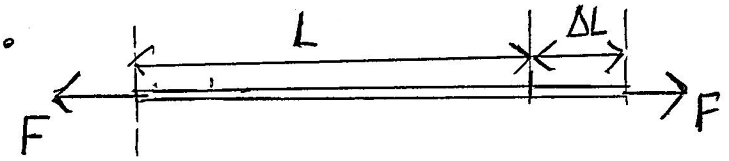
Tutorial 9
Mechanical Properties of Matter
solutions

Q1.



Data:
$F = 20\,N$, $L = 4\,m$
$\Delta L = 0.24\,mm = 0.24 \times 10^{-3}\,m$

$r = \frac{d}{2} = \frac{2mm}{2} = 1\,mm = 10^{-3}\,m$, Y = Young's modulus ?

cross-sectional area $= \pi r^2 = \pi (10^{-3})^2 = \pi 10^{-6}\,m^3$

$$\text{strain} = \frac{\Delta L}{L} = \frac{0.24 \times 10^{-3}}{4} = 0.06 \times 10^{-3} = 6 \times 10^{-5}$$

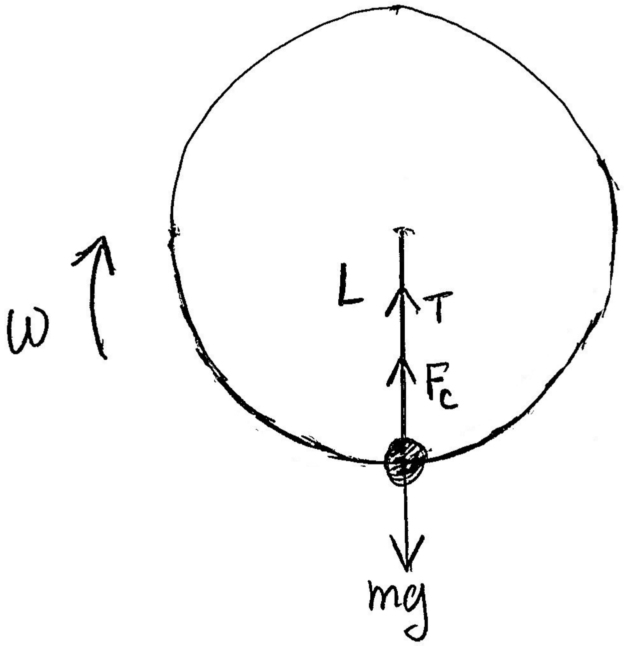
$$\text{stress} = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{20}{\pi 10^{-6}} = 6.366 \times 10^{6}\,Nm^{-2}$$

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{6.366 \times 10^{6}}{0.06 \times 10^{-3}} = 1.056 \times 10^{11}\,Nm^{-2}$$

Ans. strain $= 6.00 \times 10^{-5}$, stress $= 6.37 \times 10^{6}\,Nm^{-2}$

Young's modulus $= Y = 1.06 \times 10^{11}\,Nm^{-2}$ (3sf)

Q2



Length of unstretched steel wire $= L = 0.5 m$

$\omega = 2 \frac{rev}{s} = 2 \frac{(2\pi \, rad)}{s} = 12.57 \, rad.s^{-1}$

$F_c = m r \omega^2$

Cross-section of the steel wire $= 0.02 cm^2$
$= 0.02 (10^{-2} m)^2$
$= 2 \times 10^{-6} m^2$

$Y = 2 \times 10^{11} Pa$

$m = 15 Kg$

$g = 9.81 ms^{-2}$

At the lowest point, the tension T balances mg and provides the centripetal force F_c:

$$T = mg + mL\omega^2 = m(g + L\omega^2) = 15[9.81 + (0.5)12.57]$$
$$= 1332 N$$

→ T is the force which stretches the wire.

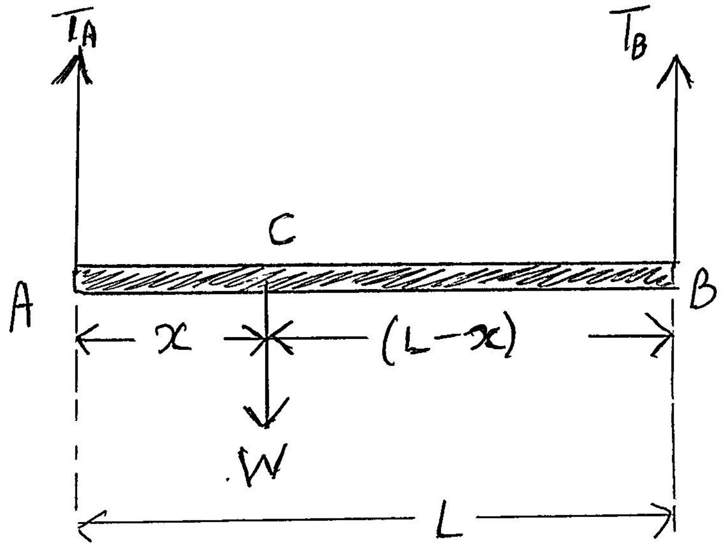
→ We can now calculate ΔL using Young's modulus:

$$Y = \frac{stress}{strain} = \frac{T/A}{\Delta L/L} = \frac{TL}{A\Delta L}$$

$$\Delta L = \frac{TL}{YA} = \frac{(1332)(0.5)}{(2 \times 10^{11})(2 \times 10^{-6})} = 166.5 \times 10^{-5} m$$
$$= 166.5 \times 10^{-5} (10^3 mm) = 1.665 mm$$

Ans. Elongation at lowest point $= \Delta L = 1.67 mm$ (3sf)

Q3.



$L = 1.05\ m$

$A_A = 1\ mm^2 = 1 \times (10^{-3} m)^2$
$= 10^{-6} m^2$

$A_B = 4\ mm^2 = 4 \times 10^{-6} m^2$

$AC = x$
$CB = L - x$

$Y_A = 2.4 \times 10^{11}\ Pa$
$Y_B = 1.2 \times 10^{11}\ Pa$

→ We use the torque condition for static equilibrium and choose the axis of rotation to pass through the point C:

$$\Sigma \tau_B - \Sigma \tau_A = T_B(L-x) - T_A x = 0$$

$$T_B L - T_B x - T_A x = 0$$

$$T_B L - x(T_A + T_B) = 0$$

$$x = \frac{T_B L}{T_A + T_B} \quad (1)$$

(a) Equal stresses

$$\text{Stress}_A = \text{Stress}_B$$

$$\frac{T_A}{1} = \frac{T_B}{4}$$

$$T_B = 4T_A \quad (2)$$

3(a) cont

Sub Eq. (2) into Eq. (1)

$$x = \frac{4T_AL}{4T_A + T_A} = \frac{4}{5} \cdot \frac{T_AL}{T_A} = \frac{4}{5}L = \frac{4}{5}(1.05) = 0.84\,m$$

Ans. W should be hung $x = 0.84\,m$ from end A

3 (b) Equal Strain

$$Y_A = \frac{\text{Stress}_A}{\text{Strain}_A}, \quad Y_B = \frac{\text{Stress}_B}{\text{Strain}_B}$$

$$\text{Stress}_A = T_A/A_A, \quad \text{Stress}_B = T_B/A_B$$

$$\text{Strain}_A = \frac{\text{Stress}_A}{Y_A} \;(3), \quad \text{Strain}_B = \frac{\text{Stress}_B}{Y_B} \;(4)$$

For equal strain

$$\text{Strain}_A = \text{strain}_B$$

$$\frac{\text{Stress}_A}{Y_A} = \frac{\text{Stress}_B}{Y_B}$$

$$\frac{T_A/A_A}{Y_A} = \frac{T_B/A_B}{Y_B}$$

$$\frac{T_A}{A_AY_A} = \frac{T_B}{A_BY_B}$$

$$T_B = \frac{A_BY_BT_A}{A_AY_A} = \frac{(4\times10^{-3})(1.2\times10^{11})}{(1\times10^{-3})(2.4\times10^{11})}T_A = 1.667\,T_A \quad (5)$$

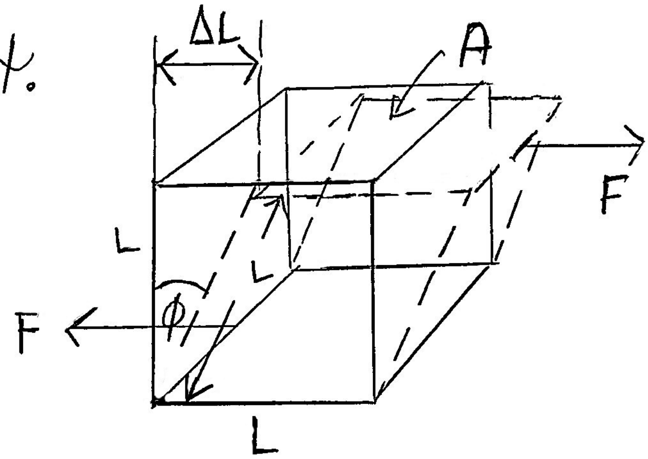
Sub. Eq. (5) into Eq. (1)

$$x = \frac{T_BL}{T_A+T_B} = \frac{(1.667\,T_A)(1.05)}{T_A + 1.667\,T_A}$$

$$= \frac{T_A}{T_A}\left(\frac{1.667\times1.05}{1+1.667}\right) = 0.6563\,m.$$

Ans. W should be hung at $x = 0.656\,m$ (3sf) from end A

4.



ϕ = Angle of shear
$F = 4000\,N$
$L = 25\,cm = 0.25\,m$
$A = L^2 = 0.0625\,m^2$
$B = 0.8 \times 10^{11}\,Pa$
B = Bulk modulus

Cube of sides L

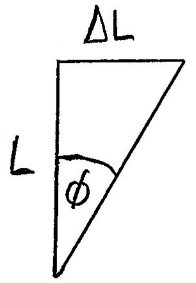
$$B = \frac{\text{stress}}{\tan\phi} \qquad (1)$$

$$\text{stress} = \frac{F}{A} = \frac{4000}{0.0625} = 64{,}000\,N$$

Rearrange Eq. (1):

$$\tan\phi = \frac{\text{stress}}{B} = \frac{64{,}000}{0.8 \times 10^{11}} = (8 \times 10^{-7})^\circ$$

For small ϕ, $\tan\phi \simeq \phi = (8 \times 10^{-7})^\circ$

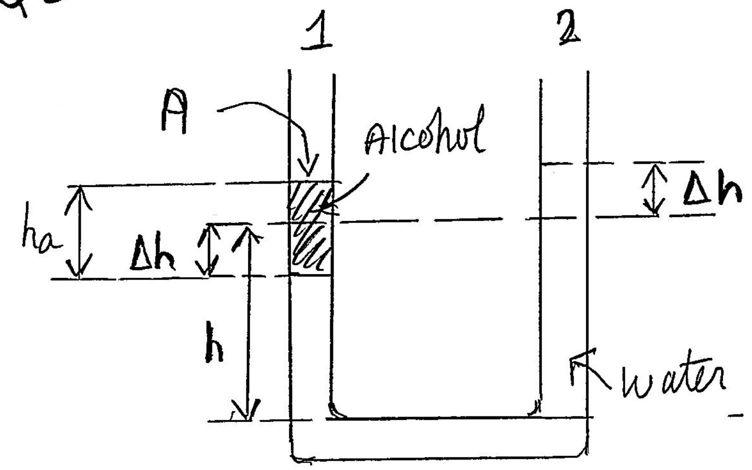


$$\tan\phi = \frac{\Delta L}{L}$$

$$\Delta L = L\tan\phi = (0.25)(8 \times 10^{-7})$$
$$= 2 \times 10^{-7}\,m$$

Ans. $\phi = (8 \times 10^{-7})^\circ$, $\Delta L = 2 \times 10^{-7}\,m$

Q5



$h_a = 3\,cm = 0.03\,m$

$h = 15\,cm = 0.15\,m$

A = cross-sectional area of tube

density of alcohol $= \rho_a = 720\,Kg\,m^{-3}$

density of water $= \rho_w = 1000\,Kg\,m^{-3}$

→ After the alcohol is poured, the water level in column 2 rises by an amount Δh, while the water level in column 1 decreases by the same amount.

→ The weight of fluids in columns 1 and 2 must balance at equilibrium, hence

$$m_{w1} g + m_a g = m_{w2} g$$

$$m_{w1} + m_a = m_{w2} \qquad (1)$$

mass of water in column 1 $= m_{w1} = \rho_w V_{w1}$

$V_{w1} = A(h - \Delta h)$, hence, $m_{w1} = \rho_w A (h - \Delta h)$

mass of alcohol in column 1 $= m_a = \rho_a V_A = \rho_a A h_a$

mass of water in column 2 $= m_{w2} = \rho_w A (h + \Delta h)$

→ Sub. into Eq. (1) we get

$$\rho_w A (h - \Delta h) + \rho_a h_a A = \rho_w A (h + \Delta h)$$

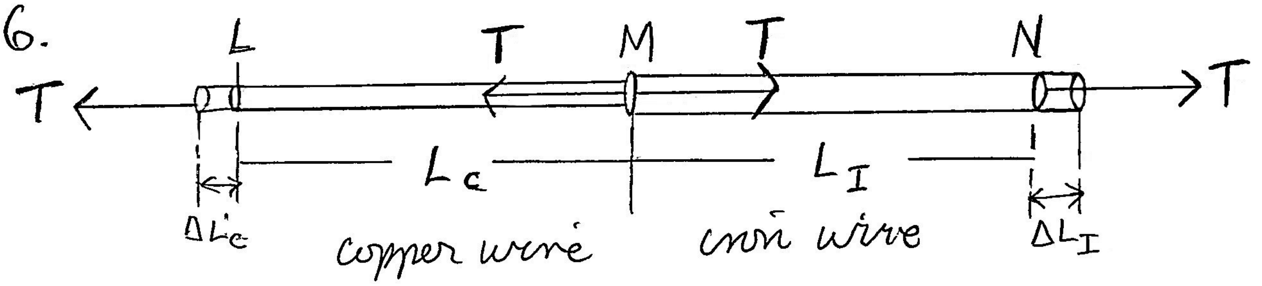
$$\cancel{\rho_w h} - \rho_w \Delta h + \rho_a h_a A = \cancel{\rho_w h} + \rho_w \Delta h$$

$$\rho_a h_a = 2 \rho_w \Delta h$$

$$\Delta h = \frac{\rho_a h_a}{2 \rho_w} = \frac{(720)(0.03)}{2(1000)} = 0.0108\,m$$

Ans. The water level in column 2 rises by $\Delta h = 0.0108\,m = 1.08\,cm$

Q6.



Data:

$L_c = 0.9\,m$, $\quad L_I = 1.4\,m$

$A_c = 0.9 \times 10^{-6}\,m^2$, $\quad A_I = 1.3 \times 10^{-6}\,m^2$

ΔL_c = extension of the copper wire, $\quad \Delta L_I$ = extension of iron wire

Total extension $= \Delta L = 0.0\,m$

$$\Delta L = \Delta L_c + \Delta L_I \qquad (1)$$

6.(a) Since the wires remain stationary, the stretching force T at each end of the compound wire is the same. Since each wire is homogeneous, the tension throughout each wire is the same, so that the forces at point M are equal and opposite as shown in the figure.

→ Thus, the force T acts at each end of each rod producing the extensions ΔL_c and ΔL_I in each respectively. In this case, we have

$$Y_c = \frac{T/A_c}{\Delta L_c / L_c}, \qquad Y_I = \frac{T/A_I}{\Delta L_I / L_I}$$

Q6 cont

$$Y_c = \frac{TL_c}{A_c \Delta L_c} , \quad Y_I = \frac{TL_I}{A_I \Delta L_I}$$

$$\Delta L_c = \frac{TL_c}{A_c Y_c} \quad (2), \qquad \Delta L_I = \frac{TL_I}{A_I Y_I} \quad (3)$$

From Eq. (1)

$$\Delta L = \Delta L_c + \Delta L_I$$

$$\Delta L_c = \Delta L - \Delta L_I \quad (4)$$

Sub. (4) into (2)

$$\Delta L - \Delta L_I = \frac{TL_c}{A_c Y_c} \Rightarrow \Delta L - \frac{TL_c}{A_c Y_c} = \Delta L_I \quad (5)$$

Equate Eq. (3) and Eq. (5)

$$\Delta L - \frac{TL_c}{A_c Y_c} = \frac{TL_I}{A_I Y_I} \Rightarrow \Delta L = \frac{TL_c}{A_c Y_c} + \frac{TL_I}{A_I Y_I}$$

$$\Delta L = T\left(\frac{L_c}{A_c Y_c} + \frac{L_I}{A_I Y_I}\right)$$

Sub. values:

$$0.01 = T\left(\frac{0.9}{(0.9\times10^{-6})(1.3\times10^{11})} + \frac{1.4}{(1.3\times10^{-6})(2.1\times10^{11})}\right)$$

$$0.01 = T(0.7692\times10^{-5} + 0.5128\times10^{-5})$$

$$0.01 = T(1.282\times10^{-5})$$

$$T = \frac{0.01}{1.282\times10^{-5}} = 780\text{N}.$$

Ans. Tension applied = 780N

Q 6 (b) Find ΔL_c :

Sub. $T = 780N$ in Eq. (2)

$$\Delta L_c = \frac{TL_c}{A_c Y_c} = \frac{(780)(0.9)}{(0.9 \times 10^{-6})(1.3 \times 10^{11})}$$

$$\Delta L_c = 600 \times 10^{-5} = 0.006\ m = 6\ mm$$

Find ΔL_I from Eq. (1)

$$\Delta L = \Delta L_c + \Delta L_I$$

$$\Delta L_I = \Delta L - \Delta L_c = 0.01 - 0.006 = 0.004\ m$$

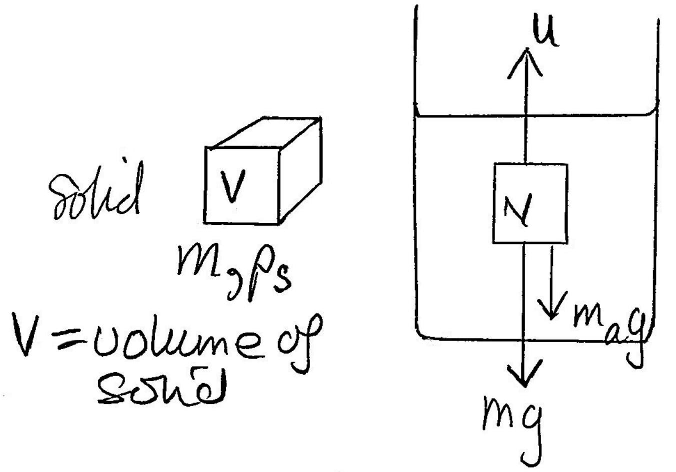
Ans. $\Delta L_c = 0.006\ m = 6\ mm$, $\Delta L_I = 0.004\ m = 4\ mm$

6 (c) $$\text{Ratio} = \frac{\Delta L_I}{\Delta L_c} = \frac{4}{6} = \frac{2}{3}$$

or 2:3 or 1:1.5

Ans. Ratio $\frac{\Delta L_I}{\Delta L_c} = 2/3$

Q7.



Data.

mass of liquid displaced $= m_\ell$

$m_\ell = \rho_\ell V$

u = upthrust

mass of solid $m = 237.5\,g$

$m = 0.2375\,kg$

apparant mass of solid $=$

$m_a = 12.5\,g = 0.0125\,Kg$

$\rho_\ell = 900\ Kg\ m^{-3}$

$\rho_w = 1000\ kg\,m^{-3}$

$\rho_s =$ density of solid ?

→ Since the solid in the liquid has an apparant mass m_a, the solid will sinc, i.e. $u < mg$.

(i) By Archimede's principle

upthrust = weight of fluid displaced

$$u = m_\ell g = \rho_\ell V g \quad (1)$$

→ The apparant mass m_a is given by

$$m_a g = mg - u \quad (2)$$

sub. Eq. (1) into Eq. (2)

$$m_a g = mg - \rho_\ell V g$$

$$m_a = m - \rho_\ell V$$

$$V\rho_\ell = m - m_a$$

$$V = \frac{m - m_a}{\rho_\ell} = \frac{0.2375 - 0.0125}{900}$$

$$V = 2.5 \times 10^{-4}\ m^3$$

Q 7(i) cont.

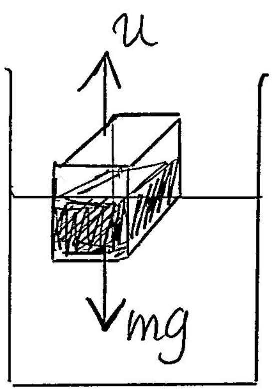
Find the density of the solid:

$$m = \rho_s V$$

$$\rho_s = \frac{m}{V} = \frac{0.2375}{2.5 \times 10^{-4}} = 950\ \text{Kg}\,m^{-3}$$

Ans. Density of the solid $\rho_s = 950\ \text{Kg}\,m^{-3}$

(ii)



volume of solid immersed $= \frac{4V}{5}$ = volume of liquid displaced

ρ' = density of the second liquid.

By Archimede's principle:

upthrust = weight of liquid displaced

$$u = m'g = \rho' \left(\frac{4V}{5}\right) g \qquad (1)$$

→ Since solid floats in the second liquid

$$u = mg \quad (2)$$

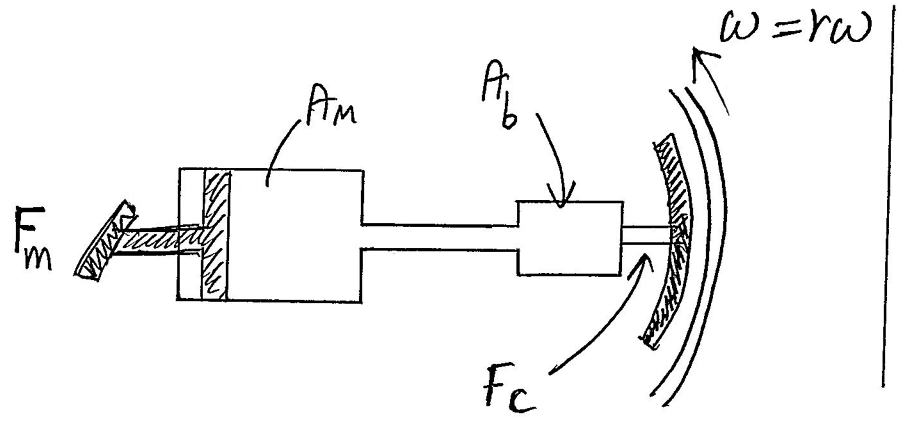
→ Equate Eq. (1) and Eq. (2)

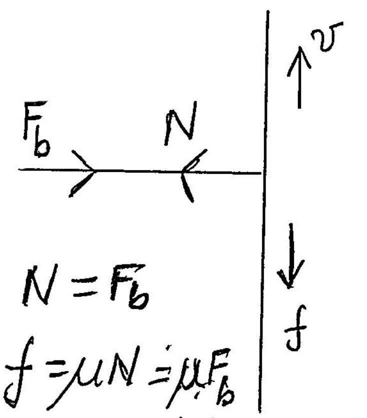
$$mg = \rho' \left(\frac{4V}{5}\right) g \Rightarrow m = \rho' \frac{4V}{5}$$

$$\rho' = \frac{5m}{V} = \frac{5(0.2375)}{4(2.5 \times 10^{-4})} = 1187.5\ \text{Kg}\,m^{-3}$$

Ans. Density of second liquid $\rho' = 1188\ \text{Kg}.m^{-3}$ (4sf)

Q8





$N = F_b$

$f = \mu N = \mu F_b$ (1)

Data:

$A_m = 6.4\ cm^2 = 6.4(10^{-2}m)^2 = 6.4\times10^{-4}\ m^2$

$A_b = 1.8\ cm^2 = 1.8\ cm^2 = 1.8(10^{-2}m)^2 = 1.8\times10^{-4}\ m^2$

$F_m = 44\ N$, radius of wheel $r = 34\ cm = 0.34\ m$

Find F_b

By Pascal's principle

pressure in master cylinder = pressure in brake cylinder

$$P_m = P_b$$

$$\frac{F_m}{A_m} = \frac{F_b}{A_b} \Rightarrow F_b = \frac{F_m A_b}{A_m}$$

$$F_b = \frac{44(1.8\times10^{-4})}{6.4\times10^{-4}} = 12.38\ N$$

Find frictional force f

$$f = \mu F_b = (0.5)(12.38) = 6.190\ N$$

Find frictional torque

$$\tau_f = fr = (6.19)(0.34) = 2.105\ Nm$$

Ans. Frictional torque $\tau_f = 2.11\ Nm$ (3 s.f.) —END—