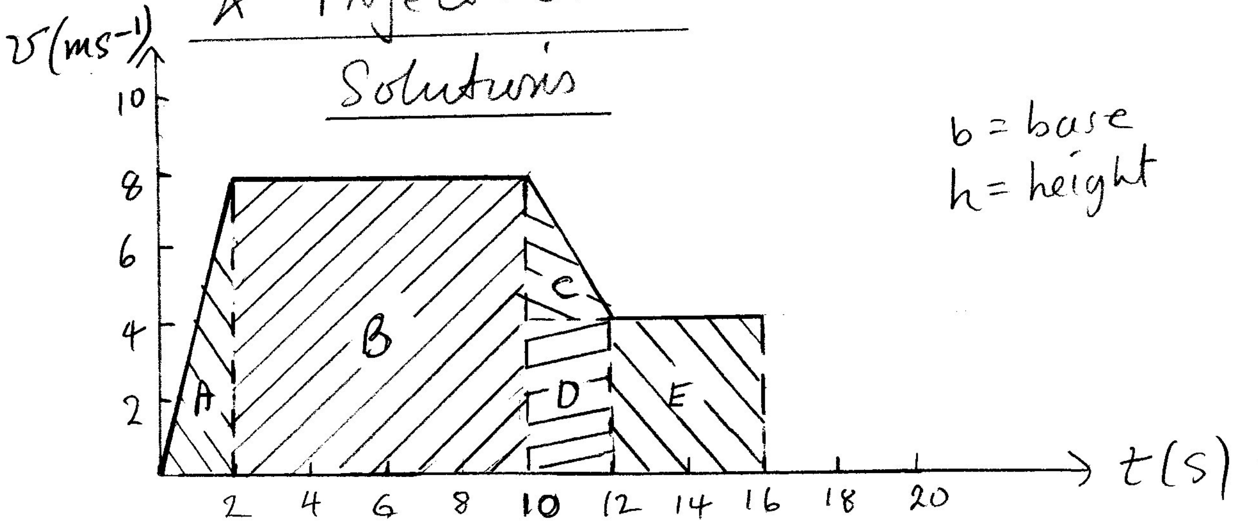
TUTORIAL 2.

Uniformly Accelerated Motion & Projectiles

Solutions

Q1



b = base
h = height

(i) d = {Distance covered by runner} = Area under the graph

$= A + B + C + D + E$

$= \frac{1}{2} b_A h_A + b_B h_B + \frac{1}{2} b_C h_C + b_D h_D + b_E h_E$

$= \frac{1}{2}(2)(8) + (8)(8) + \frac{1}{2}(2)(4) + (2)(4) + (4)(4)$

$= 8 + 64 + 4 + 8 + 16$

$= 100\ m$

Ans. Distance covered by runner $= d = 100m$

(ii) → Note that the accelerations at $t = 2s$ and $t = 11s$ are infinite since velocity changes in zero time, i.e.,

$$a = \frac{v_f - v_i}{t} = \frac{\Delta v}{0} = 0$$

→ However, the question is really asking for the

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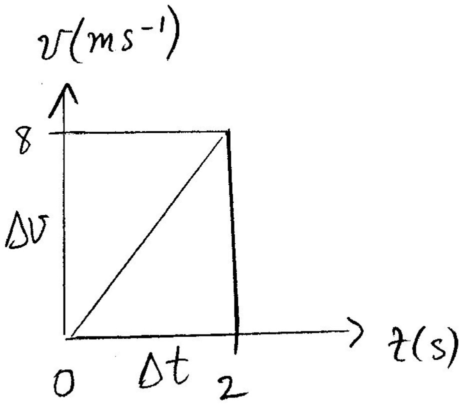
Q1 (ii) cont.

acceleration in the time intervals 0s to 2s and 10s to 12s.

acceleration a_2 from 0s to 2s:

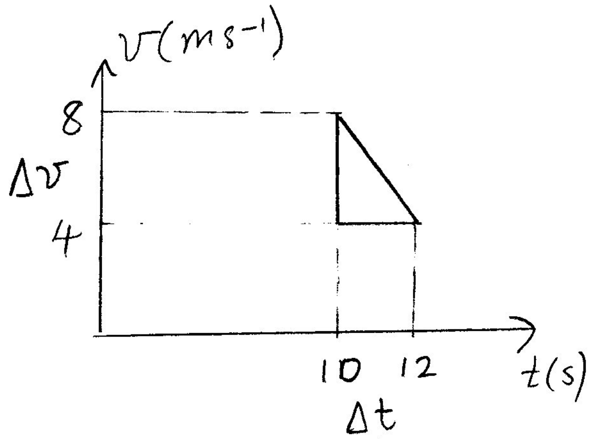
$$a_2 = \frac{\Delta V}{\Delta t} = \frac{8}{2} = 4 ms^{-2}$$



acceleration a_{10} from 10s to 12s

$$a_{10} = -\frac{\Delta V}{\Delta t} = \frac{4-8}{12-10} = \frac{-4}{2}$$

$$= -2 ms^{-1}$$



Ans. $a_2 = 4 ms^{-2}$ and $a_{10} = -2 ms^{-2}$

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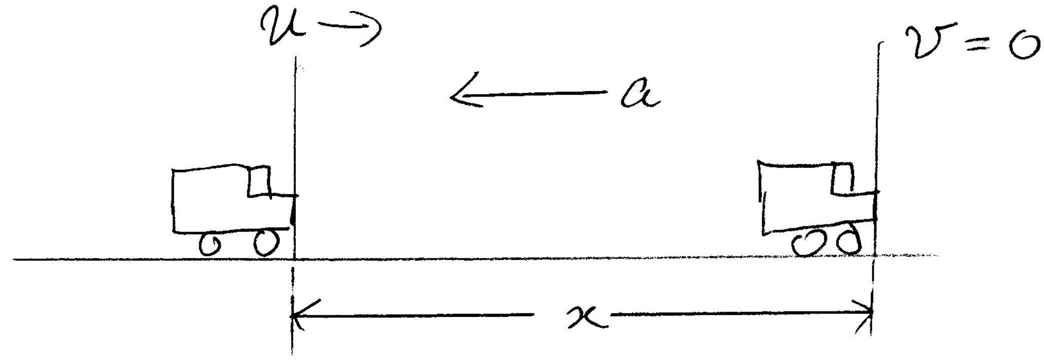
Q2.



Deceleration $= a = -2.27\ ms^{-2}$

Initial velocity $= u = 22.5\ ms^{-1}$

Final velocity $= v = 0\ ms^{-1}$

Initial position $= x_0 = 0m$

(a) t = time taken to stop

$$v = at + u$$

$$t = \frac{v-u}{a} = \frac{0-22.5}{-2.27} = 9.912\,s$$

Ans. Time taken to stop $= t = 9.91s$ (3sf)

(b) x = stopping distance

$$2ax = v^2 - u^2$$

$$x = \frac{v^2-u^2}{2a} = \frac{0-22.5^2}{2(-2.27)} = 111.5\,m$$

Ans. stopping distance $= x = 112m$ (3sf)

Q2(c) x_{23} = distance covered in the third second.

x_2 = distance covered after 2s

$x_2 = \frac{1}{2}at^2 + ut + x_0$

$x_2 = \frac{1}{2}(-2.27)2^2 + (22.5)(2) + 0 = 40.46\,m$

x_3 = distance covered after 3s

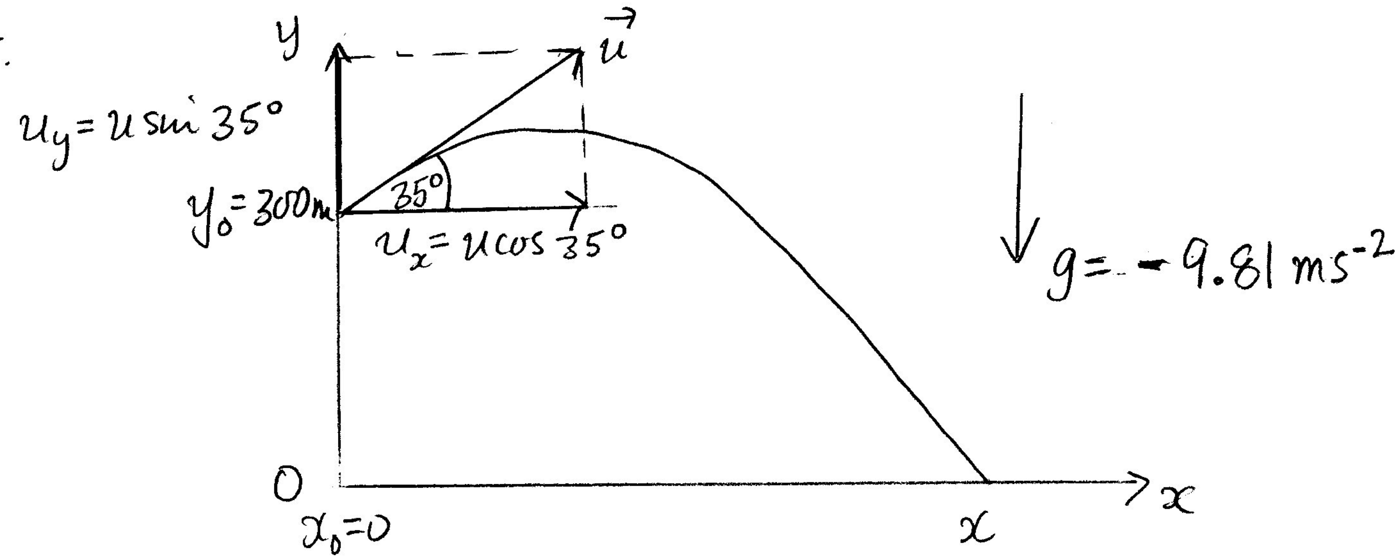
$x_3 = \frac{1}{2}at^2 + ut + x_0$

$= \frac{1}{2}(-2.27)(3^2) + (22.5)(3) + 0 = 57.29\,m$

$x_{23} = x_3 - x_2 = 57.29 - 40.46 = 16.83\,m$

Ans. Distance covered in third second $= x_{23} = 16.8\,m$ (3st)

Q4.



Data

$y_0 = 300$ m

$|\vec{u}| = u = 200$ ms^{-1}

$u_x = u\cos 35° = 200 \cos 35° = 163.8$ ms^{-1}

$u_y = u \sin 35° = 200 \sin 35° = 114.7$ ms^{-1}

$x_0 = 0$ m

$x = ?$

→ First, write the constant acceleration formulae for both the x- and y-motion.

x-motion		y-motion	
$x = u_x t + x_0$	(1)	$y = \frac{1}{2}gt^2 + u_y t + y_0$	(3)
$v_x = u_x$	(2)	$v_y = gt + u_y$	(4)
		$2ay = v^2 - u^2$	(5)

Find time of flight from Eq. (3):

$$y = \frac{1}{2}gt^2 + u_y t + y_0$$

$$0 = \frac{1}{2}(-9.81)t^2 + 114.7t + 300$$

Q4 cont.

$$0 = -4.905t^2 + 114.7t + 300$$

$$4.905t^2 - 114.7t - 300 = 0$$

Compare with

$$at^2 + bt + c = 0$$

$a = 4.905$

$b = -114.7$

$c = -300$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-114.7) \pm \sqrt{(-114.7)^2 - 4(4.905)(-300)}}{2(4.905)}$$

$t = -2.38\,s$ or $25.76\,s$

Reject the negative time as unphysical, since $t = 0$, the later time cannot be negative. Hence,

$$t = 25.76\,s$$

Find the range x from Eq. (1):

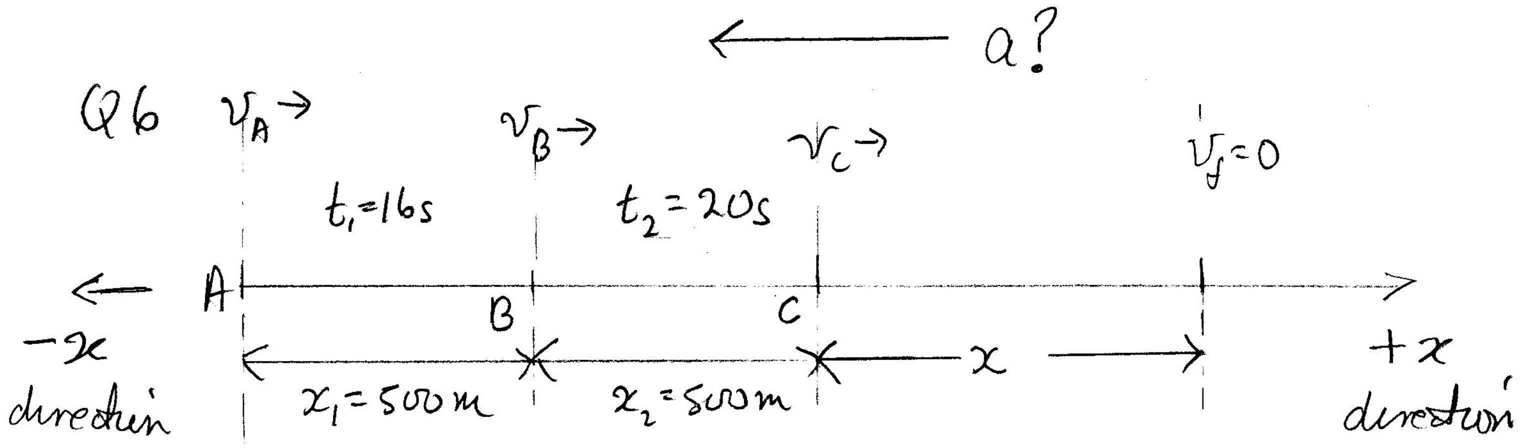
$x = u_x t + x_0$

$x = (163.8)(25.76) + 0$

$x = 4219\,m$

Ans. Time of flight $t = 25.8\,s$, and the range $x = 4220\,m$ (3st)

Q6



$$t_c = t_1 + t_2 = 36s$$

$$x_c = x_1 + x_2 = 500 + 500 = 1000\,m$$

<u>Motion from A to B</u>

$$x_1 = \tfrac{1}{2}at_1^2 + v_A t_1 + x_0$$

$$500 = \tfrac{1}{2}a(16)^2 + v_A 16 + 0$$

$$500 = 128a + 16v_A$$

$$v_A = \frac{500 - 128a}{16}$$

$$v_A = 31.25 - 8a \qquad (1)$$

<u>Motion from A to C</u>

$$x_c = \tfrac{1}{2}at_c^2 + v_A t_c + x_0$$

$$1000 = \tfrac{1}{2}a36^2 + v_A 36$$

$$1000 = 648a + 36v_A$$

$$v_A = \frac{1000 - 648a}{36} = 27.78 - 18a \qquad (2)$$

Equate Eq.(1) & Eq.(2)

$$31.25 - 8a = 27.78 - 18a$$

Q6 cont.

$$-8a + 18a = 27.78 - 31.25$$

$$10a = -3.47$$

$$a = -0.347\ ms^{-2}$$

Find v_A from Eq. 1:

$$v_A = 31.25 - 8a$$

$$v_A = 31.25 - 8(-0.347)$$

$$v_A = 34.03\ ms^{-1}$$

Find v_c :

$$v_c = at_c + v_A$$

$$v_c = (-0.347)(36) + 34.03$$

$$v_c = 21.54\ ms^{-1}$$

Find stopping distance x:

$$2ax = v_f^2 - v_c^2$$

$$x = \frac{v_f^2 - v_c^2}{2a}$$

$$x = \frac{0 - 21.54^2}{2(-0.347)}$$

$$x = 668.5\ m$$

Ans Stopping distance $x = 669\ m$ (3sf)