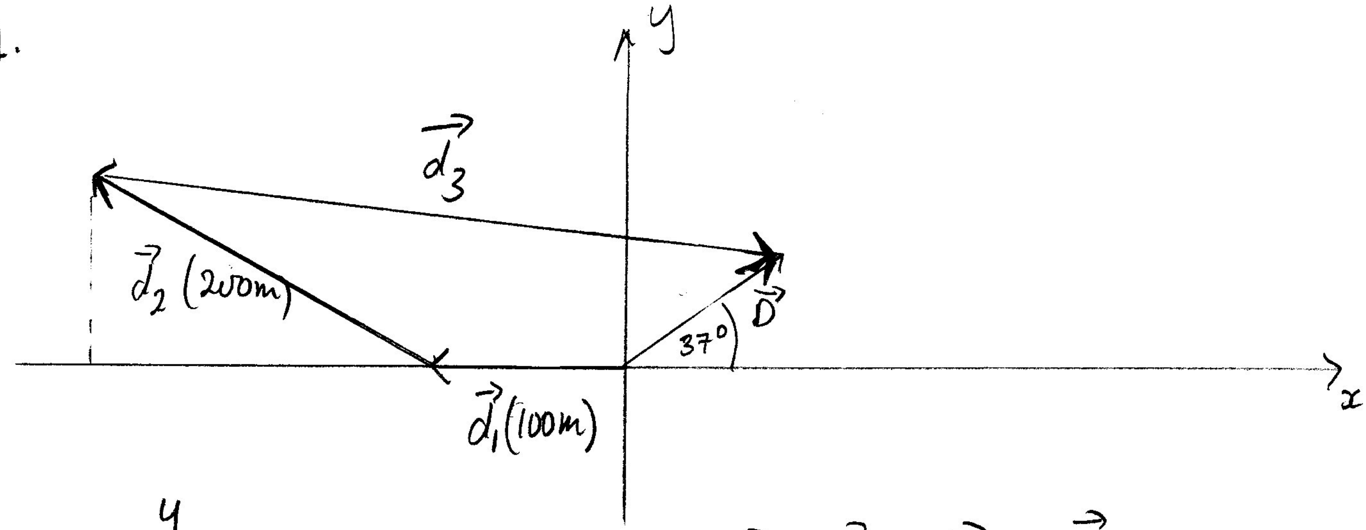
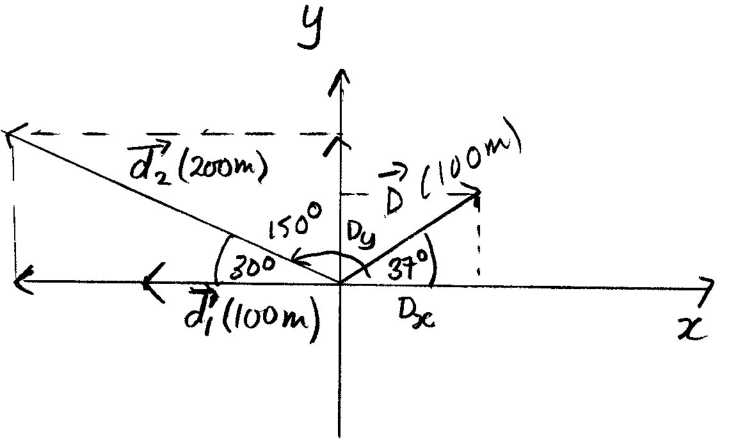
Tutorial 1. Vectors

SOLUTIONS

Q1.





$\vec{D} = \vec{d_1} + \vec{d_2} + \vec{d_3}$

$\vec{d_3} = \vec{D} - \vec{d_1} - \vec{d_2}$

$D_x = 100 \cos 37° = 79.86 m$

$D_y = 100 \sin 37° = 60.18 m$

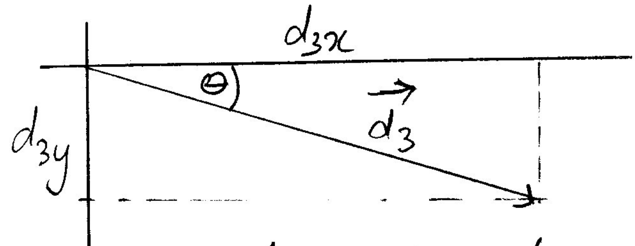
$d_{1x} = -100 m, \ d_{1y} = 0 m$

$d_{2x} = -200 \cos 30° = 200 \cos 150° = -173.2 m$

$d_{2y} = 200 \sin 30° = 200 \sin 150 = 100 m$

	x	y
$\vec{D}$	79.86	60.18
$-\vec{d_1}$	-(-100)	-(0)
$-\vec{d_2}$	-(-173.2)	-(100)
$\vec{d_3}$	353.1	-39.82
	= d_{3x}	= d_{3y}

$|\vec{d_3}| = \sqrt{d_{3x}^2 + d_{3y}^2} = \sqrt{353.1^2 + 39.82^2}$
$= 355.3 m = 355 m$ (3sf)

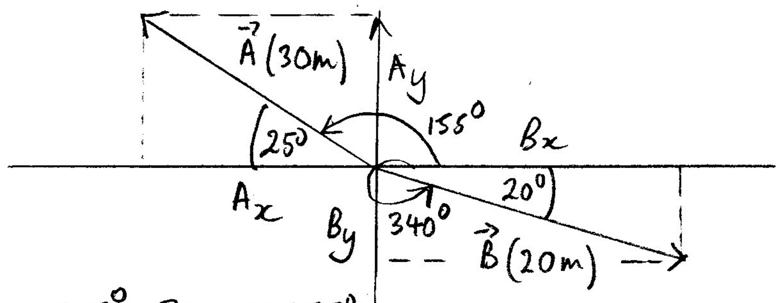


$\theta = \tan^{-1} \frac{d_{3y}}{d_{3x}} = \tan^{-1}\left(\frac{-39.82}{353.1}\right) = -6.434°$

$\phi = 360° + \theta = 360° + (-6.434°) = 353.6°$
$= 354°$ (3sf)

Ans $|\vec{d_3}| = 355 m, \ \phi = 354°$ (3sf)

Q2.



$A_x = -30\cos 25° = 30\cos 155°$
$= -27.19$ m

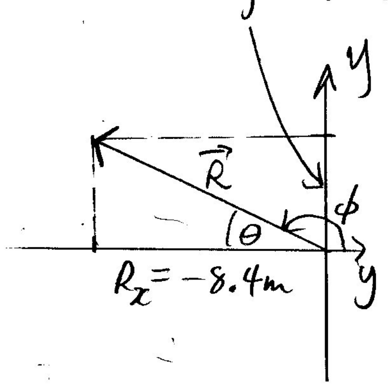
$A_y = 30\sin 25 = 12.68$m

$B_x = 20\cos 20 = 18.79$m

$B_y = -20\sin 20 = 20\sin 340°$
$= -6.840$

(i) $\vec{A} + \vec{B} = \vec{R}$

	x	y
$\vec{A}$	−27.19	12.68
$\vec{B}$	18.79	−6.84
$\vec{R}$	−8.4	5.84
	"R_x	"R_y



$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$
$= \sqrt{(-8.4)^2 + 5.84^2}$
$= 10.23$ m

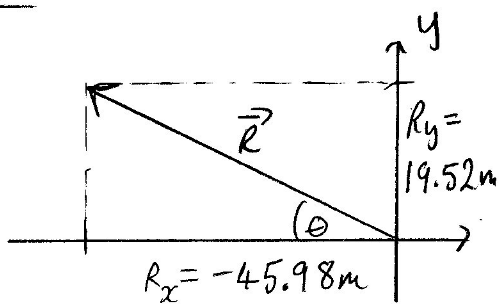
$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\left(\frac{5.84}{-8.4}\right)$
$= -34.81°$

$\phi = 180 + \theta = 180 + (-34.81) = 145.2°$

Ans $|\vec{R}| = 10.2$m, $\phi = 145°$ (3sf)

(ii) $\vec{A} - \vec{B} = \vec{R}$

	x	y
$\vec{A}$	−27.19	12.68
$-\vec{B}$	−(18.79)	−(−6.84)
$\vec{R}$	−45.98	19.52



$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$
$= \sqrt{(-45.98)^2 + 19.52^2}$
$= 49.95$ m

$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\left(\frac{19.52}{-45.98}\right) = -23.00$

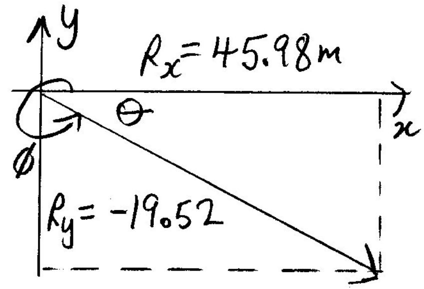
$\phi = 180° + \theta = 180° + (-23.00°) = 157°$

Ans $|\vec{R}| = 50.0$ m, $\phi = 157°$ (3sf)

Q2. cont.

(iii) $\vec{B} - \vec{A} = \vec{R}$

	x	y
$\vec{B}$	18.79	-6.840
$-\vec{A}$	-(-27.19)	-(12.68)
$\vec{R}$	45.98	-19.52



$$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{45.98^2 + (-19.52)^2}$$

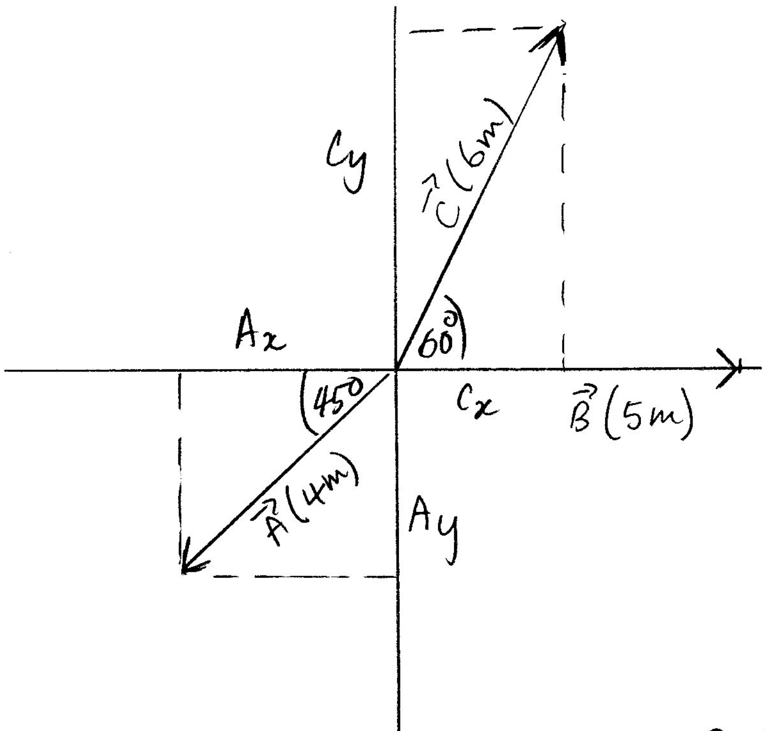
$$= 49.95\,m$$

$$\theta = \tan^{-1}\left(\frac{-19.52}{45.98}\right) = -23.00^\circ$$

$$\phi = 360 + \theta = 360 + (-23.00) = 337.0^\circ$$

Ans $|\vec{R}| = 49.95\,m$, $\phi = 337^\circ$ (3 sf)

Q3. (i)



$A_x = -4\cos 45 = -2.828\ m = -2.83\ m$ (3sf)

$A_y = -4\sin 45 = -2.828\ m = -2.83\ m$ (3sf)

$B_x = 5\ m, \quad B_y = 0\ m$

$C_x = 6\cos 60^\circ = 3\ m$

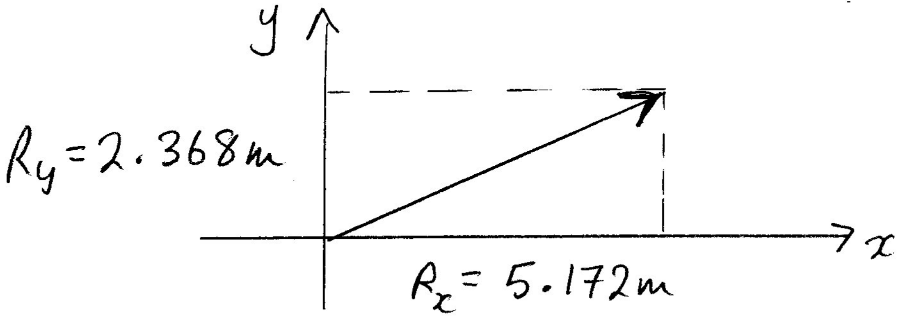
$C_y = 6\sin 60^\circ = 5.196\ m = 5.20\ m$ (3sf)

(ii) $\vec{R} = \vec{A} + \vec{B} + \vec{C}$

	x	y
$\vec{A}$	-2.828	-2.828
$\vec{B}$	5	0
$\vec{C}$	3	5.196
$\vec{R}$	5.172	2.368
	$\parallel$ R_x	$\parallel$ R_y

Ans $R_x = 5.17\ m$, $R_y = 2.37\ m$

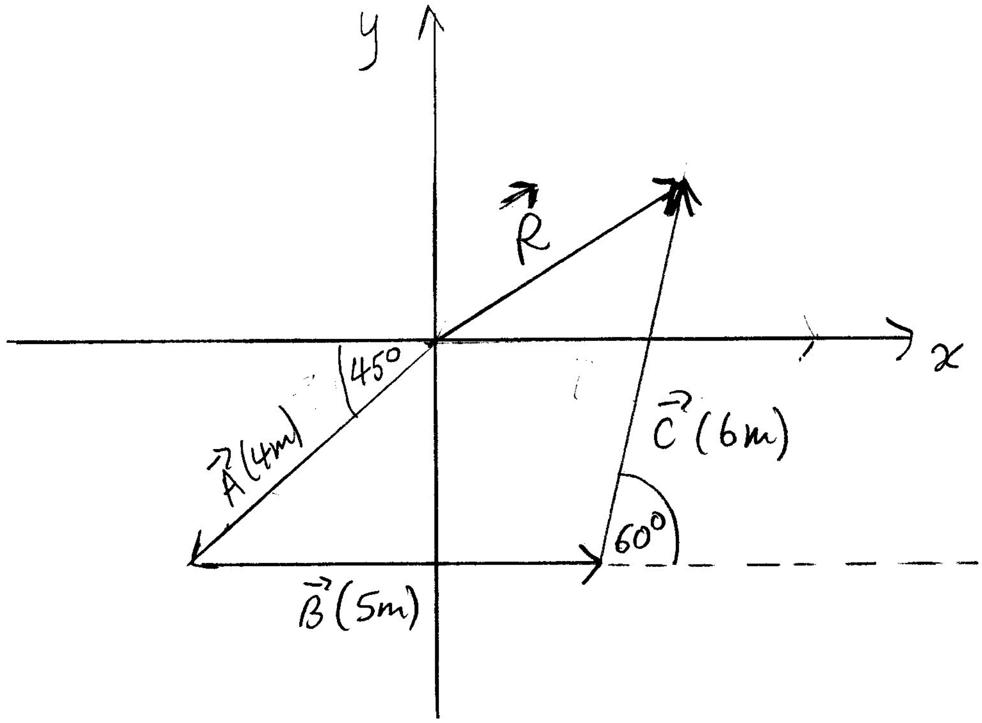
(iii) $|\vec{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{5.172^2 + 2.368^2} = 5.688\,m$



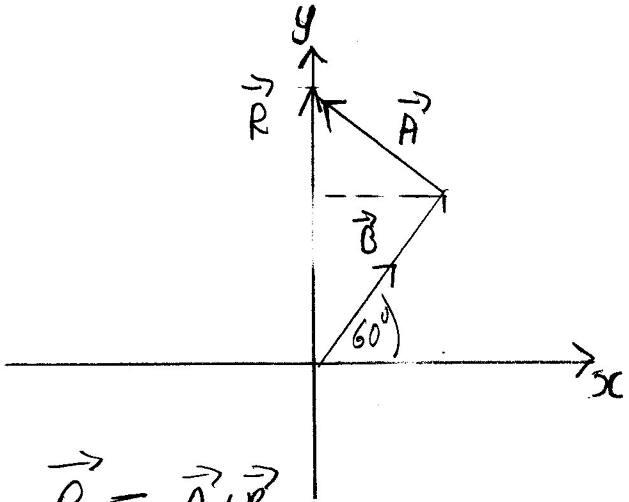
$$\theta = \tan^{-1}\left(\frac{2.368}{5.172}\right) = 24.59^\circ = \phi$$

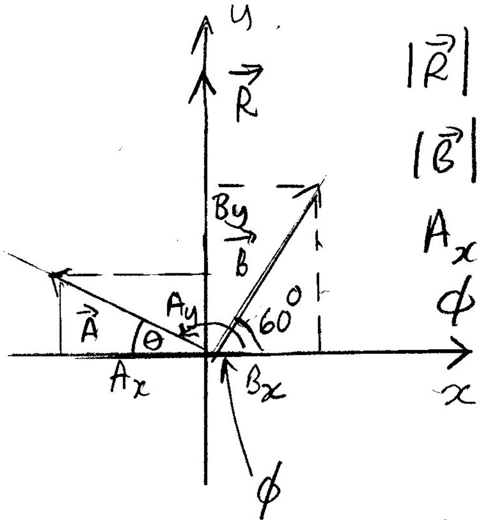
Ans $|\vec{R}| = 5.69\,m$, $\phi = 24.6^\circ$ (3sf)

(iv)



Q4





$|\vec{R}| = 150N$

$|\vec{B}| = ?$

$A_x = -60N$

$\phi = ?$

$\vec{R} = \vec{A} + \vec{B}$

$R_x = A_x + B_x = 0$, hence $B_x = -A_x = 60N$ (1)

$R_y = A_y + B_y = 150N$ (2)

Find $|\vec{B}| = B$

$$\cos 60° = \frac{B_x}{B}$$

$$B = \frac{B_x}{\cos 60°} = \frac{60}{\cos 60} = 120N$$

Find ϕ

First find A_y by finding B_y:

$B_y = 120 \sin 60 = 103.9N$

From Eq. (2)

$$A_y + B_y = 150$$

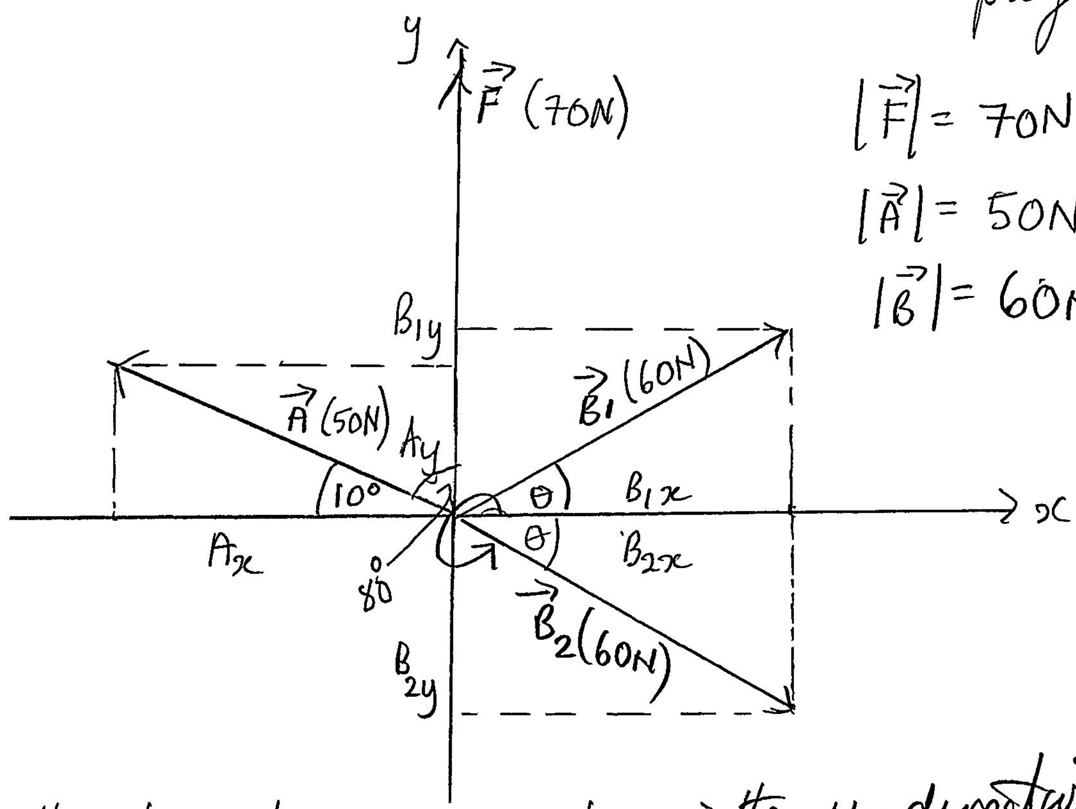
$$A_y = 150 - B_y = 150 - 103.9 = 46.1\ N$$

$$\theta = \tan^{-1}\left|\frac{A_y}{A_x}\right| = \tan^{-1}\left(\frac{46.1}{60}\right) = 37.54°$$

$$\phi = 180 - 37.54° = 142.5°$$

Ans. $|\vec{B}| = 120N$, $\phi = 143°$ (3sf)

Q7



$|\vec{F}| = 70N$

$|\vec{A}| = 50N$

$|\vec{B}| = 60N$

(i) For the box to move only in the y-direction, we must have

$$A_x = B_{1x} \quad \text{or} \quad A_x = B_{2x},$$

since there are two positions of $\vec{B}$ for which the box will move only in the y-direction

→ Now $A_x = 50 \cos 10° = 49.24N,$

hence

$$B_{1x} = B_{2x} = A_x = 49.24N$$

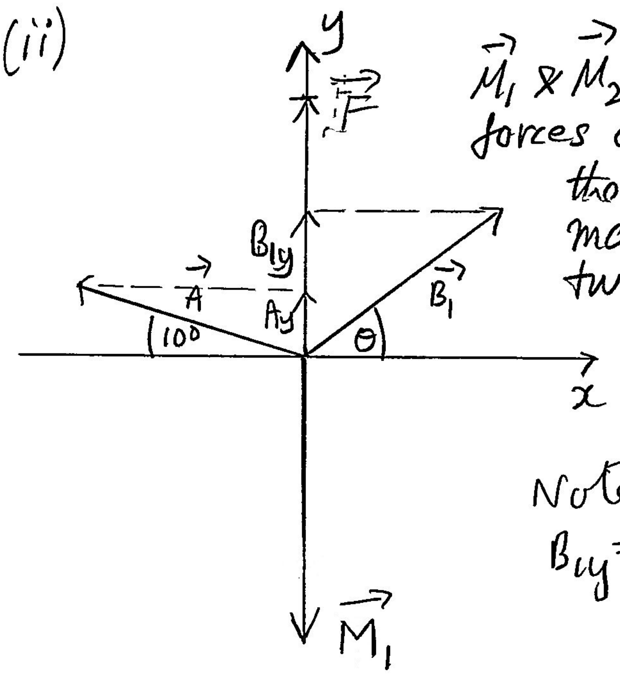
→ Hence $\cos\theta = \frac{A}{H} = \frac{49.24}{60}$

$$\theta = \cos^{-1}\left(\frac{49.24}{60}\right) = 34.84°$$

$\theta = 34.8°$ (3sf) for both $\vec{B}_1$ and $\vec{B}_2$, so

Ans. $\phi = \theta = 34.8°$ for $\vec{B}_1$ and $\phi = 360 - \theta = 325°$ for $\vec{B}_2$

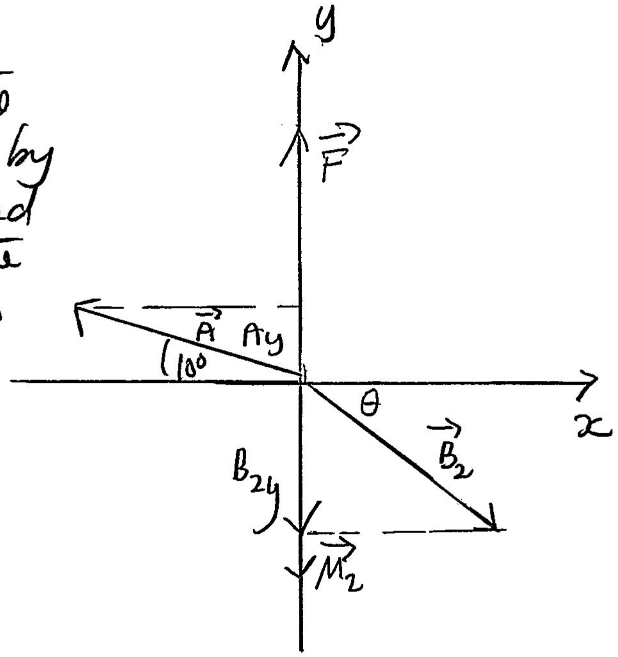
Q 7. (ii)



CASE 1

$\vec{M_1}$ & $\vec{M_2}$ are the forces applied by the second man for the two cases

Note
$B_{1y} = B_{2y}$



CASE 2

→ Both M_1 and M_2 must point in the y-direction since forces in the x-direction already balance (i.e. net force in the x-direction is zero before $\vec{M_1}$ is applied).

Case 1.

→ To stop the box moving in the y-direction, $\vec{M_1}$ must balance the y-components of $\vec{A}$, $\vec{B}$ and $\vec{F}$, hence

$M_{1y} = A_y + B_{1y} + F_y$, Noting that $|\vec{M_1}| = M_{1y}$ and $|\vec{F}| = F_y = 70N$

$A_y = 50 \sin 10^0 = 8.682N$, $B_y = 60 \sin 34.84^0 = 34.28$ N, hence

$M_{1y} = -(8.682 + 34.28 + 70) = -113.0$ N

Case 2

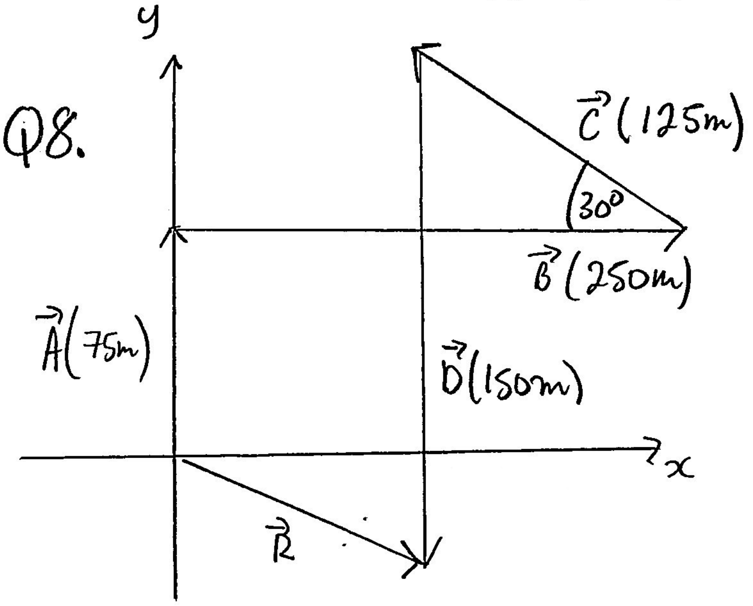
→ Note that in case 1 B_{1y} points in the same direction as F_y and A_y, so that $\vec{M_1}$ has to balance three y-components.

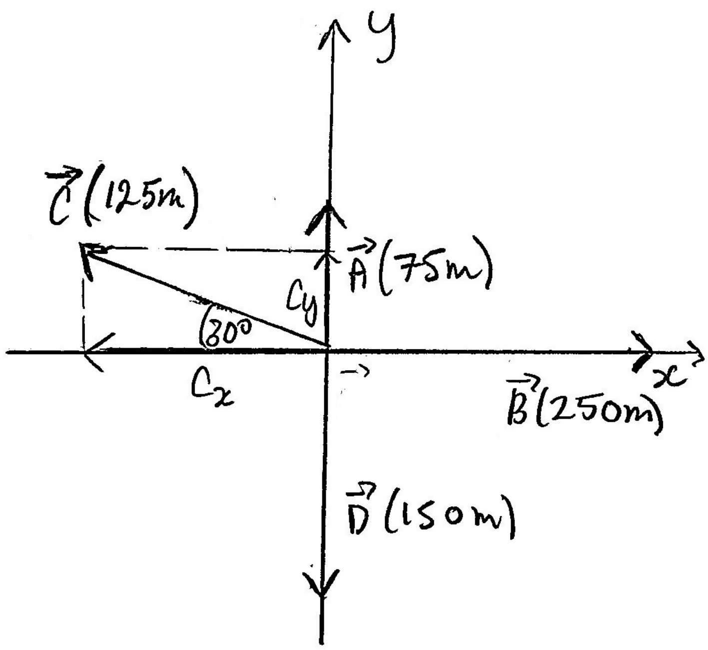
→ But for case 2, B_{2y} points in the negative y-direction, i.e., in the same direction as $\vec{M_2}$, hence $|\vec{M_2}| = M_y$ will be less than for case 1.

$M_{2y} = A_y - B_{2y} + F_y = -(8.682 - 34.28 + 70) = -44.40N$

Ans $|\vec{M_1}| = 113$ N, $|\vec{M_2}| = 44.0N$, pointing in the $-y$-direction
(3st)

Q8.





$A_x = 0m, \quad A_y = |\vec{A}| = 75m$

$B_x = |\vec{B}| = 250, \quad B_y = 0m$

$C_x = -125 \cos 30° = -108.2\,m$

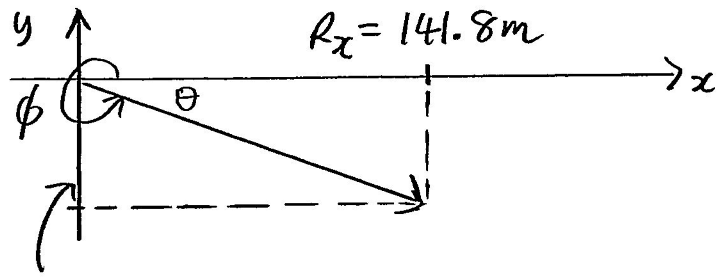
$C_y = 125 \sin 30° = 62.5\,m$

$D_x = 0m, \quad D_y = -|\vec{D}| = -150\,m$

	x	y
$\vec{A}$	0	75
$\vec{B}$	250	0
$\vec{C}$	−108.2	62.5
$\vec{D}$	0	−150
$\vec{R}$	141.8	−12.5

$|\vec{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{141.8^2 + (-12.5)^2}$

$|\vec{R}| = 142.3\,m$



$R_y = -12.5m$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-12.5}{141.8}\right) = -5.038°$

$\phi = 360 + \theta = 360 + (-5.038°) = 355.0° = 355°$ (3sf) } not asked for

Ans Displacement $= |\vec{R}| = 142\,m$ (3sf)

— END —