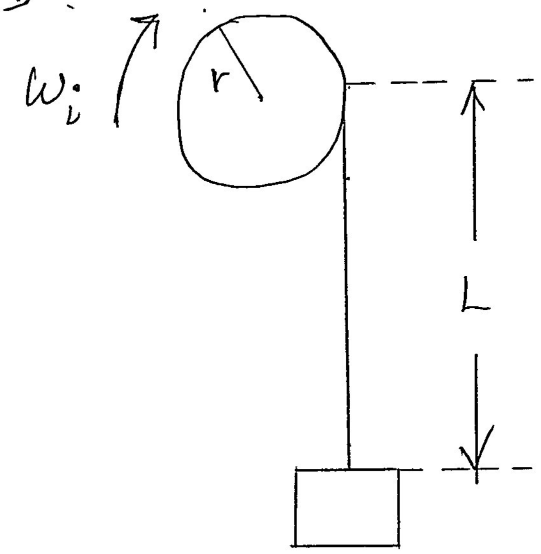
Tutorial 7
Motion in a circle
Solutions 2019/202

Q1



Data:

$r = 44\ cm = 0.44\ m$

$L = 29.5\ cm$

c = circumference $= 2\pi r$

$= 2\pi(0.44) = 2.765\ m$

$\omega_i = 1.6\ \frac{rev}{s} = 1.6(2\pi rad).s^{-1}$

$\omega_i = 10.05\ rad.s^{-1}$

$\omega_f = 0\ rad.s^{-1}$

Number of revolutions $= \theta = \frac{29.5}{2.765} = 10.67$ rev

$= 10.67\ (2\pi\ rad)$

$= 67.04$ rad

Find the acceleration:

$$2\alpha\theta = \omega_f^2 - \omega_i^2 \Rightarrow \alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta} = \frac{0 - 10.05^2}{2(67.04)}$$

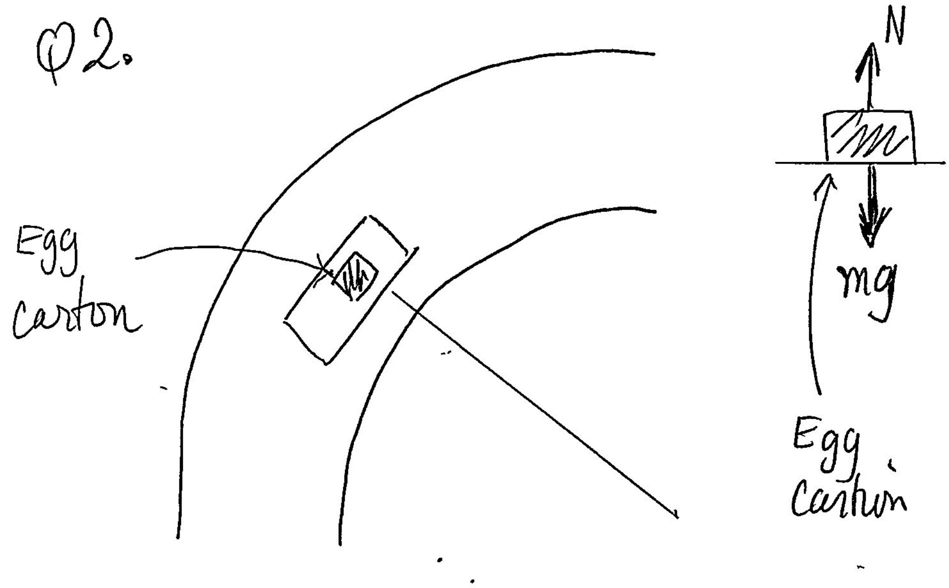
$$\alpha = 0.7533\ rad.s^{-1} = 0.7533\left(\frac{rev}{2\pi}\right).s^{-2}$$

$$\alpha = 0.1199\ rev.s^{-2}$$

Ans. Number of revolutions $= 10.7$ rev $= 67.0$ rad (3sf),

Acceleration $\alpha = 0.120\ rev.s^{-2}$

Q2.



Data:

$g = 9.81\ ms^{-2}$

$r = 26 m$

$v = 16.5 ms^{-1}$

$\mu = ?$

$f_{min} = \mu N = \mu m g$ (1)

$F_c = \frac{mv^2}{r}$ (2)

→ The only force stopping the carton from slipping is the frictional force between the carton and the car seat. Hence, this is the only force that can provide the centripetal force needed to make the cartoon turn with the car.

→ Thus, for the cartoon not to slip, the minimum frictional force f_{min} needed must equal the centripetal force F_c

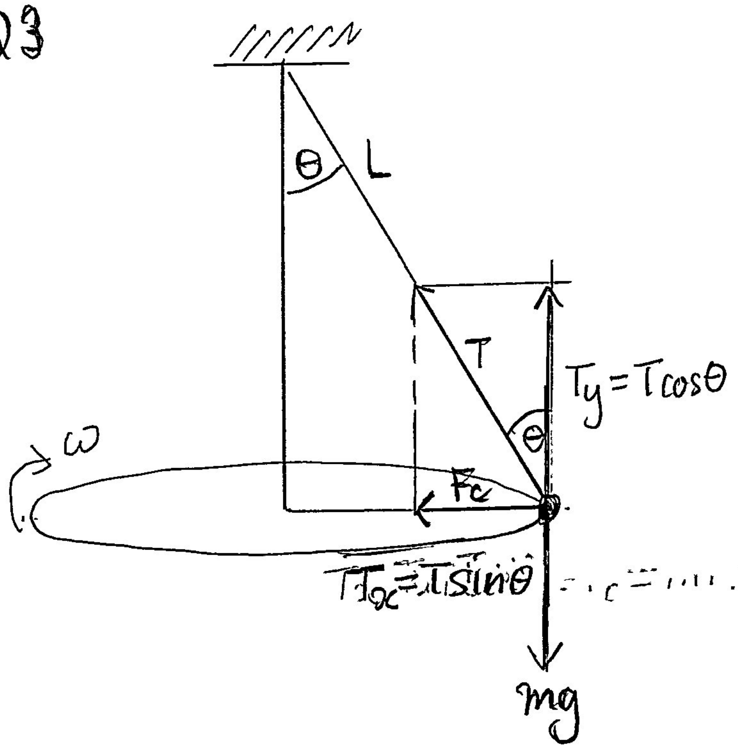
$$f_{min} = F_c$$

$$\mu m g = \frac{mv^2}{r} \qquad \text{[from Eq. (1) and Eq. (2)]}$$

$$\mu = \frac{v^2}{gr} = \frac{16.5}{(9.81)(26)} = 1.067$$

Ans. Minimum coefficient of static friction $= \mu = 1.07$ (3sf)

Q3



Data:

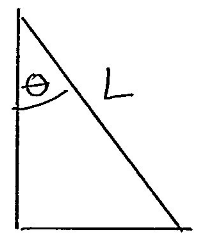
$L = 30\,cm = 0.3\,m$ = length of string

$m = 100\,g = 100(10^{-3}\,kg) = 0.1\,kg$

$\omega = 80\,\frac{rev}{min} = 80\,\frac{(2\pi\,rad)}{(60\,s)}$

$\omega = 8.378\ rad\,s^{-1}$

$r = L\sin\theta$ (1)



$r = L\sin\theta$

T_y balances mg, hence

$T_y = mg$ (2)

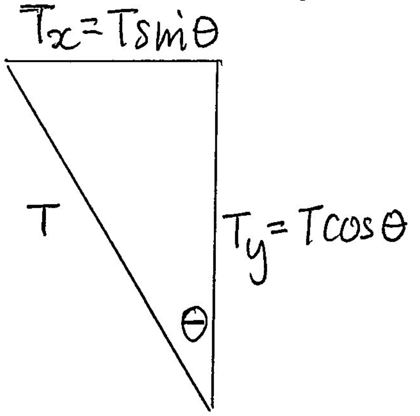
T_x provides the centripetal force, hence

$T_x = F_x = mr\omega^2$ (3)

(ii) Find T

Sub. Eq. (1) into Eq. (3)

$$T_x = m\omega^2 L\sin\theta$$



$$T\sin\theta = m\omega^2 L\sin\theta$$

$$T = m\omega^2 L = (0.1)(8.378^2)(0.3)$$

$$T = 2.106\,N$$

Ans. The tension in the string is $T = 2.11\,N$ (3sf)

Q3(i) Find θ

$$T_y = T\cos\theta$$

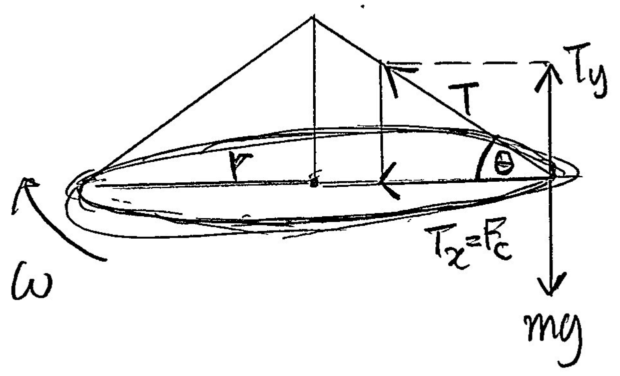
$$\cos\theta = \frac{T_y}{T} = \frac{mg}{T} = \frac{(0.1)(9.81)}{2.106} = 0.4658$$

$$\theta = \cos^{-1} 0.4658$$

$$\theta = 62.24^\circ$$

Ans. The angle of inclination of the string to the vertical $\theta = 62.2^\circ$ (3sf)

Q4



Data:
$g = 9.81\,ms^{-2}$
$m = 450g = 0.45\,Kg$
radius $= r = 1.25m$
$v = 8.5\,ms^{-1}$

$$T_x = F_c = \frac{mv^2}{r} = \frac{(0.45)(8.5^2)}{1.25} = 26.01\,N$$

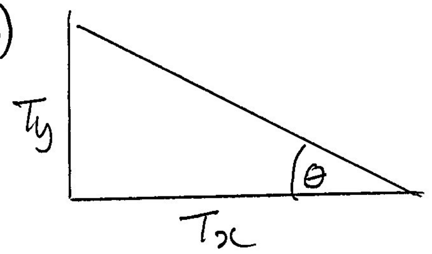
$$T_y = mg = (0.45)(9.81) = 4.415\,N$$

a) Find the tension

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{26.01^2 + 4.415^2} = 26.38N$$

Ans. The tension in the string is $T = 26.4N$ (3sf)

b)

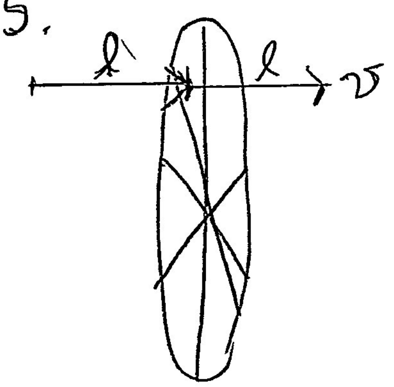


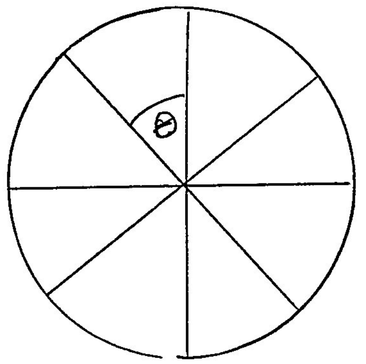
$$\tan\theta = \frac{T_y}{T_x} = \frac{4.415}{26.01} = 0.1697$$

$$\theta = \tan^{-1} 0.1697 = 9.631^\circ$$

Ans. The angle the chord makes with the horizontal $\theta = 9.63^\circ$ (3sf)

Q5.





$r = 25\,cm = 0.25\,m$

$\theta = \frac{360}{8} = 45^\circ$

or

$\theta = \frac{2\pi}{8} = 0.7854\,rad$

$v = 6\,ms^{-1}$

$l = 20\,cm = 0.2\,m$

→ Time taken for arrow to travel a distance equal to its length is

$$t = \frac{l}{v} = \frac{0.2}{6} = 0.03333\,s$$

→ For the arrow not to be hit by a spoke, the gap between spokes must remain open for at least the time it takes the arrow to travel a distance equal to its length, i.e., $t = 0.03333\,s$.

→ This minimum time $t = 0.03333\,s$ corresponds to the maximum angular velocity ω_{max} that the wheel can rotate without the spokes hitting the arrow.

→ This minimum time $t = 0.03333\,s$ is the thus the mimium time for the spokes to cover an angle of $\theta = 45^\circ = 0.7854\,rad$, hence

$$\omega_{max} = \theta/t = \frac{0.7854}{0.03333} = 23.56\,rad\,s^{-1}$$

$$\omega_{max} = 23.56\,rad\,s^{-1} = 23.56\left(\frac{rev}{2\pi}\right)s^{-1}$$

$$\omega_{max} = 3.750\,rev\,s^{-1}$$

Q5 cont.

Because angular velocity is the same for any radius inside the wheel's rim it doesn't matter how far from the center the arrow passes, so no it doesn't matter where between the axle and the rim the arrow is aimed.

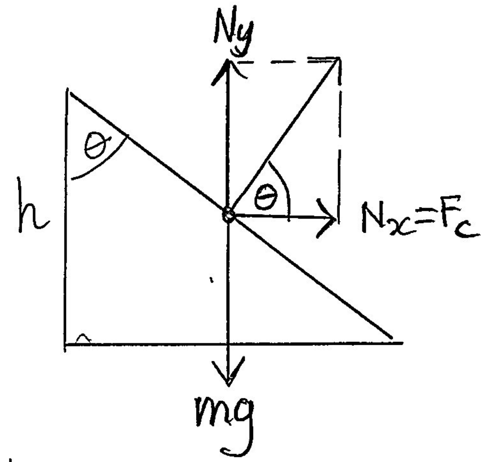
Ans. The maximum angular velocity the wheel can rotate without the spokes hitting the arrow is

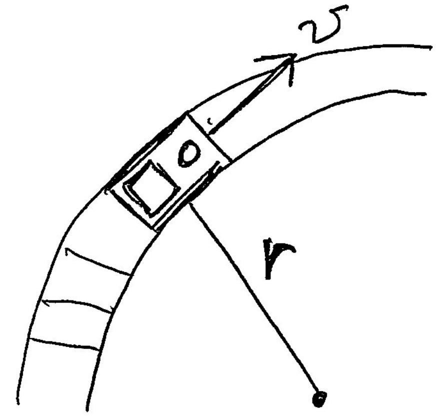
$$\omega_{max} = 23.6 \text{ rad.s}^{-1} = 3.75 \text{ rev.s}^{-1} \text{ (3sf)}$$

and

No it doesn't matter where in the gap between spokes the arrow is aimed.

Q6.





Data:

Radius of curve $= r = 500m$

Velocity of train $= v = 50 \frac{Km}{h} = 50\left(\frac{1000m}{3600s}\right) = 13.89 ms^{-1}$

Distance between rails $= d = 1.5m$

Centripetal force $= F_c = \frac{mv^2}{r}$ (1)

$F_c = N_x = N\cos\theta$ (2)

$mg = N_y = N\sin\theta$ (3)

→ h = height outer rail must be raised.

⇒ For the train to go round the curve without lateral thrust (tendancy to travel in a straight line and be thrown of the rails), there must be enough centripetal force F_c provided by the x-component N_x of the normal reaction N

→ As seen from the above figure, the smaller θ

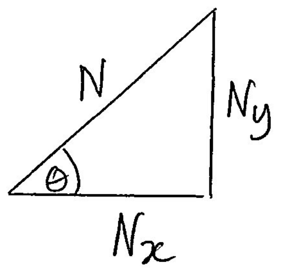
Q6. cont.

the bigger N_x. By equating $N_x = F_c$ we will find the largest value of θ corresponding to the smallest value of h the outer rail must be raised.

→ The condition for maximum θ and minimum h is thus

$$N_x = N_c = \frac{mv^2}{r}$$

Find maximum θ

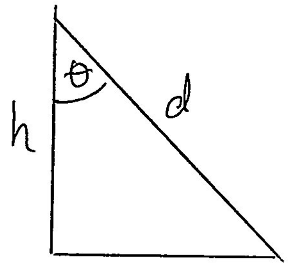


$$\tan\theta = \frac{N_y}{N_x} = \frac{mg}{mv^2/r} = \frac{gr}{v^2}$$

$$\tan\theta = \frac{(9.81)(500)}{(13.89)^2} = 25.42$$

$$\theta = \tan^{-1}(25.42) = 87.75^\circ$$

Find h

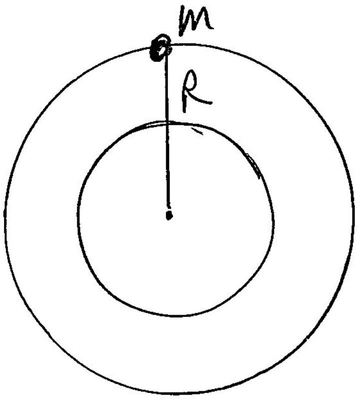


$$h = d\cos\theta = 1.5\cos 87.75$$

$$h = 0.05889 \text{ m} = 5.889 \text{ cm}$$

Ans. Outer rail must be raised to a height of atleast $h = 5.89$ cm (3sf)

Q7



$T = 87\,min = 87\,(60s) = 5220\,s$

$R = 6500\,Km = 6500\,(10^3 m)$

$= 6.5 \times 10^6 m$

$G = 6.67 \times 10^{-11} \dfrac{Nm^2}{Kg^2}$

mass of satellite $= m$

→ The centripetal force is provided by the attraction of the earth, hence

$$F_c = F_{earth}$$

$$mR\omega^2 = \frac{GmM}{R^2}$$

$$M = \frac{R^3\omega^2}{G} \quad (1)$$

Find ω:

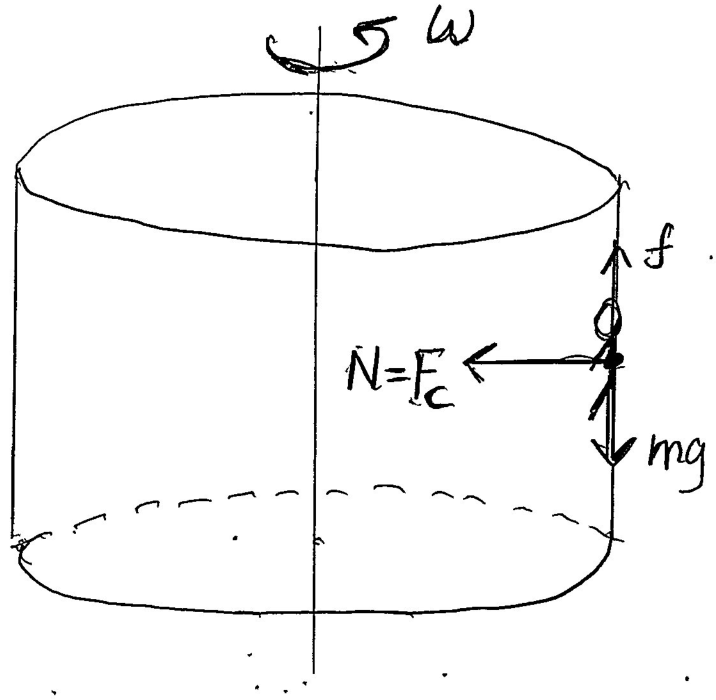
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{5220} = 1.204 \times 10^{-3}\ rad\,s^{-1}$$

Sub. ω into Eq. (1)

$$M = \frac{(6.5 \times 10^6)^3 (1.204 \times 10^{-3})^2}{6.67 \times 10^{-11}} = 5.969 \times 10^{24}\,Kg$$

Ans. Mass of the earth $M = 5.97 \times 10^{24}$ Kg

Q8



Data:

$r = 2.5\,m$

$\mu = ?$

$\omega = 0.6\,\frac{rev}{s}$

$= 0.6\,\frac{(2\pi\,rad)}{s}$

$= 3.770\,rad.s^{-1}$

$g = 9.81\,ms^{-2}$

N = normal reaction

frictional force $f = \mu N$ (1)

Centripetal force $F_c = mr\omega^2$ (2)

→ The normal reaction provides the centripetal force, hence

$$N = F_c = mr\omega^2 \quad (3)$$

Sub. Eq. (2) into Eq. (1):

$$f = \mu m\omega^2 \quad (4)$$

→ For the person not to fall, the frictional force must balance the person's weight, hence

$$f = mg \quad (5)$$

→ sub. Eq. (5) into Eq. (4):

$$mg = \mu mr\omega^2$$

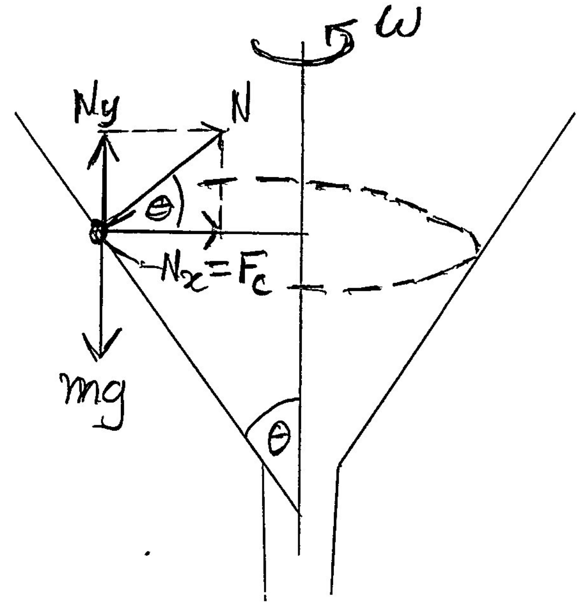
$$g = \mu r\omega^2$$

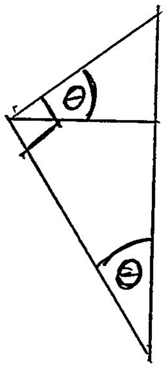
$$\mu = \frac{g}{r\omega^2} = \frac{9.81}{(2.5)(3.77^2)} = 0.2760$$

<u>Ans. the minimum coefficient of friction is</u>

<u>$\mu = 0.276$ (3s.f)</u>

Q9





$N_x = N\cos\theta$ (1)

$F_c = \frac{mv^2}{r}$ (2)

$N_x = N_c$ (3)

Sub. Eq.(1) and (2) into Eq.(3)

$N\cos\theta = \frac{mv^2}{r}$

$N = \frac{mv^2}{r\cos\theta}$ (4)

$N_y = N\sin\theta$ (5)

$N_y = mg$ (6)

Equate Eq.(5) and Eq. (6):

$mg = N\sin\theta$

$N = \frac{mg}{\sin\theta}$ (7)

Equate Eq.(4) and Eq.(7)

$\frac{mv^2}{r\cos\theta} = \frac{mg}{\sin\theta} \Rightarrow \frac{v^2}{r\cos\theta} = \frac{g}{\sin\theta} \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{rg}{v^2}$

$\tan\theta = \frac{rg}{v^2}$

$v = \sqrt{\frac{rg}{\tan\theta}}$

Ans. $v = \sqrt{\frac{rg}{\tan\theta}}$ is the required speed for particle to maintain its motion in a circle.

-END-