 - \omega_o^2\right) \qquad 8.5$$

Equation 8.5 is simplified by using equation of angular motion $\omega_f^2 = \omega_o^2 + 2\alpha\theta$ re-written as $\omega_f^2 - \omega_o^2 = 2\alpha\theta$ by substituting the term on the right into equation 8.5. This results in:

$$\tau\theta = \tfrac{1}{2} I\left(2\alpha\theta\right)$$

$$\tau = I\alpha \qquad 8.6$$

Equation 8.6 gives the torque applied on a wheel as a product of the moment of inertia and angular acceleration, not the similarities with Newton's second law of motion $F = ma$. The moment of inertia can be likened to mass and angular acceleration to the linear acceleration.

8.2 Rotational inertia

It is obvious from our everyday experiences with rotating objects that they have inertia, since once the driving force causing them to rotate is removed the motion continues for some time. If there were no opposing forces the motion would continue indefinitely. We can find the moment of inertia of rotating objects by considering the fact that these are made up of several particles with masses $m_1, m_2\ m_3 \ldots\ldots m_n$ rotating about some axis and each mass at a radial distance of $r_1, r_2, r_3 \ldots\ldots r_n$. To calculate the moment of inertia of the object we sum up the product of each mass and the square of its radial distance.

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots m_nr_n^2 = \sum m_ir_i^2 \qquad 8.7$$

Equation 8.7 can be used to compute the moments of inertia of various objects; however, the method is alright for regular geometrical objects but for complex objects Integral Calculus methods are used.

Example 8.1

Find the moment of inertia of a thin ring of mass M and an average radius R about its centre. If we consider that it rotates about an axis which passes its centre perpendicular to the plane of the ring as shown in figure 8.2.

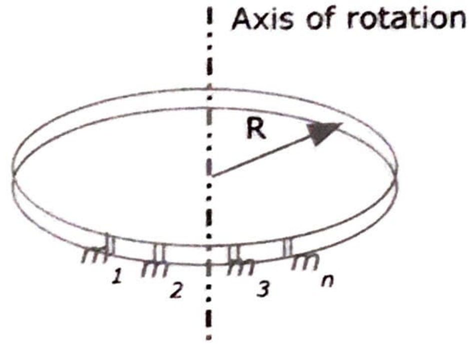


Figure 8.2 Thin ring

Solution

We can sub-divide the ring into *n* small segments whose average distance from the axis of rotation is *R*. Using equation 8.7.

$$I = m_1R_1^2 + m_2R_2^2 + m_3R_3^2 + ...m_nR_n^n$$

Where we have assumed that $R_1 = R_2 = ...R_n$.

$$I = (m_1 + m_2 + m_3 + ...m_n)R^2$$

$$\text{But } m_1 + m_2 + m_3 + ...m_n = M$$

$$\therefore I = MR^2$$

Thus we see that with simple geometrical shapes the value of the moment of inertia can be obtained more accurately by making the masses very small. This method may be used to find moments of inertia for simple geometrical shapes, but for complex shapes Integral Calculus methods are used.

Moments of Inertia of Simple Objects

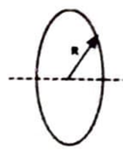
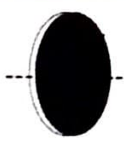
Object	Axis of rotation	Moment of inertia
Small mass in a circular path		mr^2
Hoop		mR^2
Solid disk		$\frac{1}{2}mR^2$
Solid sphere		$\frac{2}{5}mR^2$
Solid cylinder		$\frac{1}{2}mR^2$

Table 8.1

Example 8.2

A heavy solid metal wheel has a moment of inertia of 250 kgm^2. What is the needed torque to produce an acceleration of 3 rad/s^2?

Solution

Equation 8.5 gives torque as: $\tau = I\alpha$ $\tau = 250\text{kgm}^2 \times 3\text{rads}^2$ = 750 Nm

Example 8.3

A 1.5 kg grinding stone of 8 cm radius is turning at 130 rad/s. The driving motor is switched off and an axe is pressed tangentially against the grinding stone with a force of 2 N. How long will it the grinding stone to stop?

Solution

The grinding stone is a solid disc and its moment of inertia from table 8.1 is $\frac{1}{2}mR^2$.

$$I = \frac{1}{2}mR^2 = \frac{1}{2}\times 1.5\times 0.08^2 = 4.8\times 10^{-3}\ \text{kgm}^2$$

The torque the axe exerts on the grinding atone is given by equation 8.2.

$$\tau = -Fr = -2\times 0.08 = 0.16\ \text{Nm}$$

The minus sign on the force F symbolizes the slowing down effect of the force and therefore, the torque applied slows the grinding wheel. We need to find the deceleration of the grinding stone in order to get the time the grinding stone takes to stop.

The equation that links angular acceleration, torque and moment of inertia is equation 8.5 which can be re-written in terms of angular acceleration as:

$$\alpha = \frac{\tau}{I} = \frac{-0.16}{4.8\times 10^{-3}} = -33.33\ \text{rad/s}^2$$

Recall that the final angular velocity is given by the equation $\omega_f = \omega_i + \alpha t$. We set ω_f equal to zero since the grinding stone comes to rest.

$$0 = \omega_i + \alpha t \text{ or } \alpha t = -\omega_i$$

$$t = \frac{-\omega_i}{\alpha} = \frac{-130\text{rads}^{-1}}{-33.33\text{rads}^{-2}} = 3.90\ \text{s}$$

8.3 Combined Rotation and Translation

Let us consider a boy rolling a wheel on a level road. The wheel will be rotating and at the same time it will be moving horizontally along the road. If we take the wheel to be made of several small masses then each small mass is rotating at the same time it moving forward. We can conclude that the small mass has rotational kinetic energy and at the same time it has translational kinetic energy. To get the total energy of the wheel, we have to add the energies of all the small masses that make up the wheel. In this case we take the ground level as the reference point, in which case the wheel does not posses potential energy with respect to the on which it is rolling. If the wheel is rotating with angular speed ω, the tangential speed of each particle is v_T of a small mass at a distance of r from the centre of rotation is $v_T = r\omega$ (Equation 7.5). Suppose the small mass is m, its kinetic energy is:

$$\text{KE}_{\text{rot}} = \frac{1}{2}mv_T^2 = \frac{1}{2}mr^2\omega^2$$

$$\text{KE}_{\text{rot}}\ \text{all masses} = \sum\left(\frac{1}{2}mr^2\omega^2\right) = \frac{1}{2}\left(\sum mr^2\right)\omega^2 = \frac{1}{2}I\omega^2$$

$$\text{KE}_{\text{trans}} = \frac{1}{2}mv^2$$

Hence the total combined energy of rotation and translation of the mass is:

$$KE_{total} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 \qquad 8.8$$

Example 8.4

A solid disk starts rolling from rest at the top of a slope 12 m long and reaches the bottom 12 seconds later as shown in figure 8.4. What is the angle of the slope with respect to the horizontal? (Use the energy conservation method)

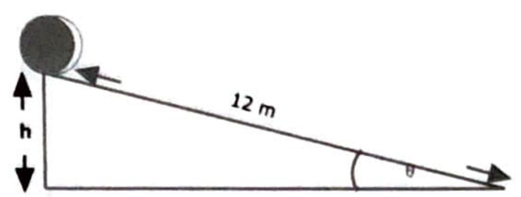


Figure 8.4 Solid disk rolling on the incline.

Solution

Let the length of the slope be L. At first we determine the final linear velocity v_f of the disk at the bottom and the time needed to reach the bottom.

From equations of linear motion:

$$L = \frac{(v_i + v_f)}{2}.t\text{, starts from rest therefore, } v_i = 0$$

$$\therefore v = \frac{2L}{t} = \frac{2\times12}{10} = 2.4\text{ m/s}$$

The moment of inertia for a solid disk $I_{disk} = \tfrac{1}{2}mR^2$ from table 8.1 and angular velocity $\omega = \frac{v}{R}$. Using the conservation of energy we have:

$mgh = \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2$, then we substitute for I and ω getting:

$$mgh = \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mR^2\right)\left(\frac{v_f^2}{R^2}\right)$$

$$gh = \tfrac{3}{4}v_f^2 \text{ or } h = \frac{0.75v_f^2}{9.8} = \frac{(0.75)(2.4^2)}{9.8} = 0.441\text{ m}$$

The angle of the slope with respect to the horizontal is then:

$$\grave{e} = \sin^{-1}\left(\frac{h}{L}\right) = \sin^{-1}\frac{0.441}{12} = 2.1^0$$

8.4 Angular Momentum

In our daily lives we not that rotating objects such fan blades, spinning grinding disks etc do not come to an abrupt stop when power is switched off. These objects continue rotating with decreasing speed until they eventually stop due to frictional forces, thus in the absence of friction we expect these objects to continue rotating or spinning forever. We can only explain this observation if we compare with objects performing linear motion that possess linear momentum. It can equally be said tha rotating objects also posses angular momentum that is a product of angular velocity ω and the object's moment of inertia$I$. It can be concluded that the conservsation of angular momentum tends to keep rotating objects in motion. Angular momentum has been given capital L as its symbol.

$$L = I\omega \qquad 8.9$$

Angular momentum is a vector quantity just like its linear equivalent. In the case of linear momentum we said that if no net force acts on an object then linear momentum is constant. We can similarly say that if there is no net torque (force) on a rotating object then its angular momentum is constant. The law of conservation of angular momentum can be stated as:

When the sum of exaternal torques acting on a system of particles is zero, the total angular momentum of the system remains constant.

$$\sum \tau = 0 \text{ then } I\omega = \text{constant, or}$$

$$I_i\omega_i = I_f\omega_f \qquad 8.10$$

This fact is exploited by ballet dancers when they need to vary their speed during performances. When they need to spin slowly they stretch out the arms thus increasing the moment of inertia$I$and reducing the angular speed ω, and when they need to speed up they bring the arms close to the body to reduce the moment of inertia and increase the angular speed. See illustration in figure 8.9.

Figure 8.9 Ballet dancer spinning on one leg.

Example 8.5

An ice-skater has a moment of inertia I = 3.5 kg.m² when her arms are extended and I = 1.2 kg.m² when her arms are at her sides. She develops an angular velocity of 3 rad/s when the arms are eextended, and then she suddenly drops them to her sides.

i) What is her final angular velocity?

ii) How much work was done in performing this move?

Solution

We use the conservation of angular momentum to find the final angular velocity by employing equation 8.10.

$$I_i\omega_i = I_f\omega_f$$

$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{3.5\text{kg.m}^2}{1.2\text{kg.m}^2}.3\text{rad/s} = 8.75\text{rad/s}$$

Note that the ice-skater's angular velocity increases three times as before whereas the momentum remains the same.

According to the work-energy theorem, any work done is equal to the change in her kinectic energy. We then calculate the change in in her kinetic energy before and after and find the the difference.

$$\text{KE}_i = \tfrac{1}{2}I_i\omega^2 = \tfrac{1}{2}(3.5)(3^2) = 15.75\text{J}$$

$$\text{KE}_f = \tfrac{1}{2}I_f\omega_f^{\,2} = \tfrac{1}{2}(1.2)(8.75^2) = 45.94\text{J}$$

$$\text{Work done, } W = \Delta\text{KE} = \text{KE}_f - \text{KE}_i$$

$$W = 45.94\text{J} - 15.75\text{J} = 30.19\text{J}$$

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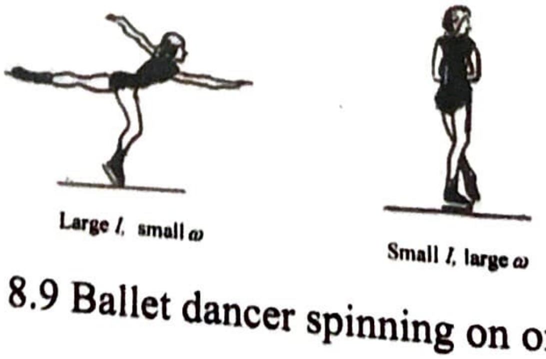

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ii) How much work was done in performing this move?

Solution

We use the conservation of angular momentum to find the final angular velocity by employing equation 8.10.

$$I_i\omega_i = I_f\omega_f$$

$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{3.5\text{kg.m}^2}{1.2\text{kg.m}^2}.3\text{rad/s} = 8.75\text{rad/s}$$

Note that the ice-skater's angular velocity increases three times as before whereas the momentum remains the same.

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$$\text{KE}_f = \tfrac{1}{2} I_f\omega_f^2 = \tfrac{1}{2}(1.2)(8.75^2) = 45.94\text{J}$$

$$\text{Work done, W=}\Delta\text{KE= KE}_f - \text{KE}_i$$

$$\text{W=45.94J-15.75J=30.19J}$$

Chapter 9

9.0 Equilibrium

9.1 Translational Equilibrium

Our earlier discussions were on the movment of objects when acted on by a net force. We now turn to look at objects that forces act on but are not moving. For such objects we can generally say that for an object at rest, it cannot have a a net force acting on it. We will first examine objects on which the forces are acting on the same point and are in the same plane. For such objects we can say that, if the vector sum of forces is equal to zero, then the object is in translational equilibrium.

$$\sum F = 0 \qquad 9.1$$

Translational equilibrium can also be obtained in cases where there is no linear acceleration for objects following a straight line path at constant speed.

The forces acting on an object in a plane are resolved into x and y components and summed up and for an object in translational equilibrium respective sums in the x and y directions are equal to zero. The vector sum in equation 9.1 is replaced by two scalar equations:

$$\sum F_x = 0 \text{ and } \sum F_y = 0 \qquad 9.2$$

The equation 9.2 is used in dealing with problems of translational equilibrium and is the condition for an object to be in static equilibrium. In working with this equation we have to follow some steps listed below:

1. Draw a freebody diagram indicating the forces exerted on the object and exclude forces the object exerts on anything else as such forces do not affect equilibrium.
2. Choose an appropriate coordinate system, resolve the forces of interest into components and attaching the correct signs on the components.
3. Sum the forces along each coordinate axis and set the sum thereof equal zero. For a 2-dimensional system this process will generate two equations which can be solved for the unknowns.
4. Use the known numerical quantities to get the required answers.

For example, let us consider a package of weight W suspended from a rigid beam by two strings of negligible weight as shown in figure 9.1. Resolve the forces (tension) acting in the strings.

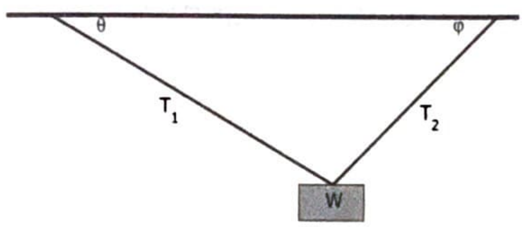


Figure 9.1 Weight suspended from a rigid beam by strings.

We can resolve the forces on free-body diagram as shown in figure 9.2.

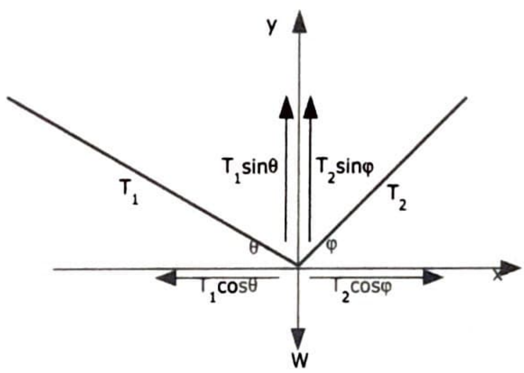


Figure 9.2 Resolution of forces at the centre of the weight.

The suspended weight W is in translational equilibrium, therefore, the sum of forces along the respective axes are as follows:

$$\sum F_x = -T_1 \cos\theta + T_2 \cos\varphi = 0 \qquad 9.3$$

$$\sum F_y = T_1 \sin\theta + T_2 \sin\varphi - W = 0 \qquad 9.4$$

If we know the angles θ, φ and the weight then we use equations 9.3 and 9.4 to find the tensions in the strings T_1 and T_2.

Exaample 9.1

In figure 9.1, let the hanging mass be 5 kg, the angle θ be 30^0 and the angle φ be 40^0. Find the tensions in the supporting strings.

Solution

From equations 9.3 we have that:

$$-T_1 \cos\theta + T_2 \cos\varphi = 0$$

$$\text{i.e. } T_2 \cos\varphi = T_1 \cos\theta$$

$$T_2 = T_1.\frac{\cos\theta}{\cos\varphi} = T_1.\frac{\cos 30^0}{\cos 40^0} = 1.13T_1 \text{ and}$$

from equation 9.4 we have that:

$$T_1 \sin\theta + T_2 \sin\varphi - W = 0$$

$$0.500T_1 + 0.643T_2 - 49 = 0$$

We eliminate T_l from the above by substitution:

$$0.500T_1 + (0.643 \times 1.13T_1) - 49 = 0$$

$$1.23T_1 - 49 = 0$$

$$T_1 = \frac{49}{1.23} = 39.8\,\text{N}$$

$$T_2 = 1.13T_1 = 1.13 \times 39.8 = 45.0\,\text{N}$$

Example 9.2

In figure 9.3, the system is in equilibrium and the pulleys are frictionless and the strings are of negligible mass. The weight of the object on the left is 100 N. Find the values of the weights of W_2 and W_3.

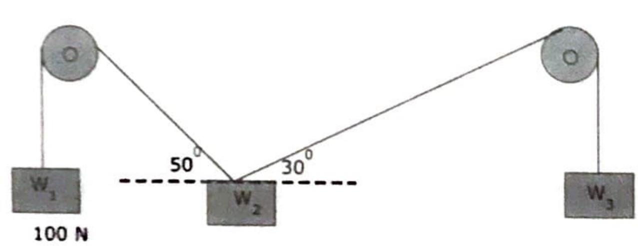


Figure 9.3 Three weights in equilibrium

Solution

We first resolve the forces (tensions) exerted by the weights on the suspending strings.

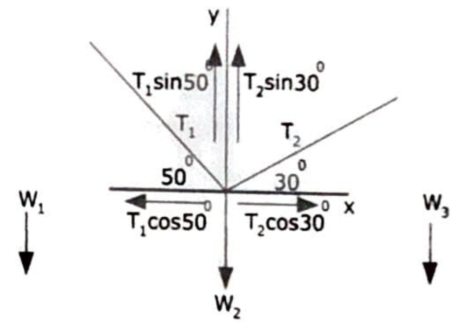


Figure 9.4 Resolution of forces about weight W_2.

Starting with weight W_1 we write theequations for equilibrium at each position.

$$\sum F_{y1} = T_1 - W_1 = 0 \text{ or } T_1 = W_1 = 100\,\text{N}$$

$$\sum F_{y2} = T_1 \sin 50^0 + T_2 \sin 30^0 - W_2 = 0$$

$$T_1 \sin 50^0 + T_2 \sin 30^0 = W_2$$

$$100\times 0.766 + 0.500T_2 = W_2$$

$$76.60 + 05.00T_2 = W_2$$

$$\sum F_x = T_2 \cos 30^0 - T_1 \cos 50^0 = 0$$

$$T_2 \cos 30^0 = T_1 \cos 50^0$$

$$T_2 = T_1 . \frac{\cos 50^0}{\cos 30^0} = 100.\left(\frac{0.643}{0.866}\right) = 74.25\,\text{N}$$

We substitute this value in the previous eaquation above to get W_2.

$$W_2 = 76.60 + (0.500\times 74.25) = 113.73\,\text{N}$$

Lastly we find the value of weight W_3.

$$\sum F_{y3} = T_2 - W_3 = 0 \text{ or } T_2 = W_3 = 74.25\,\text{N}$$

It is important to observe the correct signs on the various components used in the calculation.

9.2 Rotational Equilibrium

It is a common experience that if we apply a force tangentially to a wheel mounted on an axle on which it is free to rotate as in figure 9.5, the wheel will rotate in the direction of the applied force.

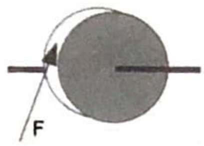


Figure 9.5 Force **F** appled tangentially to aa freely rotating wheel.

We note in this situation that the force **F** is unbalanced and therefore the wheel rotates. But suppose we applied a second force equal in magnitude on the opposite side of the wheel, figure 9.6. The wheel still rotates even though the forces are equal in magnitude and opposite in direction satisfying the condition for translational equilibrium $\sum F = 0$.

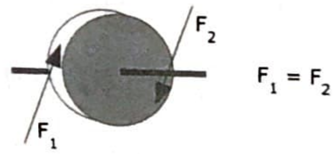


Figure 9.6

It follows that there must be another condition for the wheel to achieve static equilibrium. Note that forces $\mathbf{F_1}$ and $\mathbf{F_2}$in figure 9.6 produce torques that are in the same direction, therefore, the wheel will only be stationary if the torques are in opposing directions. The condition for the wheel to remain in rotational equilibrium is that the sum of torques acting on it is equal zero.

$\sum \tau = 0$ Condition for rotational equilibrium.

The convention used to assign signs to torques is that anti-clockwise torques are taken to be positive and clockwise torques are negative. In figure 9.7a, $F_1.r = \tau_1$ is negative and $F_2.r = \tau_2$ from equation 8.2 is positive and hence the sum is equal to zero.Figure 9.7b is illustrates the cancellation of the two torques applied on the wheel.

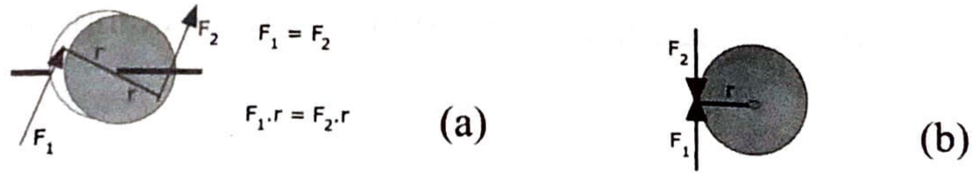


Figure 9.7 Torques applied at different points on the wheel.

It can be concluded from the above discussion that for an object to be in static equilibrium the forces must satisfy the two conditions:

$$\sum F = 0 \quad \text{(i)}$$

$$\sum \tau = 0 \quad \text{(ii)}$$

In general if the forces or torques do not lie in the same plane then sums in the three mutually perpendicular directions must be equal to zero for static equilibrium to be attained.

9.3 Lever Arm

In our discussion of work we defined torque τ as the product of the tangential force ***F*** and the radius in figure 8.1. It should be noted that the angle between the line of action of the force ***F*** is perpendular to the radius ***r***. In general a torque is calculated from the effective lever arm i.e. the perpendicular distance between the line action of a force and the centre of rotation as illustrated in figure 9.8 for an object whose length is L and the axis of rotation is x from F_1.

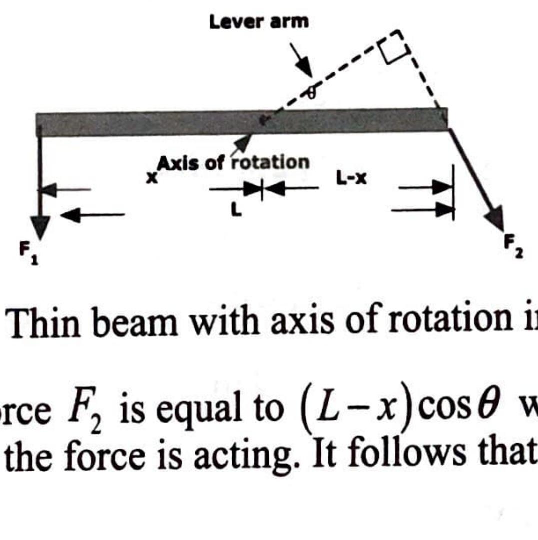


Figure 9.8 Thin beam with axis of rotation in the middle.

The lever arm for force F_2 is equal to $(L-x)\cos\theta$ where L is the length of the object on which the force is acting. It follows that the torque in this case would be given by:

$$\tau_2 = F_2 \times (L-x)$$

The object in figure 9.8 would be in rotational equilibrium if the clockwise torque τ_2 is counter balanced by the anticlockwise torque τ_1 i.e. $F_2 \times (L-x) = F_1 \times x$.

Example 9.3

A thin rod 1 m long has masses of 1 kg and 1.5 kg hanging at its ends as shown in figure 9.9. Find the point x m from the 1.5 kg mass at which the rod can be suspended without the rod tending to rotate. Assume the mass of the rod to be negligible.

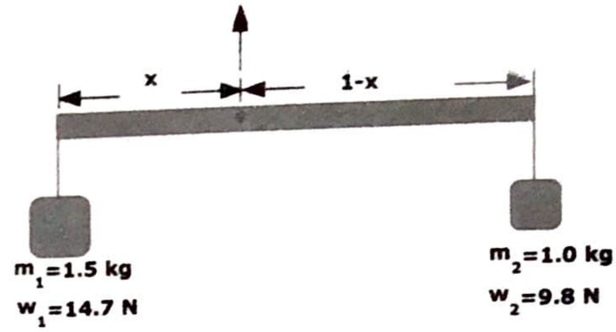


Figure 9.9 Weights hanging from a rod with off centre axis of rotation.

Solution

The condition for rotation equilibrium is that the sum of torques acting on the system should be equal to zero i.e. $\tau_1 + \tau_2 = 0$.

We first calculate the individual torques remembering to tag them with the correct signs. Recall that the anticlockwise torque is positive and the clockwise torque is negative.

$$\tau_1 = w_1 x = 14.7x \text{ Nm}$$

$$\tau_2 = w_2 x = -\left[(9.8)(1-x)\right]\text{Nm}$$

We then set τ_1 against τ_2:

$$14.7x = (9.8)(1-x)$$

$$14.7x = 9.8 - 9.8x$$

$$24.5x = 9.8$$

$$x = 0.4 \text{ m}$$

9.4 Centre of Gravity

The centre of gravity is defined as that point from which an object can be suspended without the object tending to rotate. In terms of rotational equilibrium the latter statement tells us that the clockwise and anticlockwise torques cancel out. Since an object is composed of several small elements, each element exerts a torque about whatever point an object is suspended. There exists only one point in the object about which all the torques cancel out in spite of its orientation. This unique point is the centre of gravity. For an object in equilibrium it is taken that all its weight acts through the centre of gravity. In a number of regularly shaped objects such as spheres, cubes, uniform rods, rings e.t.c. the centre of gravity is the geometrical centre. Note that the centre of gravity may lie within or outside the object.

Example 9.4

A 5.0 m ladder of weight 250 N leans against a smooth wall. The term "smooth" in this case implies a frictionless surface. The lower end of the ladder is 1.7 m away from the wall as shown in figure 9.10.

i) What is the horizontal reaction force F_1 that the wall exerts on the ladder?

ii) What is the total force F_2 that the ground exerts on the ladder?

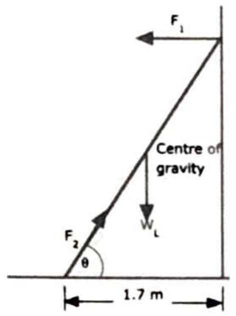


Figure 9.10 Ladder leaning against a smooth wall.

Solution

At first we draw free-body diagram of the ladder including the lever arms of the forces acting on it.

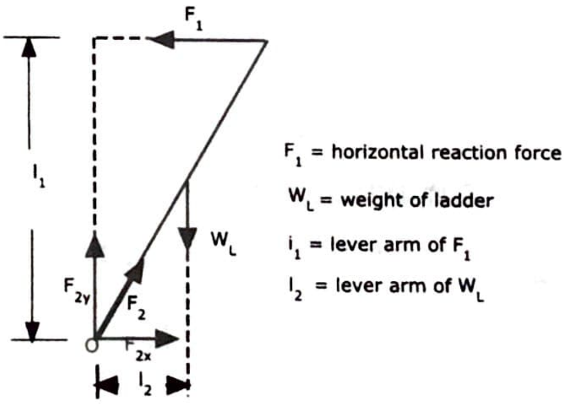


Figure 9.11 Resolution of forces about the bottom of the ladder.

The ground is not frictionless, so the force F_{2x} preventing the ladder from sliding is produced by the ground. The ladder is in equilibrium by meeting the two conditions for static equilibrium i.e. the $\sum F = 0$ and $\sum \tau = 0$. We start with the equations for translation equilibrium.

$$\sum F_x = 0$$

$$F_{2x} - F_1 = 0 \text{ or } F_{2x} = F_1$$

$$\sum F_{2y} = 0$$

$$F_{2y} + (-W_L) = 0 \text{ or } F_{2y} = W_L = 250\,\text{N}$$

We need to find the angle θ the ladder makes with the ground. We know the ladder is 5.0 m long and the bottom of the ladder is 1.7 m from the wall it is leaning against. We use trigonometry to work out the angle.

$$\cos\theta = \frac{1.7}{5.0} = 0.34$$

$$\cos^{-1}\theta = \cos^{-1} 0.34 = 70.1^{0}$$

To find the force $\boldsymbol{F_1}$, we need to use the second condition for static equilibrium ($\sum \tau = 0$). In this problem there are only two torques to consider if we place the axis of rotation as point O in figure 9.11.

$$\tau_1 = F_1 l_1, \text{ where } l_1 = 5.0 \sin 70.1^{0} = 4.7$$

$$\tau_1 = 4.7 F_1 \text{ N and}$$

$$\tau_2 = -W_L l_2, \text{ where } l_2 = \tfrac{1}{2}(5.0)\cos 70.1^{0} = 0.85$$

Where l_2 is the lever arm for W_l and W_l acts at a distance 0.85 m the from the axis of rotation O.

$$\tau_2 = -212.5\,\text{N}$$

Weset the sum of the torques equal to zero.

$$\sum \tau = 0 \text{ or } 4.7F_1 + (-212.5) = 0$$

$$F_1 = \frac{212.5}{4.7} = 45.2\,\text{N}$$

This is the horizontal force that the wall exerts on the ladder. We are left with the total force $\boldsymbol{F_2}$ that the ground exerts on the ladder. Since forces $\boldsymbol{F_{2x}}$ and $\boldsymbol{F_{2y}}$ are known, we can use Pythagorus theorem to find the total force the ground exerts on the ladder.

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2} = \sqrt{45.2^2 + 250^2} = 254\,\text{N}$$

Note that in this example we had only two torques acting on the ladder, however, if we had a man standing on the ladder we can have an an additional weight and torque that must be taken into account in formulating our equations.

Chapter 10

10.0 Properties of Matter

10.1 Mechanical Properties of Matter

The wide range of materials we encounter in everyday situations exhibit different properties in terms of their content, behavior under varying conditions, strength, state, etc. It is common knowledge that these materials are composed of matter. At a microscopic scale we talk of matter as a combination of atoms/molecules in certain configurations that make any given material. Matter exists in three forms i.e., as solids, liquids and gasses. These substances behave the way we see them due to the different forces acting between the atoms and molecules. Note that the temperature at which a substance is subjected to determines whether it will exist as solid, liquid or gas. In gasses the intermolecular forces between atoms/molecules are very weak and hence they move freely whereas in liquids/solids the forces are strong enough to constrain movement to a certain extent.

10.2 Temperature and Pressure Effects

The state of a substance depends on the temperature and pressure it is subjected to. For example if we take a common substance such as water, we observe that it solidifies at low temperature; becomes a liquid at ordinary temperature and turns into gas at high temperature. The identity of the molecules does not change in any of the three states. Pressure also affects the transitions into the above three states. Most solids have a well-defined structure in which the molecules are arranged in a well ordered pattern (crystalline structure). There are also solids in which there is no pattern or order (amorphous structure). Common salt (sodium chloride) is an example of crystalline solid in which the sodium and chlorine atoms are arranged in a well-defined pattern and glass is an example of a solid which has no ordered structure.

10.3 Density

Solid objects with the same dimensions made of different materials when placed on scale give different readings of the weight. The reason this difference arises is from the fact that the molecules are either tightly packed (densely) or loosely packed. The object with densely packed arrangement will weigh more than the loosely packed one. This characteristic is denoted by the density ρ, which is defined as a measure of mass per unit volume (in kg/m^3 or g/cm^3). Density is one of the properties of matter that is unique for each substance and may be used in identification of substances.

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{V} \qquad 10.1$$

The relative difference in heaviness of materials is due to their densities. For instance, for the same volume of lead and copper, lead has a greater mass than copper since lead has a higher density than copper. Numerically lead has a density of 11,300 kg/m^3 and copper has density of 8,890 kg/m^3.

Example 10.1

A 600 g block of a certain type of wood is 20 cm long, 15 cm wide and 10 cm thick. What is the density of this wood?

Solution

At first we find the volume of the wooden block.

$$\text{Volume} = \text{length} \times \text{width} \times \text{thickness}$$

$$V = (0.20)(0.15)(0.10) = 0.003\,\text{m}^3$$

$$\rho = \frac{m}{V} = \frac{0.60}{0.003} = 200\,\text{kg/m}^3$$

10.4 Elasticity

We know that in the absence of external forces acting on a solid material it maintains it shape and size. The application of external forces on a solid can lead to the alteration of its shape and size. It is observed that for some materials when the force causing the deformation is removed, the material regains its original shape and size. Materials which regain shape and size are said to be elastic. There are also other materials that remain permanently deformed after removal of deforming forces, such materials are said to be plastic.

Some common materials such as springs, rubber bands; etc. exhibit elasticity when stretched. However, these materials only regain shape and size when stretched within a limited range. Beyond the limited range, the material may get deformed permanently and if the stretching force is not removed it may lead to breakage. Experiments show that the amount of deformation is directly proportional to the applied force provided it is within the limit of elasticity.

10.5 Hooke's Law

Sir Robert Hooke investigated the elastic properties of materials and came up with the following law (conclusion):

When an elastic material is stretched or otherwise deformed, the amount of deformation is linearly proportional to the deforming force.

$F \propto \Delta x$, where F is the applied force and Δx is the amount of stretch.

The above relationship is re-written with a proportionality constant which is unique for each material as:

$$F = -k\Delta x \qquad 10.2$$

In equation (10.2) the negative sign is introduced to signify the fact that the restoration force is in the opposite direction to the amount of stretch Δx. Equation 10.2 holds for a limited range beyond which the proportionality does not hold as illustrated in figure 10.1.The region beyond the elastic limit is the region of plastic deformation.

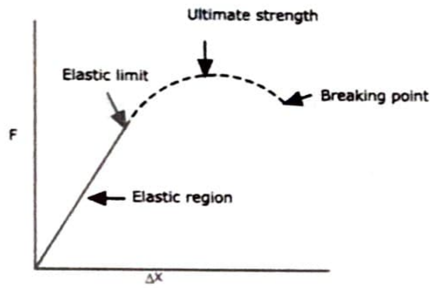


Figure 10.1Stretching force versus stretch

There are two terms that are used to describe the elastic properties of materials, namely, stress and strain. These are best described by referring to figure 10.2 where we have a wire of cross- sectional area A and length L.

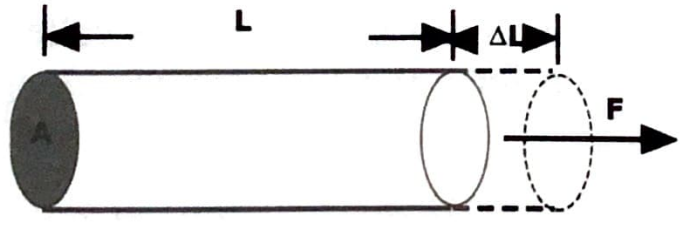


Figure 10.2 Wire under a stretching force.

When a force F is applied to the end of a wire of cross-sectional area A parallel to its length,

$$\text{Stress} = \frac{\text{force}}{\text{area}} = \frac{F}{A} \qquad 10.3$$

$$\text{Strain} = \frac{\Delta L}{L} \qquad 10.4$$

The quantities in the above equations are defined as follows:

F – force

A – cross sectional area

ΔL – change in original length

L – original length

We see from equation 10.3 that stress is a quantity that is related to the force responsible for the deformation and strain in equation 10.4 is a measure of the degree of deformation of the material. The units of stress are newtons per square metre N/m^2 and strain is a dimensionless quantity.

For small values of strain the stress is proportional to the strain and the constant of proportionality is called the elastic modulus ie,

$$\text{stress} = \text{constant} \times \text{strain} \qquad 10.5$$

$$\text{Elastic modulus} = \frac{\text{stress}}{\text{strain}} \qquad 10.6$$

10.6 Young's Modulus

The elastic modulus is a measure of a material's rigidity (stiffness). The elastic modulus in the region of proportionality is called Young's modulus when the force is a pulling force (tension) applied along the length of the material.

$$Y = \frac{F/A}{\Delta L/L} \qquad 10.7$$

We can consider Young's modulus as a measure of the resistance of material to change in its length.

There are two other types ofelastic moduli that measure the behaviour of solid materials, namely, the shear modulus and the bulk modulus.

Example 10.2

A copper wire of 1.5 mm diameter and 2.0 m length has a mass of 8.0 kg suspended from it. How much does this load stretch the wire given that Young's modulus for copper is 110 x 10^9 N/m^2?

Solution

Equation 10.7 gives Young's modulus as $Y = \frac{F/A}{\Delta L/L}$, which can be re-written in terms of the change in length ΔL.

$$\Delta L = \frac{L.F}{Y.A}$$

The radius of 1.5 mm diameter wire is $\frac{1.5\times10^{-3}}{2} = 7.5\times10^{-4}\,\text{m}$

$$\therefore \text{ cross-sectional area} = \pi r^2 = \pi\times(7.5\times10^{-4})^2\,\text{m}^2$$

$$\Delta L = \frac{2\times(8\times9.8)}{110\times10^9\times7.77\times10^{-6}} = 1.83\times10^{-4}\,\text{m}$$

$$\text{or } \Delta L = 0.18 \text{ mm}$$

10.7 Shear Modulus

A shear stress changes the shape of an object and not its volume. In solids the molecules are arranged in a regular pattern and we can consider that a solid material is composed of several layers. A shear stress arises when an object such as a rectangular block has the bottom surface fixed and a force parallel to its top surface is applied to it as in figure 10.3.

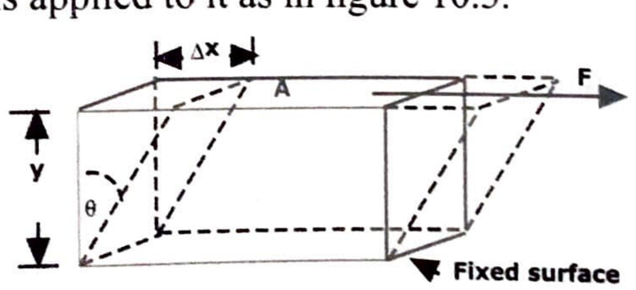


Figure 10.3 Solid block under a shear force.

As can be seen the shape of the block changes under a shear force from rectangular one to a rhombic shape. In figure 10.3, the angle θ is used as a measure of the distortion and is referred to as the angle of shear. The shear angle is normally very small so that its value in radians can be obtained from the ratio of the displacement of the the block's faces Δx to the distance y between them.

$$\grave{e} = \frac{\Delta x}{y} \qquad 10.8$$

If the area $A$to which the shear force is applied is large, then the resulting displacement of the top face is small. It follows that the shear stress by definition is:

$$S = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} \qquad 10.9$$

The shear strain is equal to the angle of shear θ which is given in equation 10.8. The shear modulus can be obtained from the definition of elastic modulus given in equation 10.6.

$$\text{Shear modulus} = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F/A}{\theta} = \frac{F/A}{\Delta x/y} \qquad 10.10$$

In figure 10.3, $\tan\theta = \frac{\Delta x}{y}$ and therefore we can express the shear modulus as:

$$S = \frac{F/A}{\tan\theta} \qquad 10.11$$

The higher the value of the shear modulus the more rigid the material is to shear stresses.

Example 10.3

Normal mild steel will yield (break) when subjected to a shear stress of 3.5×10^8 N/m². What is the force needed by a metal punching machine to make a 8.0 mm hole in a sheet 3.0 mm thick.

Solution

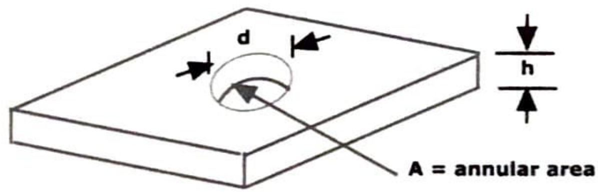


Figure 10.4 Steel plate with hole punched in it.

Note that a 8 mm diameter punching piston exerts a shear stress on cylindrical inner surface of the hole. The area we consider in this situation is that of the annular surface.

$$A = \pi dh = \pi \times 8.0\times10^{-3} \times 3.0\times10^{-3} = 7.54\times10^{-5}\ \text{m}^2$$

The maximum shear stress of mild steel is given as $3.5\times10^{8}\ \text{N/m}^2$.

$$\therefore\ F = S \times A \text{ from equation 10.9}$$

$$F = 3.5\times10^{8} \times 7.54\times10^{-5}\ \text{N} = 2.6\times10^{4}\ \text{N}$$

10.8 Bulk Modulus

A bulk stress reduces the volume of an object, for volume to reduce the object in question must be subjected to uniformly distributed forces from all directions as in figure 10.4. This could be the case for example a cube which is immersed in water at a great depth.

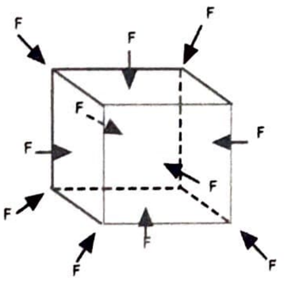


Figure 10.5 Cube of material immersed in water.

In this case we can consider the pressure to be uniform on all the faces of the cube. The compressive force per unit area *F/A* is uniform over the entire cube. Let the volume of the cube be *V* and shrinkage be ΔV, then we can say that:

$$\text{Volume stress} = \frac{\text{Force}}{\text{Area}} = -\frac{F}{A} \qquad 10.12$$

$$\text{Volume strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V} \qquad 10.13$$

It follows that the bulk modulus by definition is

$$\text{Bulk modulus} = -\frac{\text{Volume stress}}{\text{Volume strain}}$$

$$B = -\frac{F/A}{\Delta V/V} \qquad 10.14$$

We get from equation 10.14 that the relative change in volume for uniform compression is:

$$\frac{\Delta V}{V} = -\frac{1}{B} \cdot \frac{F}{A} \qquad 10.15$$

Where the negative sign has been introduced to take care of the reduction in volumedue to the increase in the compression force. Since the perpendicular stress *F/A* defines pressure *P*, we write the relative change in volume as:

$$\frac{\Delta V}{V} = -\frac{P}{B} \qquad 10.16$$

Examples of Elastic Modulus for selected materials.

Material	Young's (10^9N/ m^2)	Shear (10^9N/m^2)	Bulk (10^9N/ m^2)
Copper	110	42	140
Brass	91	35	61
Steel	200	84	160
Rubber	0.004	0.001	3
Glass	55	23	37
Tungsten	350	140	200
Water	-	-	2.1

Table 10.1

10.9 Pressure in Fluids

A fluid in our context is any substance composed of particles that are able to move freely and change ralative position without the separation of mass and easily yield to pressure. In other words substances that are capable of flowing like water are called fluids. The definition encompasses liquids such as water, gases and slow flowing substances such honey.

When an object is totally immersed in water at a given depth and is neither floating upwards nor sinking, we can say the object is in static equilibrium and hence it experiences equal forces from all directions. The forces it experiences are perpendicular to its surfaces that have a tendency of reducing its volume. The average pressure *P* in water at the level at which the object is submerged is the ratio of the perpendicar forces to its surface area.

$$P = \frac{F}{A} \qquad 10.17$$

Note that the outward pressure on any unit area at the same level on the walls is also uniform, as the force exerted by the water is perpendicular to the contact surfaces as shown figure 10.6. The net force on any volume element for water at rest is zero.

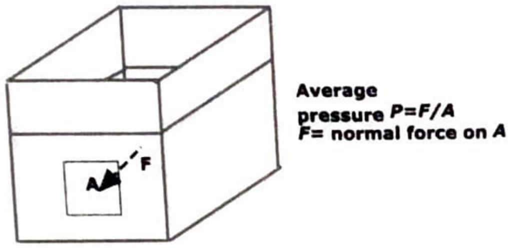


Figure 10.6 Container with water.

The definition of pressure then is force per unit area, it has units of N/m^2 which have been given a special name pascal (Pa).

10.10 Pressure at Depth

In a liquid at rest in a container, all parts of the the liquid must be in static equilibrium. It can further be stated that all points at the same depth must be at the same pressure. The latter fact can be confirmed by a simple experiment in which a cylindrical vessel is filled with water and holes are opened at the same level on either side, we observe that the jets of water land at the same distance from the vessel, figure 10.7.

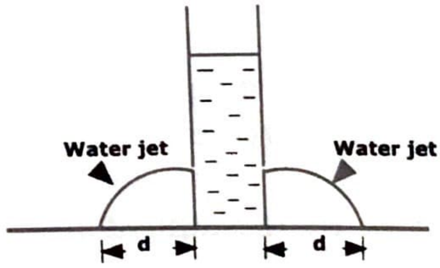


Figure 10.7 Cylindrical vessel with holes punched at same height.

At depth h below the surface, the pressure at that point is due to the weight column of liquid above it(figure 10.8).

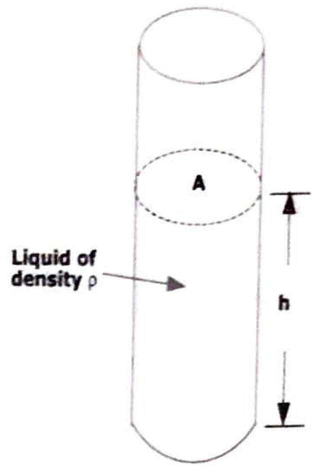


Figure 10.8 Cylindrical vessel with some liquid of hieght h.

Let M be the mass of the column h of the liquid, then its weight is the product of M and acceleration due gravity g and from equation 10.1 $M = \rho V$ hence the weight is:

$$W = Mg = \rho Vg \qquad 10.18$$

The pressure at the bottom surface that has an area A is given by:

$$P = \frac{F}{A} = \frac{W}{A} = \frac{\rho Vg}{A} \qquad 10.19$$

In figure 10.8 the volume of the column of liquid is the product of the area A and the height h. It follows that we can simplify equation 10. 19 by substituting for the volume.

$$P = \frac{\rho Vg}{A} = \frac{\rho(Ah)g}{A} = \rho gh$$

$$P = \rho gh \qquad 10.20$$

Equation 10.20 is used to calculate the pressure at specific depths in fluids of known density. Note that in this derivation, the surface are has been eliminated. In figure 10. 8 the vessel is open,the total pressure obtained inludes the air pressure exrted by the atmosphre. The total pressure is the sum of external pressure and the pressure due to the column of liquid.

$$P_{Total} = P_{external} + \rho gh \qquad 10.21$$

Example 10.4

The pressure at a certain depth in a lake is 2.0×10^5 Pa. The density of of water is 1000 kg/m^3 and atmospheric pressure at the surface of the lake is 1.01×10^5 Pa. How deep is this point?

Solution

The pressure at depth in this case is inclusive of the atmospheric pressure, therefore, we can use equation 10.21 to find the depth h.

$$P_{Total} = P_{external} + \rho gh$$

$$2.0\times10^5 = 1.01\times10^5 + (1.0\times10^3)(9.8)(h)$$

$$h = \frac{2.0\times10^5 - 1.01\times10^5}{1.0\times10^3\times9.8} = 10.10 \text{ m}$$

The point is 10.10 m below the surface of the lake.

10.11 Pascal's Principle

The pressure in a liquid depends on depth and the value P_{Total}, any additional pressure at the surface is transmitted to every point in the liquid. Pascal's principle states that:

The increase in pressure at any point in a confined incompressible liquid (fluid) results in the same pressure change at every point in the liquid.

This principle is used in hydraulic systems such as motor vehicle braking systems, hydraulic lifts used to raise heavy things such vehicles, hydraulic presses e.t.c. In practical systems such as hydraulic lifts, the design consists of two pistons that are interconnected as in figure 10.9. The force F_1 applied to the smaller piston is transmitted to the second larger piston which supports what is being lifted.

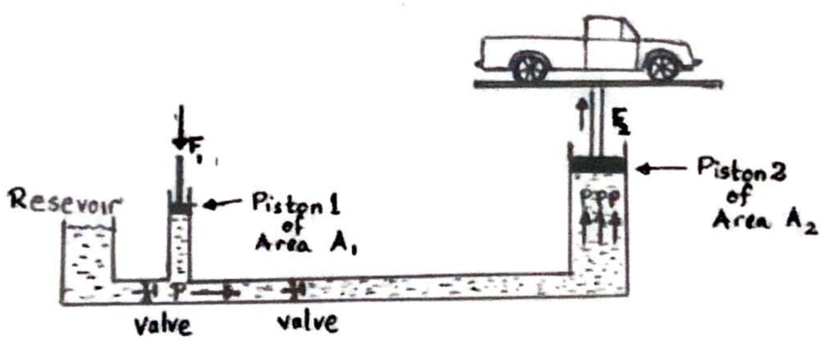


Figure 10.9 Pick-up truck on a hydraulic lift.

The force F_1 applied on small piston generates a pressure P_1. This pressure can be calculated from:

$$P_1 = \frac{F_1}{A_1} = P$$

According to Pascal's principle, pressure P_2 at the large piston is the same as at the small piston. It follows that:

$$F_2 = P_1 A_2 = \frac{F_1}{A_1} \times A_2 = \left(\frac{A_2}{A_1}\right) F_1 \qquad 10.22$$

Since A_2 is much larger than A_1, the upward force F_2 is very large as shown in equation 10.22 and this is used to lift heavy loads. Thus the hydraulic lift uses the ratio of the piston areas to generate large forces on the lifting side.

Example 10.5

A hydraulic vehicle lift is designed to lift vehicles with a maximum gross mass of 3500 kg. The cross section area of the piston of the lifting platform is 400 cm^2. What would be the maximum pressure that the smaller piston will have to withstand?

Solution

The pressure on a piston carrying a load is given by equation 10.17.

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{3500 \times 9.8}{400 \times 10^{-4}} = 8.56 \times 10^5 \text{ Pa}$$

The smaller piston has to withstand the pressure equal to that of the platform piston according to Pascal's priciple of 8.56×10^5 Pa.

10.12 Archimede's Principle

We experience in daily encounters with water that some objects tend to sink in water whereas others float on top and others maybe partially submerged. These observations were explained by a Greek mathematician and physicist, Archimedes. He stated that:

An object immersed in a fluid is buoyed up by a force which is equal to the weight of the fluid displaced.

$$\text{Buoyant force} = \text{Weight of displaced fluid}$$

$$F_B = W_{fluid} \qquad 10.23$$

Any object placed in a fluid experiences an upward force which tends to reduce its weight. The effect of the upward force is called buoyancy. For example, balloons float in air, boats and ships float in water because they are buoyed up (supported) by the fluid in which they are.The phrase "weight of fluid displaced" means the the weight of the fluid that would spill out if the container was full to the brim before an object is placed in the fluid. Objects will float if the buoyant force F_B is greater than the weight W_{fluid} of displaced and they will sink into the fluid if F_B is lesser than W_{fluid}. The latter statement is true for floating or sunk objects. It follows that the buoyant force is dependant on its strength compared to other froces acting on the object.

Example 10.6

A pontoon 15 m long and 8 m wide is ferrying cars across a river. When two cars are driven onto the pontoon, the pontoon sinks 5 cm into the water. What is the combined weight of the two cars?

Solution

We first take note that the weight of the displaced water is equal to the additional bouyant force needed to keep the pontoon afloat and this is equal to the combined weight of the cars from equation 10.23.

$$\text{Volume of water displaced} = \text{Area} \times \text{depth sunk}$$

$$V = (15 \times 8) \times 0.05 = 6.0\,\text{m}^3$$

$$\text{Weight of displaced water} = \rho A g$$

$$W = 1000 \times 6 \times 9.8 = 58{,}800\,\text{N}$$

Example 10.7

A balloon filled with helium has a volume of 1500 m³. The density of helium is 0.178 kg/m³, and has a gross weight (balloon material, ropes and basket) of 3500 N. What is the maximum load that this balloon can lift? The average density of air is taken as 1.293 kg/m³.

We first find the total weight of helium and the balloon W_{He}.

$$W_{He} = \rho V g = (0.178)(1500)(9.8) = 2616.6\,\text{N}$$

Let W_T be the total force acting downwards, then:

$$W_T = W_{He} + W_{Balloon} = 2616.6 + 3500 = 6116.6\,\text{N}$$

The lifting force (buoyant force) is due to the displaced air.

$$F_B = \rho V g = (1.293)(1500)(9.8) = 18963\,\text{N}$$

The maximum load the balloon can take is the difference between the buoyant F_B and the total weight W_T. Let L represent the maximum load, then:

$$L = F_B - W_T = 18963 - 6116.6 = 12846.4\,\text{N}$$

Chapter 11

11.0 Thermal Properties of Matter

11.1 Introduction

Ordinarily when we discuss temperature, we associate it with how cold or hot an object is relative to some standard reference device such as a thermometer. We humans evaluate the hotness or coldness of objects with a sense of touch, however, this method cannot be relied on when we need reproducible results since two different people will have different sensations on how hot or cold an object is. The sensation of hotness or coldness is produced by heat energy.

11.2 Thermometer

A thermometer is a device used to measure the temperature of an object which in turn can be used to quantify the heat of the object. To construct temperature measuring devices, many physical properties which vary linearly with temperature change are used such as volume of a liquid, length of metallic objects, the electrical resistance of a wire and so on. These properties have become handy in the construction all kinds of thermometers. The commonest thermometers are based on the expansion of mercury or alcohol in sealed and evacuated capillary tube with a reservoir bulb at one end as shown in figure 11.1.

Figure 11.1Thermometer

11.3 Temperature Scales

Common thermometers are graduated using two reference points. In the SI unit system, the standard sea level points are the freezing point of water taken as 0 ^{0}C and the boiling point of water taken as 100 ^{0}C. This scale is referred to as the *Celsius* scale and was originally called the *Centigrade* scale (because of the 100 divisions between 0 and 100). Another scale not commonly used in physics is the *Fahrenheit* scale whose lower point is the freezing point of water is taken as 32 ^{0}F and the boiling point of water is taken as 212 ^{0}F. There are 180 divisions between the two points.

A third scale is the *Kelvin,* or absolute scale used mostly in the scientific world. The SI unit on this scale is Kelvin (K) and it does not use the term degree as with the previous two scales. The freezing and boiling points of water are 273.15 K and 373.15 K. This scale moves away from defining scale based on the property of a substance. It uses a single point the triple point of water. This is the temperature at which water, water vapour and ice can co-exist in thermal equilibrium. The triple point of water occurs at 0.01 ^{0}C and a pressure of 0.006 atm. On this scale, a temperature of -273.15^{0}C is taken to be absolute zero and there can be no lower temperature than this theoretically.

11.4 Temperature Scales Relationship

Suppose T_c is the temperature of an object on the Celsius scale and T_F the temperature on the Fahrenheit scale, the relationship is:

$$\frac{T_C}{100} = \frac{T_F - 32}{180} \qquad 11.1$$

We can simplify equation 11.1 to:

$$T_C = (T_F - 32)100/180 = (T_F - 32)5/9 \qquad 11.2$$

Example 11.1

A weather man predicts the temperature as 104 ^{0}F at noon in Lusaka. What is the temperature on the Celsius scale?

From equation 11.1, $T_C = (T_F - 32)5/9$

$$T_C = (104 - 32)5/9 = 40\,^0\text{C}$$

However, the relationship between the Celsius scale and the Kelvin scale is much simpler as the magnitude of 1^{0}C is equal to 1 K. This being the case, we simply add 273.15 to the temperature in Celsius.

$$T = T_c + 273.15 \qquad 11.3$$

Note that in most calculations the fraction on the Kelvin is dropped.

11.5 Thermal Expansion of Solids, Liquids and Gases

We know from observations that matter exists in three states: solid, liquid and gas. A chunk of ice (solid) left at surrounding (ambient) room temperature melts into water, and if the water (liquid) is heated it gradually evaporates changing into vapour (gas). The three states of matter mentioned above react differently when subjected to heat. In the discussion of the thermometer we noted that the mercury or alcohol in the capillary expands when heated and hence we can conclude that liquids expand when heated. This behavior can be extended to all the three states of matter.

11.6 Solids

A rod made of metal will change its dimensions when heated. The change is more noticeable in its linear dimensions.

Suppose we have a rod of some material whose length is L_o, its change in length noted is ΔL after its temperature changes by ΔT. The equation that governs these three quantities is given by:

$$\Delta L = \alpha L_0 \Delta T \qquad 11.4$$

where α is called the coefficient of linear expansion. It is a constant whose value depends on the type of material the rod is made of. It is also true that if a material is cooled by the same temperature change ΔT, then it will shrink in length by ΔL. From equation 11.4 we see that $\Delta L = L - L_0$ and $\Delta T = T - T_0$. Since the expansions of various materials are unique to each material, it follows that for the same temperature change the expansions will be different according to the coefficient of linear expansion of each material. Equation 11.4 in terms of the new length L can also be written as:

$$L = L_0 (1 + \alpha \Delta T) \qquad 11.5$$

Table 11.1 has some selected coefficients of linear expansions.

Table 11.1: Coefficients of Linear Expansion of Some Common Materials

Substance	Coefficient ($\times 10^{-6}\ /^{0}C$)
Aluminium	23
Brass	19
Concrete	12
Glass	3.2
Steel	12
Silver	51
Copper	17

Example 11.2

What is the increase in length of a copper rod 50 cm long at 20 ^{0}C when its temperature is raised to 50 ^{0}C?

Solution

The coefficient of linear expansion of copper from the table is $17\times10^{-6}\ /^0\,C$. From equation 11.4

$$\Delta L = \alpha L_0 \Delta T = (17\times10^{-6}\ /^0\,C)(0.5m)(50^0C\text{-}20^0C)$$

$$= 2.55 \times 10^{-4}\ m = 0.255\ mm$$

Example 11.3

A steel shaft has a diameter of 30.00 mm. The inner diameter of a steel pulley to be fitted on the shaft is deliberately made smaller so that it can be shrunk in place for a tight fit over the shaft. If the pulley's inner diameter is 29.86 mm at 20 ^{0}C. Find the temperature to which it must be heated to fit over the shaft. The coefficient of linear expansion of steel is $12\times10^{-6}\ /^0\,C$.

Solution

Let the inner diameter be L_o, then L_o= 29.86 mm and $\Delta L = (30.00 - 29.86)\,\text{mm} = 0.14\,\text{mm}$. From equation 11.4 we have:

$$\Delta T = \frac{\Delta L}{\alpha L_o} = \frac{0.14mm}{(12\times10^{-6}\ /^0\,C)(29.86\ mm)}$$

$$= 390.71\ ^0C$$

The required temperature is therefore 20 ^{0}C + 390.71 ^{0}C = 410.17 ^{0}C.

11.7 Fluids

A fluid is a substance that has no fixed shape and yields easily to external pressure; a gas or (especially) a liquid. When such substances are subjected to increase in temperature, we expect the volume of such substances to increase. We can say they undergo volume expansion. An equation similar to equation 11.4 used for volume expansion is:

$$\Delta V = V\beta\Delta T \qquad 11.6$$

where β is the coefficient of volume expansion. Table 11.2 is gives values of coefficients of volume expansion for some substances.

Table 11.2: Coefficients of Volume Expansion of Some Selected Materials

Substance	Coefficient ($\times 10^{-6}/^0C$)
Glycerin	485
Mercury	182
Water	210
Air	3670*
Methyl Alcohol	1134

*At constant pressure

Example 11.4

A certain liquid has a volume of 25.000 cm^3 at 20 ^0C. When the temperature of the liquid is raised to 40 ^0C, its volume is found to be 25.567 cm^3. What is the coefficient of volume expansion? In addition identify this liquid from table 11.2.

Solution

We can re-arrange equation 11.6 as follows:

$$\beta = \frac{\Delta V}{V \Delta T}$$

$$= \frac{0.563}{(25)(40-20)} {}^0C = 1134 \times 10^{-6}/^0C$$

Checking this value in table 11.2, we find it corresponds to methyl alcohol.

11.8 Gases

Gases in general have some special properties that are very different from solids and liquids. The thermal expansion of gases is very large in comparison to solids and liquids and in addition the volume change of a given mass of gas is very considerable when subjected to pressure changes. For us to state the physical condition of a sample of gas we need to take into account the pressure, volume and temperature.

The three parameters, namely, temperature, pressure and volume in relation to gases have been extensively studied experimentally. These studies have resulted in three important relationships between these parameters:

- How the pressure and volume of a mass of gas relate to each other at constant temperature.
- How the temperature and volume of a mass of gas relate to each other at constant pressure.
- How the temperature and pressure of a mass of gas relate to each other at constant volume.

11.9 Constant temperature

Empirical studies have shown that at a given temperature the volume (V) of a given mass of gas is inversely proportional to its pressure (P). That is

$$V \propto \frac{1}{P} \qquad 11.7$$

Equation 11.7 is normally written at constant temperature as:

$$PV = \text{constant} \qquad 11.8$$

Since $P_1V_1 = \text{constant}$, and $P_2V_2 = \text{constant}$ where the constant is the same. Suppose that for a gas P_1 and V_1 represent the initial pressure and volume, and P_2 and V_2 its final pressure and volume respectively, then we can say that:

$$P_1V_1 = P_2V_2 \qquad 11.9$$

A gas which obeys the condition above is said to be an ideal gas. This ideal behavior breaks down under certain extreme conditions due physical limitations.

11.10 Constant Pressure

It is observed that at constant pressure, the volume of a given mass of gas is directly proportional to its absolute temperature (Kelvin). The relationship can be expressed as:

$$V \propto T$$

or

$$\frac{V}{T} = \text{constant} \qquad 11.10$$

It follows that

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad 11.11$$

If a graph of the volume of a given mass of gas versus temperature (Celsius) at constant pressure is plotted, it gives a straight line whose intercept is at -273.15 ^{0}C as shown in figure 11.2.

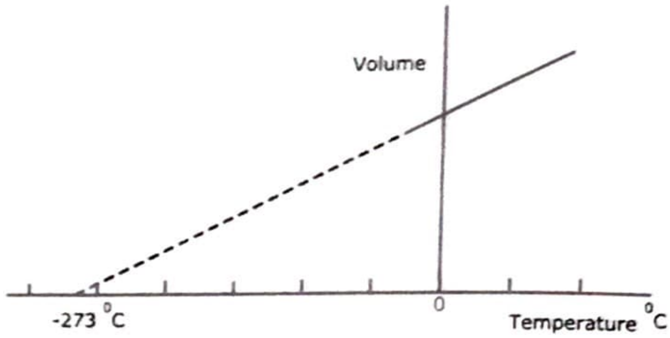


Figure 11.2: Volume-temperature plot of a gas at constant pressure

The plot tells us that the volume of the gas at this temperature will be zero. However, this is not so in reality as the gas will freeze into a solid well before reaching this point. The intercept temperature is called absolute zero (0 K).

11.11 Constant Volume

It has been observed experimentally that at constant volume, the pressure of a given mass of gas is directly proportional to its absolute temperature.

$$P \propto T$$

or

$$\frac{P}{T} = \text{constant} \qquad 11.12$$

and it follows that

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad 11.13$$

11.12 The Ideal Gas Equation

An ideal or perfect gas should obey the previously stated observations. However, in reality we find that gases do not follow the expected trend at all temperatures. In general gases liquefy well above absolute zero and as such they deviate from ideal behaviuor.

The three gas equations, namely, 11.9, 11.11, and 11.13 can be summarized into one ideal gas equation.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \qquad 11.14$$

In all these equations the temperature must be in Kelvin. Equation 11.4 as a direct consequence can be written:

$$\frac{PV}{T} = \text{constant} \qquad 11.15$$

A more common general form of equation 11.15 that involves the number moles n of a given quantity of gas and the universal gas constant R is given as follows:

$$PV = nRT \qquad 11.16$$

where R is 8.314 J/(mol.K) and the quantity of gas $n = \frac{m}{M}$, where m is the mass of gas sample and M is the atomic or molecular mass of the gas.

11.13 Mole and Avogadro's Number

Experimentally it has been determined that 12 g ofcarbon-12 isotope contain 6.02214×10^{23} atoms and this number is called Avogadro's number N_A. It follows that if a substance has N_A atoms or molecules, then it is a gram-mole of that of that substance or simply a mole.

To find the number of moles n in any substance, we divide the number of atoms or molecules N in a substance by the number of atoms or molecules per mole N_A (Avogadro's number):

$$n = \frac{N}{N_A} \qquad 11.17$$

It should be noted that although the mole is defined in terms of carbon-12 isotope, the general idea of a mole may be applied to any number of similar items. For example, one mole of molecular oxygen contains 6.022×10^{23} oxygen molecules. We can also find the number n of moles in substance from its mass. The product of the atomic/molecular mass and Avogadro's number gives us the atomic/molecular mass of a mole of the substance. Thus to get the number of moles n of a substance, we can multiply the numerator and denominator of the right-hand side of equation 11.17 by the mass m of a single atom or molecule, expressed in grams:

$$n = \frac{m_p N}{m_p N_A} = \frac{m}{M}$$

$$n = \frac{m}{M} \qquad 11.18$$

Where m is mass of the substance and M is the atomic or molecular mass of the substance.

Example 11.5

A gas tank contains 0.15 kg mass of oxygen. Calculate the number of moles in the tank, given that the atomic mass number of oxygen is 16.

Solution

Oxygen in nature exists as diatomic molecule hence, the mass number of O_2 is 2 times the mass of one atom.

$$M = 2 \times 16 = 32 \text{ g/mol}$$

$$\text{Number of moles } n = \frac{m}{M}$$

$$n = \frac{150\text{g}}{32\text{g/mol}} = 4.69 \text{ mol}$$

Example 11.6

A small 500 cm³ gas container is filled with pure nitrogen gas at a pressure of 2.5 atm and is at a temperature of 25 ⁰C.

i) Calculate the number of moles of the mass of nitrogen gas in the container. Given that 1 mole of N_2 = 28 g at 1 atm = 1.013×10^5 Pa.

ii) If the temperature of the gas is raised to 90 ⁰C, what will be the new pressure?

Solution

The ideal gas law states that $PV = nRT$, therefore, knowing *P*, *V* and *T*, we can calculate the amount of nitrogen in moles. We re-write this equation in terms of *n*.

$$n = \frac{PV}{RT} = \frac{(1.013\times10^5\text{Pa})(5.00\times10^{-4}\text{m}^3)}{(8.134\text{J}/(\text{K.mol}))(298\text{K})} = 0.021 \text{ mol}$$

When the temperature rises to 90 ⁰C (363 K), the new pressure in the gas container will be:

$$P = \frac{nRT}{V} = \frac{0.021\text{mol}\times8.134\text{J/K.mol}\times363\text{K}}{5.00\times10^{-4}\text{m}^3} = 1.24\times10^5 \text{ Pa}$$

Alternatively, we calculate the new pressure using equation 11.14:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

But $V_1 = V_2$, therefore:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\therefore P_2 = P_1 . \frac{T_2}{T_1} = (1.013 \times 10^5 \text{ Pa})\left(\frac{363\text{K}}{298\text{K}}\right) = 1.24 \times 10^5 \text{ Pa}$$

11.14 Heat

In our daily encounters we come across situations in which there is flow of heat from one object to another. For example when one wraps a palm on a mug of hot coffee, the palm feels the warmth from the mug and depending on the temperature of the coffee the heat may become unbearable. We can look at this situation as a case in which the hot mug is transferring heat from it to the palm and surroundings. This process can only be explained if we assume that there is a flow of heat energy from the hot object (the mug) to the cooler object (the palm). If this process is observed for a longer time, we find that the coffee mug's temperature will go lower as the temperature of the palm rises. We can conclude that there is a flow of heat energy from the mug to the palm. Additionally, this process is governed by the temperature difference between the objects and experiments can be performed to justify this statement. Suppose we put a hot object in contact with a cold object and we leave them like that for a long time, if we check the temperatures of the two objects we will find that they stabilize at some common lower temperature. We can say that the objects have reached thermal equilibrium. In short there is energy flow from the hot object to the cold object until they attain the same temperature. The energy flow between the two objects is called heat or heat energy.

We can define heat as the flow of energy from one system to another by virtue of the temperature difference between them.

11.15 Units of Heat

The old unit of heat energy is the calorie (cal) which is defined as the amount of heat energy required to raise the temperature of 1 g of water by 1°C. However, it should be noted that there is a small variation of the specific heat capacity with temperature. It has been observed that the specific heat capacity of water varies by 1% in the temperature range between 0 °C to 100 °C. Hence the precise definition of calorie is taken as the heat needed to raise the temperature of 1 g of water from 14.5 °C to 15.5 °C. Since heat is a measure of quantity of energy transferred, it was later decided that it should have an equivalent unit to work whose unit in the SI system is the joule (J). It follows that the calorie in terms of the joule is:

$$1\ \text{cal} \equiv 4.186\ \text{J} \qquad 11.19$$

11.16 Heat Capacity

The term heat capacity has to do with the amount of heat a substance can store or absorb. Ordinarily we have observed that different substances absorb different quantities of heat when subjected to same conditions that provide them with heat energy. For example, if we have two identical pans, in one we fill it with sand and the other water; then we place these pans in the sun for the same period of time. It will be found that the pan with water will be much cooler than the pan with sand; this physically means that the temperature of the sand rises more than that of the water. This observation reveals that water requires much more heat to raise its temperature to the same level as the sand. We can conclude that water has a higher capacity of absorbing heat energy than sand.

Since the heat capacity varies with substance under consideration, we normally define a quantity called the specific heat capacity. The specific heat capacity of a substance is the heat capacity per unit mass, usually measured in joules per kilogram per degree Celsius. The symbol for specific heat capacity is c.

$$c = \frac{Q}{m\Delta T} \qquad 11.20$$

Where Q is the quantity of heat energy, m is the mass of the substance and ΔT is the change in temperature. The unit of the specific heat capacity of a substance in the SI system of units is J/kg.^{0}C. However, other unit used for specific heat capacity is calorie per gram per degree Celsius (cal/g.^{0}C). One can calculate the amount of heat required to change the temperature of substance up or down by re-writing equation 11.20 as:

$$Q = mc\Delta T \qquad 11.21$$

Equation 11.21 implies that the quantity of heat Q is directly proportional to the change in temperature but in practice this is for a limited range of temperature.

Table 11.3: Specific Heat Capacities of Some Substances

Substance	Specific Heat Capacity, c J/(kg.^{0}C)
Aluminium	900
Copper	387
Steel	452
Silver	234
Ethyl Alcohol	2430
Glycerin	2410
Water	4186

Example 11.7

How much heat does a 30 g block of aluminium absorb as its temperature rises from 25 ^{0}C to 100 ^{0}C? The specific heat capacity of aluminium is 900 J/(kg.^{0}C).

Solution

We use equation 3.21 to work out the amount of heat energy needed.

$$Q = mc\Delta T$$

$$Q = (0.030\text{kg})\left(900\text{J/(kg.}^0\text{C)}\right)\left(100^0\text{C-}25^0\text{C}\right)$$

$$Q = 2025 \text{ J}$$

Example 11.8

An amount of heat is added to a mass of aluminium ($c = 0.215$ cal/g.^{0}C), and its temperature rises to 60 ^{0}C. Suppose that the same amount heat is added to silver of the same mass ($c = 0.056$ cal/g.^{0}C). How much will the temperature of the silver rise?

Solution

In this example the quantity of heat energy Q is the same for both. We can set the Q quantities against each other.

$$mc_{Al}\Delta T_{Al} = mc_{Ag}\Delta T_{Ag}$$

or

$$\Delta T_{Ag} = \left(\frac{c_{Al}}{c_{Ag}}\right)(\Delta T_{Al}) = \left(\frac{0.215}{0.056}\right)(60^0\text{C}) = 230.4^0\text{C}$$

Note from this example that the temperature of silver rises to almost four times that of aluminium for the same quantity of heat input. The physical meaning of this observation is that aluminium has higher capacity of absorbing heat than silver.

11.17 Calorimetry

Calorimetry is a method used in measuring the specific heat capacities of both solids and liquids. In the simplest method, the substance whose specific heat capacity is to be determined is raised to some temperature. The substance is quickly transferred into an insulated vessel containing cold water of known mass and temperature after which temperature readings are taken until equilibrium is reached (the water temperature stops rising). The vessel must have good insulation properties so that no heat is lost to the surroundings. Under such conditions the system can be assumed to be an isolated system. Such vessels are known as calorimeters and the evaluation done on the heat transfers is known as calorimetry. Figure 11.3 shows a typical set up of a calorimeter which essentially is an insulated vessel like a flask for storing hot or cold liquids.

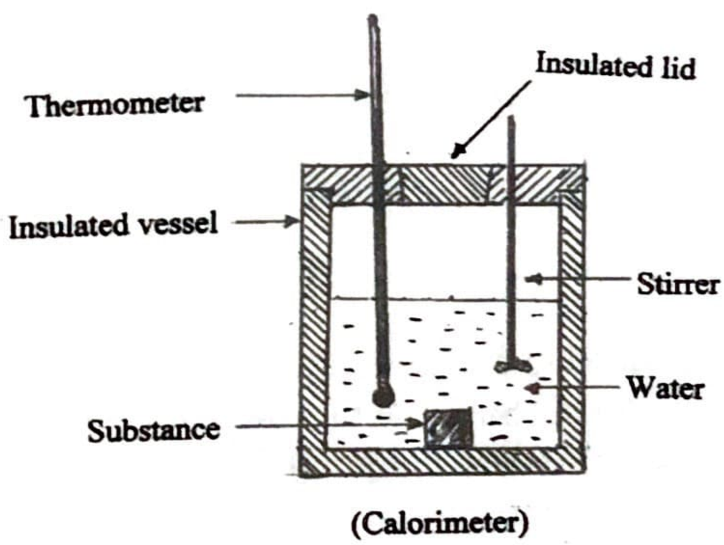


Figure 11.3: Calorimeter

In such an isolated system we can apply the principle of conservation of energy which requires that the heat energy lost by the hot substance is equal to the heat energy gained by the water and the vessel. The principle can be stated in equation form as:

$$Q_{cold} = -Q_{hot} \qquad 11.22$$

The term Q_{cold} is positive because heat energy is entering the water and the vessel whereas Q_{hot} is negative because heat energy is leaving the hot substance. In general, hot must balance cold at all times so that

$$Q_{cold} + Q_{hot} = 0$$

Example 11.9

A 150 g chunk of unknown material at a temperature of 95 ^{0}C is quickly placed in a thermos flask containing 300 g of water at a temperature of 25 ^{0}C. The system reaches an equilibrium temperature of 27.2 ^{0}C. What is the specific heat capacity, c_m, of the unknown material if we neglect the heat capacity of the flask?

Solution

The water gains heat energy Q_{cold}, while the chunk of material loses heat energy -Q_{hot}. We use equation 11.22, and expand it using equation 11.21 to solve for the unknown material's specific heat capacity c_m.

$$Q_{cold} = -Q_{hot}$$

$$m_w c_w (T - T_w) = -m_m c_m (T - T_m)$$

$$c_m = \frac{m_w c_w (T - T_w)}{-m_m (T - T_m)}$$

$$c_m = \frac{(0.300)(4186)(27.2 - 25.0)}{(0.150)(27.2 - 95)} \text{ J/kg.}^0\text{C}$$

$$271.7 \text{ J/kg.}^0\text{C}$$

Here T is the equilibrium temperature, whereas T_w and T_m are the initial temperature of the water and the material respectively.

11.18 Change of Phase

The addition or removal of heat does not always result in a temperature change. For example, if you have a water-ice mixture at a temperature of 0.0^0 C and leave it in the sun while monitoring the temperature. It will be observed that as the ice gradually melts the temperature of the mixture will remain static at zero in spite of the heat input from the surrounding. The temperature will only start rising when all the ice has melted. The heat from the surrounding is used to melt and later it causes the rise in temperature. Note that the substance (water) in this observation exists in more than one phase i.e. as a liquid as well as a solid. If we leave the substance long enough we note that the level of the liquid will start falling. The fall in the level is due to evaporation of the water molecules. This latter process involves the transformation of water from a liquid state to a gaseous state. The stages mentioned above involve water existing as a solid, liquid and gas.

It should be recognized that substances (matter) can change from one state to another, and heat plays a role in the change. These changes of state take place when heat is added or subtracted from a substance. A solid will melt or fuse into a liquid when supplied with sufficient heat, while a liquid can solidify (freeze) into a solid when heat is removed. In a similar manner, a liquid will evaporate into a gas if heat is supplied, whereas the gas can be liquefied (condense) when heat is removed from it. These phase changes take place at definite temperature and pressure for the majority of substances that have a crystalline structure. A few substances that are termed as not true solids such changes take place gradually. For example, glass, wax, butter etc. are not true solids as they have an amorphous structure, the molecules in these substances do not have a definite pattern.

11.19 Melting and Freezing Points

The melting point is the unique temperature at which a solid changes into a liquid. The process of melting takes place at a well-defined temperature and it remains constant until the entire solid is liquefied.

The freezing point is the temperature at which a liquid changes into a solid. It also takes place at definite temperature.

The changes of state discussed above are shown in figure 11.3

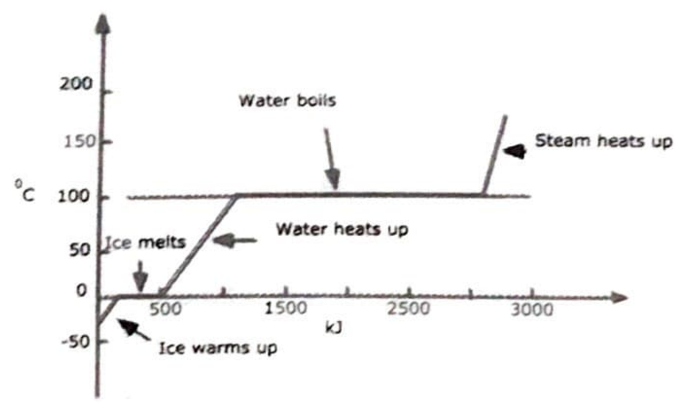


Figure 11.4: Phase change processes

In the case of ice at some temperature lower than 0 °C, the ice has first to heat up to 0 °C before the ice starts melting as shown in figure 11.4. When the ice starts melting the temperature will remain the same until all the ice has melted, this is the change of phase region. Almost all solid materials follow this behaviuor followed by the stage at which the temperature starts rising again. For water the temperature will rise until it reaches 100 °C where it remains constant until all of it changes into steam at 100 °C (another region of change of phase). Beyond this region the temperature of the steam will start rising. As can be seen from the above illustration different substances have different unique temperatures at which they melt (fuse) and boil.

11.20 Latent Heat of Fusion

In order to melt a substance such as ice, a given quantity of thermal energy to melt a unit mass of the ice is required, the flat portion shown in figure 11.4. This quantity of energy is required to break-down the crystalline structure of the ice. The same amount of thermal energy will be given off in changing water back into ice. The thermal energy required to change a unit mass of a substance from the solid phase to the liquid phase is called the latent heat of fusion H_f. The quantity of heat to be supplied or removed (in melting or solidifying) of a substance is

$$Q = mH_f \qquad 11.23$$

where m is the mass of the substance and subscript f denotes fusion. The SI unit of the latent heat of fusion is J/kg.

11.21 Latent Heat of Vaporization

To change a liquid such as water to a vapour completely, a given quantity of energy to vaporize a unit mass of water is required. The amount of thermal energy used to change a unit mass of a substance from a liquid phase to a gaseous phase will be the same amount released when the substance condenses back to a liquid. The thermal energy required to change a unit mass of a substance from the liquid phase to the gaseous phase is called the latent heat of vaporization H_v. The quantity of heat to be supplied or removed (in vaporization or condensation) of a substance is

$$Q = mH_v \qquad 11.24$$

where m is the mass of the liquid and subscript v denotes vaporization.

Table 11.4: Latent Heats of Fusion and Vaporization of Some Selected Substances

Substance	Heat of Fusion (kJ/kg)	Heat of Vaporization (kJ/kg)
Aluminium	396	10,500
Ammonia	332	1,370
Benzene	126	394
Copper	205	4,790
Iron	267	6,290
Mercury	11	295
Silver	105	2,370
Water	335	2,260

Example 11.10

A 60 g chunk of ice at 0 ^{0}C is left in a container at room temperature. After sometime it is observed that all the ice has melted. How much thermal energy did the ice get from the surroundings for it to melt?

Solution

In order for the ice to melt the ice has to get energy from the surroundings which can be calculated from equation 11.23 and using the fusion constant for water from table 11.4. At first convert the mass of ice to kilograms

$$Q = mH_f$$

$$Q = (0.06\text{kg})(335\text{kJ/kg}) = 20.1\text{J}$$

Example 11.11

At a birthday party, a 500 g piece of ice at -12 ^{0}C is placed in 2.8 kg of Orange drink at 22 ^{0}C. At what temperature and in what phase will the final mixture be? The Orange drink can be considered to be practically water. Ignore any heat flow to the surroundings, including the container. Given that the specific heat capacity of water is 4184 j/kg, specific heat capacity of ice is 2100 J/kg and the heat of fusion for ice is 335 kJ/kg.

Solution

We first have to verify whether it will be a mixture of ice and Orange drink before applying the law of conservation of energy. The subscripts in the equations *w* and *I* represent water and ice respectively.

To cool the Orange drink from 22 °C to 0 °C requires:

$$m_w c_w \Delta T_w = (2.8\text{kg})(4184\text{J/kg}^0\text{C})(22^0\text{C}) = 2577344.4\text{J}$$

To raise the temperature of ice from -12 °C to 0 °C requires:

$$m_i c_i \left(0^0 - \left(-12^0\right)\right) = (0.5\text{kg})\left(2100\text{J/kg.}^0\text{C}\right)\left(12^0\text{C}\right) = 12600\text{J}$$

To melt the piece of ice into water at 0° C requires the heat of fusion given by:

$$m_i H_f = (0.5\text{kg})\left(335\times10^3\,\text{J/kg}\right) = 167500\text{J}$$

The total energy gained by ice from -10° C to water at 0° C is:

$$\text{Energy gained} = 12600\text{J} + 167500\text{J} = 180100\text{J}$$

Since the energy needed to cool the Orange drink from 22 °C to 0 °C is much greater than the energy needed to melt the ice, the final the final temperature T will be between 0 °C and 22 °C. We can calculate this temperature by applying the law of conservation of energy.

$$\text{Heat gained} = \text{Heat lost}$$

(Q to raise ice to 0°) + (Q to change ice to water) + (Q to raise melted ice to T) = (Q lost by drink)

$$12600\text{J} + 167500\text{J} + (0.5\text{kg})(4184\text{J/kg})(T\text{-}0^0\text{C}) = (2.8\text{kg})(4184\text{J/kg})(22^0\text{C} - T)$$

$$180100 + 2092T\text{J} = 25773.44\text{J} - 11715.2T$$

$$T = \left(\frac{154326.56}{13807.2}\right)^0\text{C} = 11.2^0\text{C}$$

11.22 Heat Transfer

In cooking we normally place a pot on the source of heat energy such as a cooker plate. The heat from the cooker plate is somehow transferred to the contents of the pot. Gradually the temperature of the contents will start rising. The temperature rises because there is a net flow of heat energy from the hot cooker plate to the cool pot and its contents. We can conclude that there is heat energy transfer due the difference in temperatures between the cooker plate and the pot. Equally if one placed the hand near the cooker plate without touching it, a sensation of warmth (hotness) will be felt.

The question to be answered is how to explain the heat energy transfer from one object to the other. Our starting point will be to examine the events at a molecular scale. It is a fact that as an object's temperature rises, the kinetic energy of its atoms/molecules increases and they begin to vibrate more and more about some mean position; in so doing they relay these vibrations to the adjacent atoms/molecules and hence transfer the energy. This first scenario applies to solids, however, the second scenario is in fluids where the transfer of kinetic energy is through collisions of atoms/molecules. There is another transfer that does not involve the flow of energy through physical media, for instance, the warmth experienced from the sun through a glass window even though the objects involved that is the sun, a glass window, and a human body are not in contact with each other. The processes by which heat energy is transferred from one object to the other are classified into three categories: conduction, convection and radiation.

11.23 Conduction

It is common experience that when one dips a cold spoon in hot water and holds the handle for some time, the spoon handle gradually becomes hot. The heat energy transferred from the water flows through the spoon handle to the hand. The heat transfer through the spoon handle is known as conduction subsequently to the hand. In conduction the atoms of the material are in "fixed" positions but relay the heat energy through vibrations to the cooler neighbours which in turn start vibrating because of the increased energy. The process is repeated throughout the material with the heat energy moving in the direction of the colder section. This process can occur in solids, liquids, and gases. Thus the transfer is a result of inter-atomic interactions in most materials. In metals there are free electrons which drift through the material and are responsible for most transfer of heat energy between atoms.

There are four factors that govern the rate at which heat is conducted through a slab of material i.e. the temperature difference between the hot and cold side, the thickness of the material, the area in contact, and the nature of the material. Let us consider a slab of material as shown in figure 11.4.

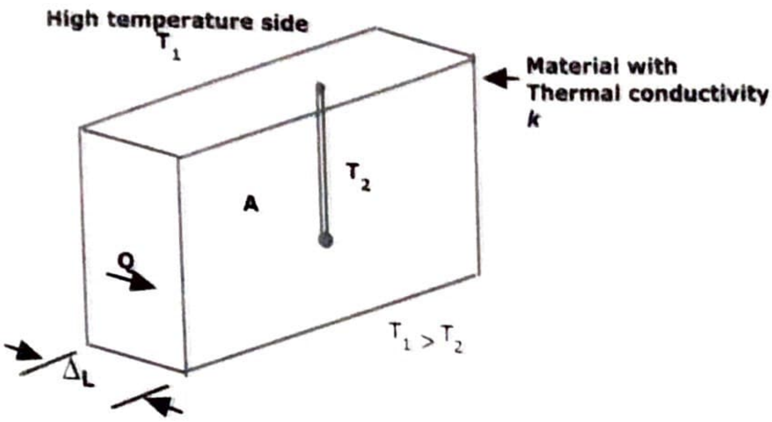


Figure 11.4 Slab of material

The difference between the high temperature side and the low temperature side effects the flow rate of heat energy proportionately to the difference. The effect of the temperature difference is to increase the flow rate as the difference increases. When the thickness ΔL is large the rate at which heat energy Q flows from one side of the slab to the other is low, which practically means that there is an inverse relationship between the heat flow and the thickness ($\Delta Q/t \propto 1/\Delta L$). The area A of the slab also proportionately increases the amount of heat energy transmitted across the thickness of the slab of material. Lastly, the material of the slab has a direct bearing on the flow of energy in it as different materials have varying thermal conductivities. For example, metals have good thermal conductivities compared to wood or glass. The above observations are summarized in equation 11.23.

$$\frac{Q}{\Delta t} = \frac{kA\Delta T}{\Delta L} = kA\left(\frac{\Delta T}{\Delta L}\right) \qquad 11.23$$

where k is the thermal conductivity and the quantity in brackets is called the thermal gradient, being one of the quantities that determine the rate of flow of heat energy. The units of the rate of flow of heat energy are watts (W) if the area A is in metres squared, L is in metres and the temperature T in Kelvin (K).

The value of the thermal conductivity is large for good thermal conductors and it is small for bad thermal conductors. Table 11.4 gives values of some selected materials.

Table 11.4: Thermal Conductivities* of Some Materials

Material	Thermal Conductivity, ($W/K.m$)
Copper	390
Aluminium	240
Silver	420
Hard wood	0.15
Concrete	1.1
Glass	0.80
Water	0.60

*Measurements made around 20 ^{0}C.

Note that good thermal conductors are also good conductors of electricity and poor thermal conductors are poor conductors of electricity.

Example 11.12

Find the amount of heat transferred into a room in 2 hours through a large glass window 1.5 m high, 3 m long and 6 mm thick if the outside temperature is 10 ^{0}C and the inside is 23 ^{0}C.

Solution

Since we want to find the quantity of heat lost through the glass window, we re-write equation 11.23 as:

$$Q = kA\left(\frac{\Delta T}{\Delta L}\right)\Delta t$$

We convert non-SI quantities to SI unit and then use them in the above equation.

$$Q = (0.80)(1.5\times3)\left(\frac{296-283}{0.006}\right)(2\times3600)$$

$$= 5.61\times10^7 \text{ J}$$

Convection

Liquids and gases (fluids) are by nature very poor conductors of heat energy; therefore, there must be another method by which heat is transmitted in fluids. The transfer of heat in a fluid is best explained by convection. In convection the transfer of heat in a fluid is done by the actual movement of the heated fluid from one location to another. The heated fluid carries with it the heat energy. The flow of the fluid is from a high temperature region to a low (cold) temperature region.

We can understand better this mode of heat transfer if we consider a pot of water being heated on a hot plate. Initially the pot and the water are cold, as the hot plate starts to heat up it transfers heat to the metal pot by conduction and in turn this heat is transferred to the water in contact with the pot. The heated water molecules expand and become less dense. These molecules begin to rise within the water because of the reduced density. The vacancy left by the hot water is replaced by cold water from above. The overall effect is that currents of water flow are set up within the fluid. The fluid flow current occurring in such a case is called convection current. A similar type of event occurs in a room being heated by a room heater but in this case the air in the room circulates in a similar pattern where hot air rises and cooler air descends to the bottom to replace the rising heated air. Figure 11.5 shows the typical pattern of convection currents set up in a pot of water being heated on a hot plate.

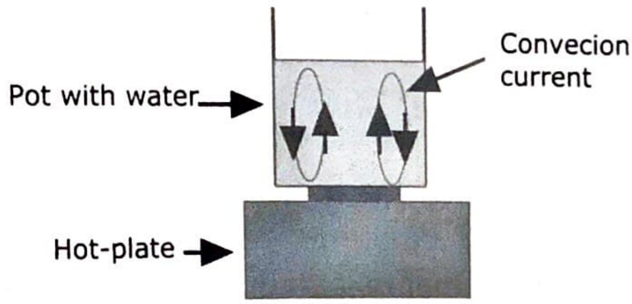


Figure 11.5 Pot of water on a hot plate

11.24 Radiation

It is a well-known fact that we receive heat energy from the Sun through empty space (vacuum). We can conclude from this observation that there must be another mode of transfer of heat that does not require a medium of transmission. The heat energy from the Sun is radiated in all directions and the Earth captures part of this radiant energy, the energy transmitted in this manner is called radiation.

11.25 Properties of Thermal Radiation

Objects emit thermal radiation on account of their possession of thermal energy, that is, they transmit heat to the surroundings through the process of radiation. Radiant energy has an electromagnetic nature and is transmitted at the speed of light in a vacuum. It behaves in a similar manner to light, it can be reflected, refracted and it can be polarized. The intensity of thermal radiation falling on a given surface area is inversely proportional to the square of the distance of the surface from the source. It obeys the inverse square law for thermal radiation.

All objects emit radiation in form of electromagnetic waves. The rate at which the radiation is emitted has been found to be proportional to the fourth power of the temperature in Kelvin. However, apart from emitting radiation, objects do also absorb radiation in amounts dictated by the temperature difference between the surroundings and the object. The net flow of radiant energy is in the direction of the lower temperature. It should be noted that good emitters of radiant energy are also good absorbers, and a poor absorber is also a poor emitter. In practice it has been observed that matt (not shiny) black surfaces are good absorbers and are also good emitters. In contrast shiny surfaces such as polished silver are poor absorbers of radiant energy and are also poor emitters. Note that the type of surface influences the rate at which radiation is emitted as well as it is absorbed. All these factors have led to the formulation of an empirical equation that gives the quantity of radiant energy either emitted or absorbed. Stephan-Boltzmann law of radiation equation reads:

$$Q = \sigma e T^4 A t \qquad 11.24$$

Where $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4 is the Stephan-Boltzmann constant, e is the emissivity factor, T is the temperature in Kelvin, A is the surface area, and t is the time in seconds. Note that σ is a universal constant as it applies to all materials irrespective of the nature of their surfaces. The values of the emissivity factor vary between 0 and 1. For a perfect emitter e has the value 1, and for shiny surfaces like a mirror it is nearly zero. The colour black is associated with almost complete of visible light; therefore, such a surface is called a black body and will have a value of emissivity almost equal to one.

Example 11.13

A standard tungsten filament light bulb develops a temperature of 2400 K during operation. It has coiled filament 540 mm long and a diameter of 0.046 mm. If it operated for 1 hour and assuming its emissivity to be 0.85. Calculate the quantity of radiant energy it radiates in his time interval.

Solution

We use equation 11.24 to work out the radiated heat energy; however, we do not have the surface area of the filament that is radiating. This is calculated from the given dimensions.

$$A = 2\pi rl = 2\times\pi\times 1.445\times 10^{-4} 0.58 = 0.84\,\text{m}^2$$

$$Q = \sigma e T^4 A t = (5.67 \times 10^{-8})(0.85)(2400^4)(0.84)(3600)$$

$$= 4.84\times 10^9\,\text{J}$$

Example 11.14

Calculate the rate of emission of thermal energy from example 11.11.

Solution

We simply re-write equation 11.24 as $\frac{Q}{t} = \sigma e T^4 A$ and solve it or divide the answer of example 11.11 by time.

$$\frac{Q}{t} = \frac{4.84\times 10^9}{3600} = 1.34\times 10^6\,\text{W}$$

Chapter 12

12.0 Thermodynamics

12.1 Introduction

Thermodynamics is primarily associated with heat and temperature and their relation to energy and work. It was developed in a quest to improve the efficiency of the early steam engines. In general terms the basic concern is the transformation of thermal energy (heat) into mechanical energy. It follows that a device or a system that converts heat into mechanical energy is called a heat engine. Devices or systems may be simple or complex. The measurable quantities used to describe a system are normally temperature, pressure and volume. In addition quantities such as internal energy, heat, work and entropy are also used. For example, n moles of gas which is in equilibrium in closed container will have a definite temperature, pressure and volume, such a system will be said to be in a thermodynamic state. Thus the variables describing a given thermodynamic state are called state variables. The properties of a given thermodynamic state will remain the same as long as the state variables have the same values. An additional important quantity that characterizes a system is internal energy (U). The internal energy constitutes of the sum of all the kinetic and potential energies possessed by its atoms or molecules. A system's temperature is an indicator of its internal energy.

12.2 Zeroth Law of Thermodynamics

When two systems are each in thermal equilibrium with a third, then they are also in thermal equilibrium with each other. The latter statement is called the zeroth law of thermodynamics. For example if the temperature of an object measured with a thermometer is found to be the same when a second object's temperature is measured, we can conclude that the objects are in thermal equilibrium with each other.

12.3 First Law of Thermodynamics

In discussing thermodynamics laws we consider heat as a form energy which it must conserved like its mechanical equivalent. If a system is in a given thermodynamic state then it can be stated that it has a definite amount of internal energy. When heat flows into a system it can do two things, one it can increase the internal energy of the system and secondly it can provide energy to the system allowing it to do work to the surroundings. This law is an extension of the law of conservation of energy that includes heat energy and internal energy of the system under consideration. The first law of thermodynamics says that the amount of energy Q going into a system is equal to the sum of the increase in internal energy ΔU and the external work done W.

$$Q = \Delta U + W \qquad 12.1$$

It follows that a system in a given state has a definite amount of internal energy. When heat flows into a system two are likely to happen:

i) The internal energy of the system increases or

ii) The heat may provide energy for the work to be done the surroundings.

For example, if we have a system comprising of a gas enclosed by a movable piston in cylindrical vessel with a closed end see figure 12.1, and then heat is added to the system. The expanding gas will move the piston and thus the gas does work on the piston. The piston moves against an external force. Suppose heat was removed from the enclosed gas we expect the piston to move inwards. The latter situation can be equated to the surroundings doing work on the system, in which case the work will be negative.

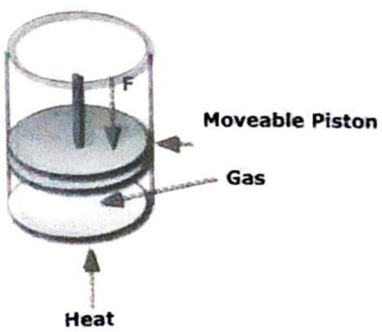


Figure 12.1 Gas tight moveable piston in a cylinder.

In general many other systems give us similar results, we can conclude that the heat added to the system should be equal to the increase in internal ΔU plus the external work done by the system W. It follows that the first law of thermodynamics is a law of conservation of energy which includes thermal energy. Convention has it that when heat flows into the system the quantity of energy Q is positive and when the heat flows out of the system then quantity of energy Q *is* negative. For example in figure 12.1 if the piston is pushed downwards by an outside force F the work done is negative. We can make an analogy with the definition of work in mechanics which states that:

$$\text{Work} = \text{Force} \times \text{Displacement} \times \cos\theta$$

Where the angle θ is the angle between the force vector and the displacement vector. Suppose that the piston is pushed by a force F downwards and it moves a distance Δy the work done W is:

$$W = F\Delta y\cos\theta = F\Delta y\cos 0^\circ = F\Delta y \qquad 12.2$$

This result is because cos0° is equal to one as the force vector and displacement vector are in the same direction. In figure 12.2 the piston is displaced by an amount Δy and if the cross section area of the cylinder is A then the volume increase is the product of this area and Δy.

$$\Delta V = A\Delta y \qquad 12.3$$

The pressure generated by the force F (arising from the weight of the piston) is $P = F/A$. This equation can be written in terms of the force as $F = PA$ and substituting in equation 12.2 we get that work by the expanding gas is:

$$W = F\Delta y = PA\Delta y = P\Delta V \qquad 12.4$$

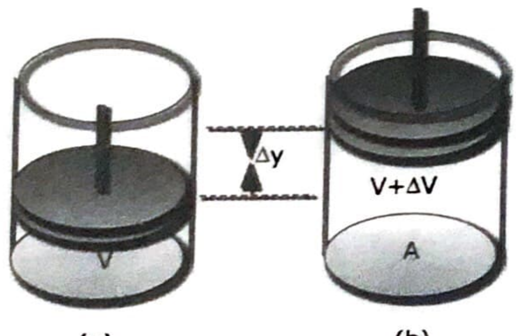


Figure 12.2 Gas expansion in a cylinder

During expansion the volume occupied by the enclosed gas increases and the work done is positive, whereas when the gas is compressed there is volume reduction i.e. ΔV is negative, hence the work done is negative. In the case of applying constant pressure on the gas, the work done is simply calculated from the equation:

$$W = P\Delta V \qquad 12.5$$

12.4 Constant Pressure

In the case where expansion takes place under constant pressure, namely, isobaric process the work done can be calculated using equation 12.5. This process can be plotted on graph of pressure versus volume (P-V diagram) as in figure 12.3.

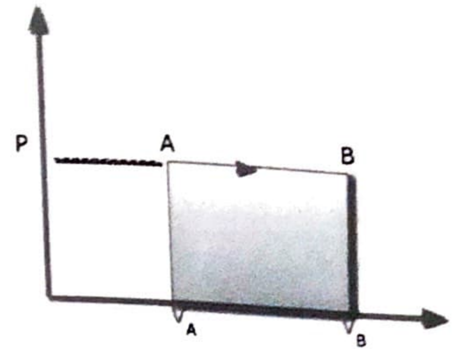


Figure 12.3 Pressure-volume plot at constant pressure.

The expansion takes place at a constant pressure P and the volume increases from V_A to V_B then the work done in this case is:

$$W = P\Delta V = P(V_B - V_A) \qquad 12.6$$

It should be noted that in equation 12.6 the term $P(V_B - V_A)$ is the area under the process path. In general the area under the process path is equal to the work done. In the case where the process is reversed (compression) the area remains the same but the work done and the work is taken to be negative.

12.5 Non-constant Pressure

The work done during a non-constant pressure process can also be worked out by finding the area under the process path. However, it is tedious for such a process as we may have to use some form of crude integration of the area on the P-V diagram as illustrated in figure 12.4.

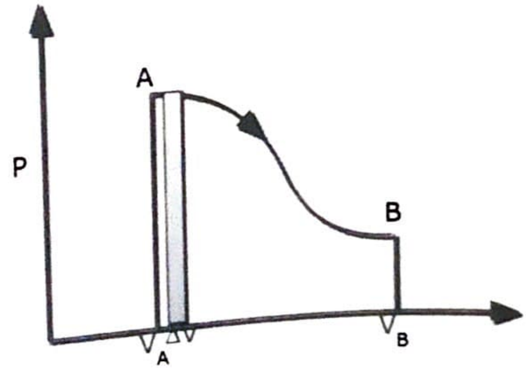


Figure 12.4 Pressure-volume plot for changing pressure.

Considering the small shaded strip in figure 12.4, we can assume that the pressure over ΔV is constant and the work done is:

$$W = P\Delta V$$

The total work done will be equal to the sum of several such strips taken over the interval V_A to V_B. Taking much smaller strips will yield a more accurate answer of the work done from A to B.

Example 12.1

A gas undergoes a process in which it is expanded from point A to B as shown in Figure 12.5. Calculate the work done by the gas.

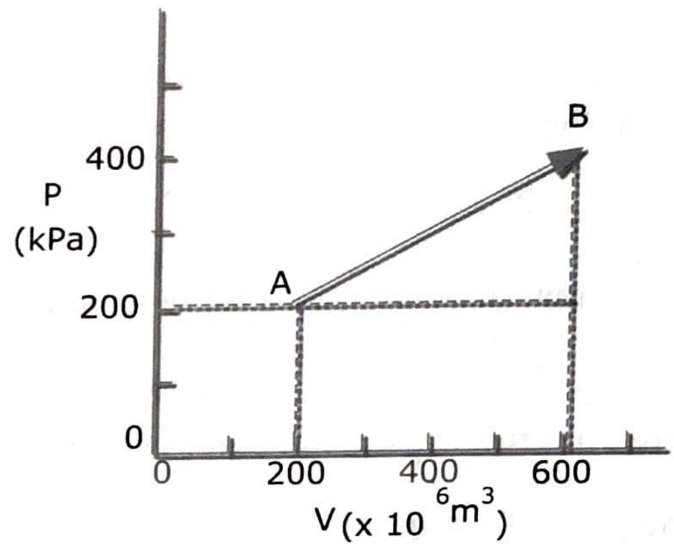


Figure 12.5 PV diagram

Solution

The work done by the gas is equal to the area under the process path *AB* and is equal to the sum of the area of the triangle and the rectangle in figure 12.5.

Area of triangle = ½ base x height

$$= \tfrac{1}{2}\left(600\times10^{-6} - 200\times10^{-6}\right)\text{m}^3 \times \left(400-200\right)\text{kPa}$$

$$= 40\ \text{J}$$

$$\text{Area of rectangle} = \left(600\times10^{-6} - 200\times10^{-6}\right)\text{m}^3 \times \left(200-0\right)\text{kPa}$$

$$= 80\ \text{J}$$

Total area under process path = 40 J +80 J = 120 J

The work done for this thermodynamic process is 120 J.

12.6 Internal Energy of an Ideal Gas

It was earlier on alluded to that the system's temperature is an indicator of its internal energy. The internal energy of a gas is mostly assigned to the kinetic energy possessed by the atoms/molecules of a gas. Thus we can see the effect of temperature on the internal energy of a monatomic gas from equation 12.7. This shows that the internal energy of an ideal gas depends only on its temperature.

$$KE = N(\overline{KE}) = \tfrac{3}{2}nRT \qquad 12.7$$

Where N is the number of molecules in a gas, n is the number moles, and R the gas constant. In the case of a monatomic gas the energy goes into mainly translational motion and its internal energy is assigned to the kinetic energy as expressed in equation 12.8.

$$U = KE_{trans.} = \tfrac{3}{2}nRT \qquad 12.8$$

It follows that the change in internal energy of a monatomic gas can be calculated from equation 12.9.

$$\Delta U = \tfrac{3}{2}nR\Delta T \qquad 12.9$$

It should be noted that for molecular gases the energy flowing into them is partitioned into other forms of motion such as rotation and vibration. These gases will have higher internal energy when at the same temperature as a monatomic gas. Equation 12.9 can be re-written in a more general form as:

$$U = K(\tfrac{1}{2}nRT) \qquad 12.10$$

Where K is an integer greater than or equal to 3. In the case of monatomic gases K is equal to 3.

12.7 Heat Transfer and Specific Heat Capacity of an Ideal Gas

The calculation of the amount of heat transferred into or out of an ideal gas depends on the process used. Two common processes used are the constant-volume process and the constant pressure process. These processes are discussed with reference to the first law of thermodynamics which is about the conservation of energy.

12.8 Constant–Volume Process

Recall, the first law of thermodynamics states that energy added to system goes into increasing the internal energy of the system plus work done, if any, by the system equation 12.1. In doing work we expect a volume change in the system. Thus in a constant volume process where there is no volume change $(\Delta V = 0)$ the work done is zero and all the energy goes into increasing the internal of the system. The first law of thermodynamic reduces to:

$$Q = \Delta U \text{ (Constant-volume)}$$

Referring to equation 12.9 for a monatomic gas we say that:

$$Q = \Delta U = \tfrac{3}{2} nRT \qquad 12.11$$

We know that the only quantity that relates Q and ΔT is the specific heat capacity of a substance in equation 11.21.

$$c = \frac{Q}{m\Delta T}$$

We can now define a new quantity the molar specific heat capacity C by replacing mass m in the last equation with the mole n.

$$C = \frac{Q}{n\Delta T} \qquad 12.12$$

To differentiate between the processes a subscript is attached to molar specific heat capacity term and for constant-volume process C_v is used. Substituting for Q in equation 12.12 term from equation 12.11 we get for a monatomic gas the following result:

$$C_v = \frac{\tfrac{3}{2} nR\Delta T}{n\Delta T} = \tfrac{3}{2} R \qquad 12.13$$

The above result can be generalized for molecular gases by appropriately changing the preceding fraction.

12.9 Constant-Pressure Process

The work done in the case of a constant-pressure is simply the product of the pressure and the change in volume ($W = P\Delta V$). It follows that the first law of thermodynamics can re-written as

$$Q = \Delta U + W = \Delta U + P\Delta V \qquad 12.14$$

At constant pressure the ideal-gas law equation 11.16 can be stated as

$$P\Delta V = nR\Delta T$$

The quantity of heat involved becomes

$$Q = \Delta U + nR\Delta T \text{ (Constant pressure)}$$

In a similar manner we defined C_v, the molar specific heat at constant pressure, C_p is defined as

$$C_p = \frac{Q}{n\Delta T} = \frac{\Delta U + P\Delta V}{n\Delta T} = \frac{\Delta U}{n\Delta T} + \frac{nR\Delta T}{n\Delta T}$$

At constant volume the ideal-gas law reduces to $Q = \Delta U$, hence if we substitute for ΔU the equation above becomes

$$C_p = C_v + R \qquad 12.15$$

This result tell us that the molar specific heat capacity at constant-pressure is always larger that the molar specific heat capacity at constant volume. This follows from the fact that in this process some heat goes into work. Recall the molar specific heat capacity at constant volume for a monatomic gas gave us the result that $C_v = \frac{3}{2}R$. It follows that if we substitute this value in equation 12.5 we get

$$C_p = \frac{3}{2}R + R = \frac{5}{2}R$$

$$C_p = \frac{5}{2}R \text{ (monatomic gas)} \qquad 12.16$$

The ratio of the two processes gives an approximately constant value γ. The value of this constant for monatomic gases is 1.67 and for diatomic gases 1.40. The values of γ for a few selected gases are given in table 12.1

$$\gamma = C_P / C_V \qquad 12.17$$

Table 12.1 Ratio of C_p to C_v of Some Gases

Gas	γ
Monatomic	
Helium (He)	1.67
Argon (Ar)	1.67
Krypton (Kr)	1.69
Diatomic	
Hydrogen (H_2)	1.41
Oxygen (O_2)	1.40
Carbon Monoxide (CO_2)	1.40
Polyatomic	
Steam (H_2O)	1.30
Sulphur Dioxide (SO_2)	1.29
Carbon Dioxide (CO_2)	1.30

12.10 Zero-Heat Transfer Process

In the zero-heat transfer process transfer no energy is moved into or out of the system and it is called an adiabatic process. For this to occur the process must be thermally isolated. In practice this is very difficult to attain, however, if a process is very fast then it can be considered to be adiabatic as there is very little time for the transfer of thermal energy? For this process $Q = 0$ and the first law of thermodynamics becomes

$$0 = \Delta U + W \text{ or } \Delta U = -W \text{ (adiabatic)}$$

This implies that work is done at the expense of internal energy of the system or that the internal energy of the system must decrease.

12.11 PV Diagram Representation of the Thermodynamic Processes

The first two processes, namely, the Constant-pressure and Constant-volume are simple as these are represented by straight lines, for example, as in figure 12.3. The other two processes are complex as for them there is an inverse relationship between the pressure and the volume arising out of the ideal gas law. It states that if the temperature T is held constant then PV is also constant. So that for the isothermal process the equation of the isotherm is

$$PV = \text{constant or } P = \frac{\text{constant}}{V} \qquad 12.8$$

A plot for an isothermal process involving compression at constant temperature is shown in figure 12.6.

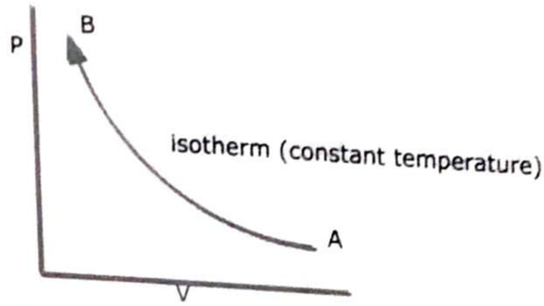


Figure 12.6 PV diagram for the isothermal process.

Note that work is done during is the isothermal compression that is given by

$$W = nRT \ln\left(\frac{V_f}{V_i}\right) \qquad 12.9$$

Where T, is the temperature at which compression takes place, V_f and V_i are the final and initial volumes respectively. The final volume during this process is less than the initial volume and we see that this is consistent with the fact that equation 12.9 will give us a negative number as the natural log of a number less than one is negative.

In the case of the last process the adiabatic process where work is done at the expense of internal energy the equation of the adiabatic is

$$PV^{\gamma} = \text{constant or } P = \frac{\text{constant}}{V^{\gamma}} \qquad 12.10$$

Where γ is as defined in equation 12.7. A plot of this process is in figure 12.7.

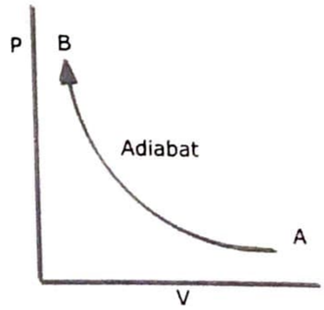


Figure 12.7 PV diagram for the adiabatic process

It should be noted that for adiabatic compression the pressure rises very rapidly when compared to the isothermal process.

Example 12.2

A sample of air $(\gamma = 1.40)$ is slowly compressed from a pressure of 1 atmosphere to 3 atmospheres. The original volume v_1 of air is 15 litres and the temperature is 300 K. The temperature is constant during compression (isothermal process). Later on the air is suddenly (adiabatically) expanded back to its original pressure of 1 atmosphere.

i) Sketch a PV-diagram of these processes,

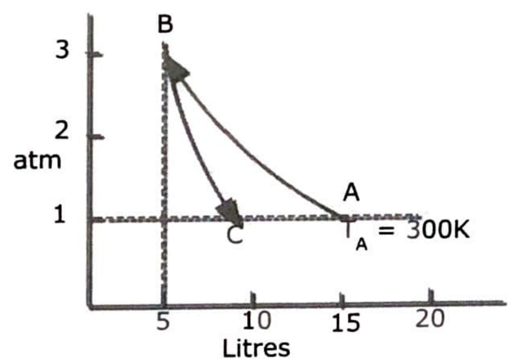
ii) find the final volume and temperature,

iii) find the $\Delta U, Q$ and W for each process, and

iv) number of moles of the air.

Solution

i)



ii) We can find the initial volume air by considering the isothermal part (AB) on the PV-diagram using the ideal gas law.

$$P_A V_A = P_B V_B$$

$$V_B = V_A\left(\frac{P_A}{P_B}\right) = 15\left(\frac{1}{3}\right) = 5\text{litres}$$

The final volume V_C at C has to be calculated using the adiabatic portion for which:

$$P_C V_C^{\gamma} = P_B V_B^{\gamma}$$

$$V_C^{\gamma} = V_C^{\gamma}\left(\frac{P_B}{P_C}\right) \text{or } V_C = V_B\left(\frac{P_B}{P_C}\right)^{1/\gamma} = 5\left(\frac{3}{1}\right)^{1/1.4} = 10.96\text{litres}$$

To get the temperature at C we use the ideal gas law equation 11.14 from which we get that $\frac{P_B V_B}{T_B} = \frac{P_C V_C}{T_C}$

$$T_C = T_B\left(\frac{P_C}{P_B}\right)\left(\frac{V_C}{V_B}\right) = 300\left(\frac{1}{3}\right)\left(\frac{10.96}{5}\right) = 219.1\text{K}$$

iii) For the isothermal process there is no change in temperature $(\Delta T = 0)$ and therefore there is no change in internal energy $(\Delta U = 0)$.

The work done is given by equation 12.9.

$$W_{AB} = nRT\ln\left(\frac{V_B}{V_A}\right) = P_A V_A \ln\left(\frac{V_B}{V_A}\right) = (1.01\times10^5)(15\times10^{-3})\ln\left(\frac{5}{15}\right) = -1664.4\text{J}$$

We can get Q_{AB} from the first law of thermodynamics.

$$Q_{AB} = \Delta U + W = 0 + (-1664.4) = -1664.4\text{J}$$

For the adiabatic process $Q_{BC} = 0$ and the result from the first law of thermodynamics is that $W_{BC} = \Delta U_{BC}$.

iv) To find the number of moles of air the ideal gas law equation 11.16 re-written as,

$$n = \frac{P_A V_A}{RT_A} = \frac{(1.01\times10^5)(15\times10^{-3})}{(8.314)(300)} = 0.61\text{mol}$$

12.12 Second Law of Thermodynamics

The second law of thermodynamics is concerned with the conversion of heat energy into usable form i.e. into mechanical energy. Heat energy can be obtained from a variety of sources such as wood, coal, oil etc. In our earlier discussion it was pointed out that heat is governed by the random motion of atoms/molecules of a substance. To convert heat into a usable form we need to extract energy from these random motions of atoms/molecules. This law limits the choice of fuels for engines in general. The second law of thermodynamics is anchored on the flow of heat energy in a given system. In everyday situations we know that ice left in the sun will melt and the temperature of the ice can never increase when left in such an environment. In other words the heat flows into the ice and a reverse flow is physically impossible.

12.13 Statement of the Second Law of Thermodynamics

It states that heat flows spontaneously from a substance at a higher temperature to a substance at a lower temperature and cannot flow spontaneously in the opposite direction. This law is best illustrated in figure 12.8.

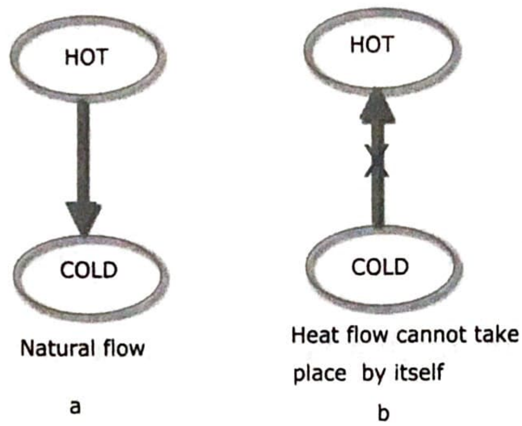


Figure 12.8

12.14 Entropy (Re-statement of the second law of thermodynamics)

The concept of entropy states that if an isolated system made up of many parts is allowed to undergo spontaneous change, it changes in such a way that the disorder increases or, at best, does not decrease. Usually entropy is interpreted in terms of order and disorder of a system. Irreversible processes lead to increase in the entropy of the universe, they lead to energy degradation meaning that part of the energy becomes unavailable to do work. Entropy S (ΔS denotes change in entropy) is a state variable whose change is defined for a reversible process at a temperature T in which a quantity of energy Q is the heat absorbed.

$$\Delta S = \frac{\Delta Q}{T} \qquad 12.11$$

Equation 12.11 is a measure of the amount of energy which is unavailable to do work or a measure of the disorder of a system. The quantity Q is the amount of heat added in a reversible manner to a system at temperature T. It should be noted that reversible processes do not change the total entropy of the universe.

Example 12.3

A 10 g mass of helium gas is expanded by the isothermal process at a temperature of -85 ^{0}C to 4 times its original volume. What is the change in entropy of the helium gas?

Solution

The process is isothermal so $\Delta T = 0$ and $\Delta U = 0$, the from the first law of thermodynamics we get that $Q = W$ and work for this process is

$$W = nRT \ln\left(\frac{V_f}{V_i}\right) = \left(\frac{0.01}{4}\right)(8314)(188)\ln\left(\frac{4}{1}\right) = 5417J.$$

$$\Delta S = \frac{Q}{T} = \frac{5417}{188} = 28.8\text{J/K}$$

Example 12.4

Find the change in entropy that occurs when 50 g of ice melts slowly at 0 ^{0}C. Consider that heat flows into the ice and regard it as an isolated system.

Solution

The ice is melting at a constant temperature hence the process is isothermal taking place at 273 K. The heat needed for this process can be calculated using equation for the heat of fusion.

$$\Delta Q = mH_f = (50\text{g})(80\text{cal/g})(4.184\text{J/cal}) = 16736\text{J}$$

Entropy can be calculated from equation 12.11.

$$\Delta S = \frac{\Delta Q}{T} = \frac{16736}{273} = 61.3\text{J/K}$$

12.15 Heat Engine

A heat engine is any device that converts heat energy into mechanical energy. We are familiar with various types of engines powered by fuels such petrol, diesel, kerosene and rarely these days by steam. All of the above have one thing in common in that they all transform heat energy into mechanical energy by employing a repetitive cycle. They employ a working substance that is returned to its initial state at the end of each cycle. Figure 12.9 is a general representation of a heat engine that operates between a hot reservoir at temperature T_h and a cold reservoir at temperature T_c. In each operation cycle the engine absorbs an amount of heat Q_h from the hot reservoir. The absorbed heat is partly used to perform work, W, and the remainder, Q_c, is transferred to the cold reservoir.

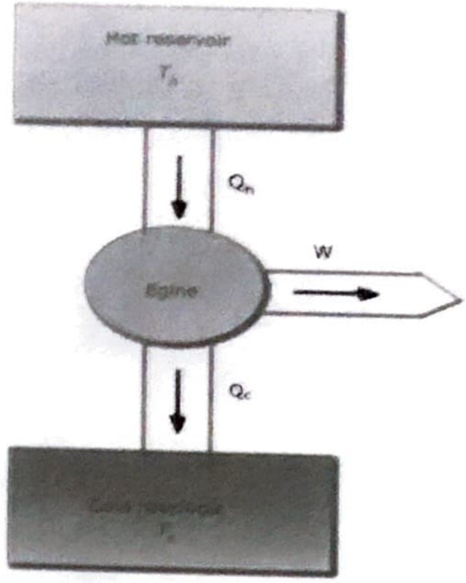


Figure 12.9Schematic heat engine.

A heat engine obeys the law of conservation of energy. It follows that the heat energy from the hot reservoir is portioned between doing work and being exhausted to the cold reservoir. In this case the cold reservoir is the surrounding area. Since a heat engine obeys the law of conservation of energy we can apply to it the first law of thermodynamics equation 12.5.

$$Q = \Delta U + W \text{ or that}$$

$$Q_{net} = Q_h - Q_c = W + \Delta U \qquad 12.12$$

Where W is the work output per cycle and ΔU is the change in the internal energy. A complete cycle results in no net change in internal energy so that ΔU is zero and equation 12.12 reduces to

$$W = Q_h - Q_c \qquad 12.13$$

To find the efficiency of a heat engine we use the definition of efficiency which states that the efficiency of a "machine" is the ratio of its output to its input energy.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input energy}} \qquad 12.14$$

Substituting for quantities in equation 12.4 and using the letter e for efficiency we get

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} \qquad 12.15$$

The heat energy Q_c exhausted to the surroundings is the amount of energy that does not take part in doing work and is responsible for the inefficiency of a heat engine. If there was no exhausted energy (i.e. $Q_c = 0$) to the surroundings then the efficiency of an engine will be hundred percent. It is well known that this is a physical impossibility. We can conclude that there are definite efficiency limits for heat engines.

Example 12.5

A certain engine absorbs 2000 J of heat energy from the hot reservoir and exhausts 800 J to the surroundings during each cycle of operation.

i) What is the efficiency of this engine?

ii) How much work does it perform during each cycle?

Solution

i) The efficiency can be calculated using equation 12.15

$$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{800}{2000}$$

$$= 1 - 0.4 = 0.6$$

i.e. the engine is 60% efficient.

ii) The work performed during each cycle is equal to the difference between the heat input and the exhausted heat.

$$W = Q_h - Q_c$$

$$W = 2000\ \text{J} - 800\ \text{J} = 1200\ \text{J}$$

12.16 Carnot Engine

A Carnot engine is an idealized engine which sets the maximum efficiency that can be obtained by any engine operating at the same temperature extremes. No machine can be considered as ideal as some heat energy is lost during operation due to friction. The friction losses can be accounted for by the law of conservation of energy. A Carnot engine consists of a gas enclosed in a cylinder with a movable piston and its operation cycle consists of two isothermal processes and two adiabatic processes. In order that we approach Carnot efficiency, the processes involved must be reversible and involve no change in entropy. Such conditions cannot be obtained in reality as real engine processes are not reversible and physical processes lead to increase in entropy. The efficiency of this engine can be calculated using equation 12.15.

12.17 Carnot Cycle

A Carnot cycle is usually represented on a *PV* diagram as, as shown in figure 12.10. Work is done by the engine during the two expansions, and work is done on the engine during the two compressions. The net work done per cycle of operation is the area enclosed by the four processes.

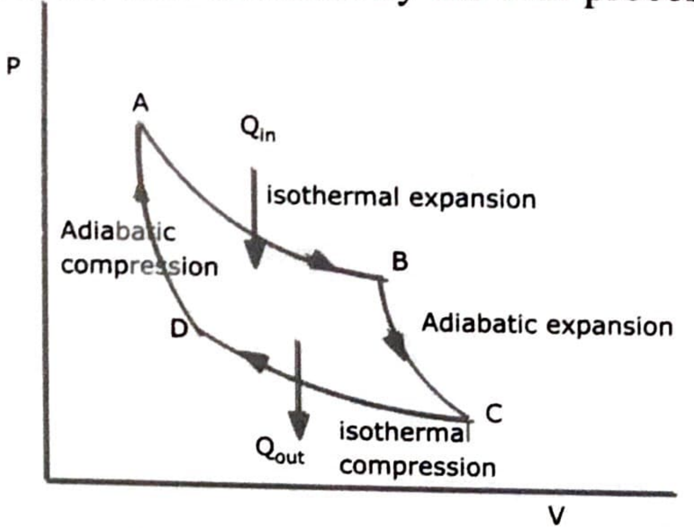


Figure 12.10 Carnot cycle.

The isothermal processes in the diagram are from *A* to *B* and from *C* to *D*. During isothermal expansion *A* to *B* at temperature T_A heat Q_{in} is absorbed from the reservoir and work is done. In the adiabatic expansion *BC* there is no further heat input but work is done at the expense internal energy and the temperature falls to T_C. During the isothermal compression *CD* the engine surroundings are at temperature T_C that is lower than T_A. In this compression heat Q_{out} is released and work is done. To complete the cycle an adiabatic compression *DA* returns the engine to its original state. The work done on the gas (compression) raises the temperature to T_A. The total work done during a cycle is equal to the area enclosed within the curves. The efficiency of a Carnot engine can be calculated from equation 12.15. We know that the net work done in a cycle is equal to the net heat transfer $(Q_{in} - Q_{out})$ as the change in internal energy is zero.

It is a fact that the amount of heat *Q* transferred from a Carnot engine is directly proportional to the absolute temperature *T*.

$$Q \propto T$$

$$\therefore \frac{Q}{T} = \text{constant i.e.}$$

$$\frac{|Q_c|}{|Q_t|} = \frac{T_c}{T_h} \qquad 12.16$$

It follows that the efficiency of a Carnot engine can be stated in terms of temperature by substituting 12.16 into 12.15. The temperature must be in Kelvin for the latter equation to give correct results.

$$e_{\text{Carnot}} = 1 - \frac{T_c}{T_h} \qquad 12.17$$

Example 12.6

A Carnot engine whose hot reservoir is at 550 °C takes 5000 J of heat per cycle and exhausts 2600 J of heat to the cold reservoir (surroundings).

i) Find the temperature of the cold reservoir, and

ii) Calculate the engine's efficiency by using the temperatures and the heat quantities.

Solution

i) We are given the following: $T_h = 550\,^0\text{C} = 823$ K, $Q_h = 5000$ J and $Q_c = 2600$ J. We can use equation 12.16 to get the temperature of the cold reservoir.

$$\frac{|Q_c|}{|Q_t|} = \frac{T_c}{T_h} \text{ i.e.}$$

$$T_c = \frac{Q_c}{Q_h}.T_h = \frac{2600}{5000}.823 = 428\text{K}$$

ii) Using equation 12.17

$$e_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$$

$$e_{\text{Carnot}} = 1 - \frac{428}{823} = 0.48$$

Using equation 12.15

$$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{2600}{5000} = 0.48$$

Note that in both cases we get the same value of the efficiency, however this only holds for Carnot engine. Any other engine operating at the same temperature extremes will result in a lower value of the efficiency.

12.18 Refrigerators and Heat Pumps

A refrigerator operates in reverse fashion to a heat engine to extract heat from a low temperature reservoir and transfer it to the high temperature reservoir. The natural tendency of heat is to flow from a hot region to colder region. The operation of the refrigerator is contrary to what the second law of thermodynamics states that heat cannot flow from a cold region to hot region on its own accord. However if energy is supplied then heat can be forced to flow from a low temperature region to a high temperature region. Refrigerators, air conditioners and heat pumps are devices that do precisely that. Figure 12.11 is a schematic representation of a refrigerator.

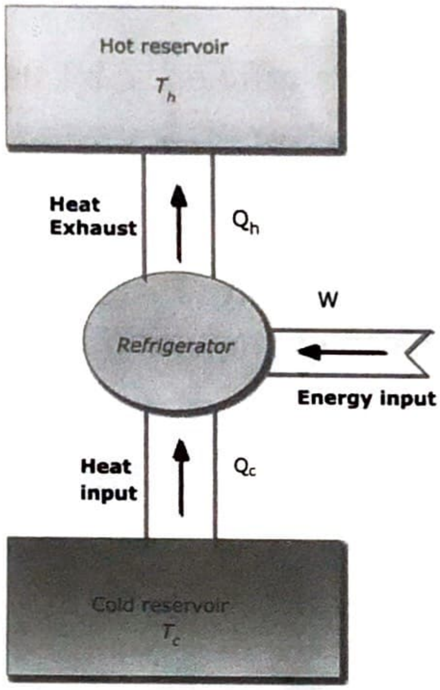


Figure 12.11 Refrigeration

The process in which work is done on the system to lower the temperature of a substance is known as a refrigeration cycle. In this case the energy flow as shown in figure 12.11 is the reverse of a heat engine. A refrigerator operates at temperatures T_c and T_h where the work input W allows heat at a low temperature Q_c to be moved to the high temperature region of the system. According to the first law of thermodynamics that is for energy to be conserved, the energy into the system must be equal to the energy out of the system.

$$\text{Energy in} = \text{Energy out}$$

$$Q_c + W = Q_h \qquad 12.18$$

In order for the refrigeration cycle to be very effective (efficient), it depends on removing large amounts of heat for very little input work.

In refrigeration we specify the effectiveness of the system in terms of the *coefficient of performance*, COP in place of the efficiency. This quantity is a ratio of the heat removed from the cold reservoir Q_c to the work input W.

$$\text{COP} = \frac{Q_c}{W} \qquad 12.19$$

If we re-arrange equation 12.18 so that $W = Q_h - Q_c$ and then we substitute this in equation 12.19 we get:

$$\text{COP}_{\text{refrigerator}} = \frac{Q_c}{Q_h - Q_c} \qquad 12.20$$

Referring to equation 12.16 where the quantity of heat energy involved is taken to be directly proportional to the absolute temperature in Kelvin, then we can re-state equation 12.20 in terms of absolute temperature.

$$\text{COP}_{\text{Max}} = \frac{T_c}{T_h - T_c} \qquad 12.21$$

The latter equation implies that when the difference between T_c and T_h is small, less work is needed to extract heat from the cold reservoir that is exhausted into the hot reservoir. This equation also gives the maximum coefficient of performance.

12.19 Heat Pumps

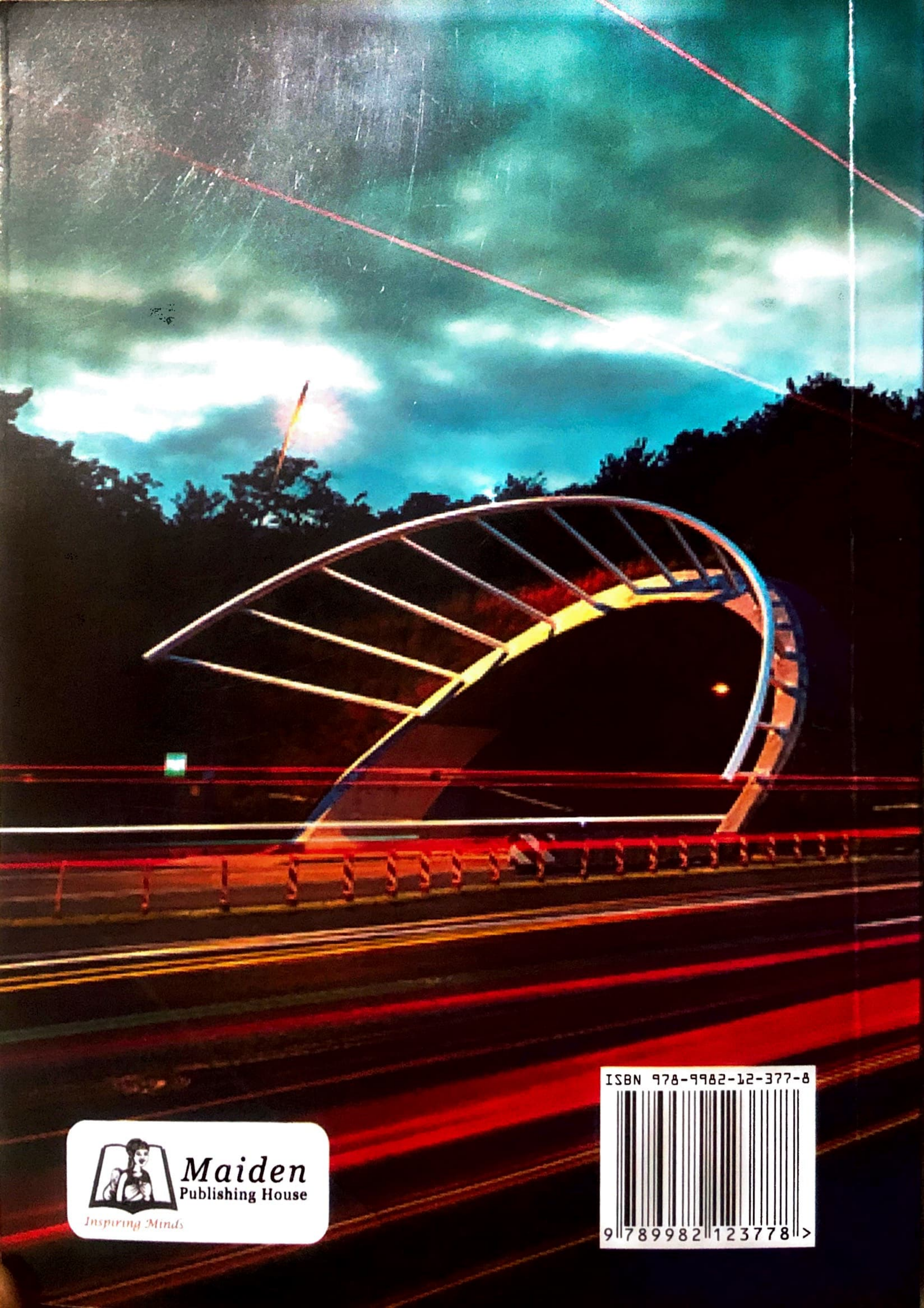
Heat pumps such as air-conditioners operate in a similar manner to refrigerators. These can also be used to heat up the interior of buildings during the cold season in moderate climates, in designs that have a mechanism of reversing the flow of energy between the reservoirs. In this case the heat pump heats up the cold interior as opposed to cooling it. It follows that the coefficient of performance is the ratio of heat Q_h delivered into the interior to the work W needed to deliver it.

$$\text{Heat pump} \quad \text{COP} = \frac{Q_h}{W} \qquad 12.22$$

Since the work done is $W = Q_h - Q_c$ we can substitute the term on the right side of this equation into equation 12.22.

$$\text{COP}_{\text{Heat pump}} = \frac{Q_h}{Q_h - Q_c} \qquad 12.23$$

We can in a similar manner to a refrigerator define the maximum COP by substituting with temperatures of the reservoirs in equation 12.23. However, this is a theoretical COP that can never be achieved in reality.



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