TUTORIAL 6
MOMENTUM
SOLUTIONS
2019/2020

Q1. Data:

$m_1 = m_2 = m$

Initial velocity of car 1 $= u_1 = 0$

Initial velocity of car 2 $= u_2 = ?$

Stopping distance $= x = 6$ m

$\mu = 0.7$

$g = 9.81\,ms^{-2}$

Stopping force = frictional force

$= f = \mu N = 0.7(2mg)$

$f = 1.4(9.81)m$

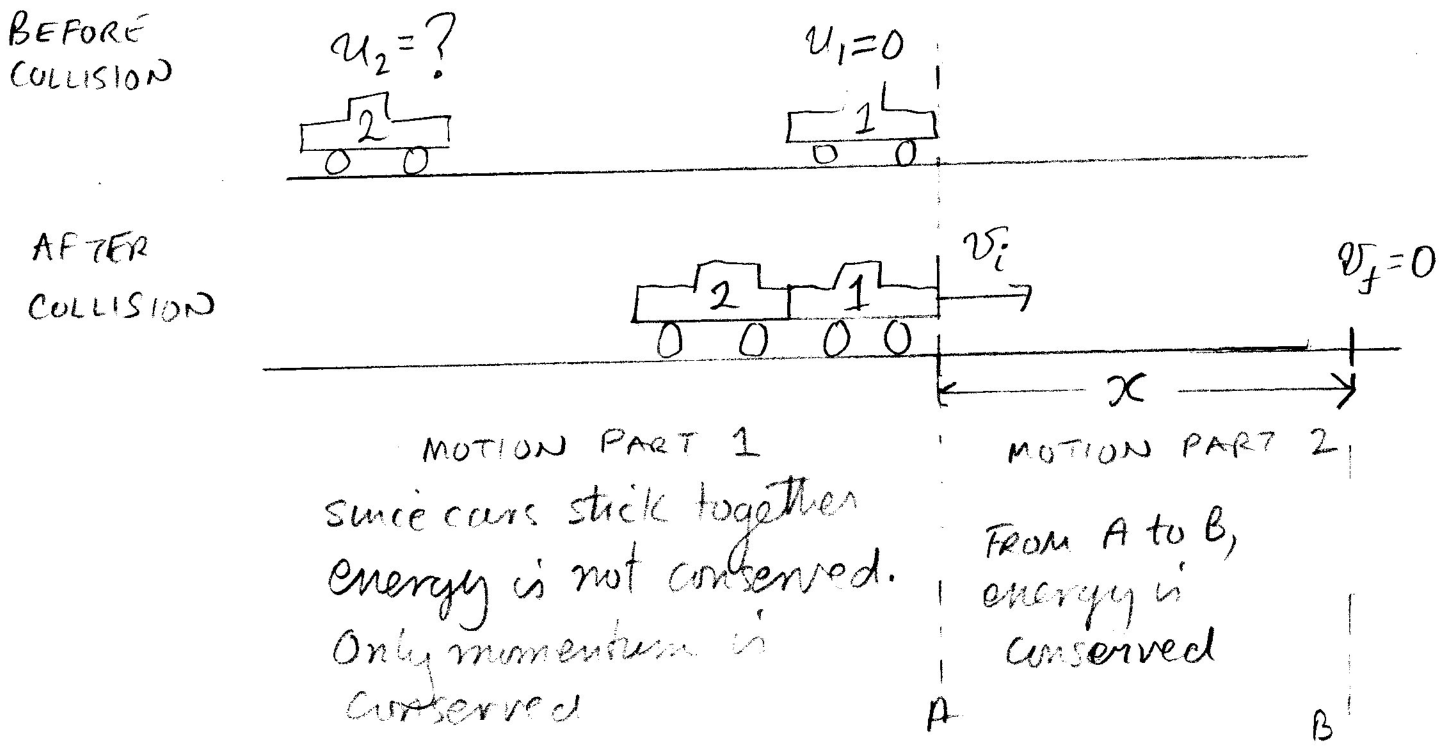
$f = 13.73\,m$

Find deceleration a:

$f = (2m)a$

$13.73m = 2ma$

$a = 13.73/2 = 6.865\,ms^{-2}$



→ By conservation of momentum:

Q1 cont

MOMENTUM BEFORE COLLISION $\equiv$ MOMENTUM AFTER COLLISION

$$mu_2 + mu_1 = 2mv_i$$

Since $u_1 = 0$, and dividing by m, we get

$$u_2 = 2v_i \qquad (1)$$

Thus, to find u_2 we must first find v_i. This can be done 2 ways.

METHOD 1.

From before

$$a = 6.865 \text{ ms}^{-2}$$

Since the stopping force f is constant, the deceleration a is constant so we can use the constant acceleration formula

$$2(-a)x = v_f^2 - v_i^2$$ (The minus sign is because a is a deceleration)

$$-2ax = 0 - v_i^2$$

$$v_i = \sqrt{2ax} = \sqrt{2(6.865)6}$$

$$v_i = 9.076 \text{ ms}^{-1}$$

METHOD 2

Use the WORK ENERGY THEOREM

$$f.x = \left|\tfrac{1}{2}(2m)v_f^2 - \tfrac{1}{2}(2m)v_i^2\right| =$$

$$f.x = mv_i^2$$

PHY1010 T6 SOL 2019/2020

Q1 cont

Sub $f = 13.73$

$$(13.73m)\,6 = m v_i^2$$

$$v_i = \sqrt{(13.73)(6)}$$

$$v_i = 9.076\ ms^{-1}$$

Sub. for v_i in Eq.(1)

$$u_2 = 2v_i = 2(9.076)$$

$$u_2 = 18.15\ ms^{-1}$$

Ans Initial velocity of car 2 = $u_2 = 18.2\ ms^{-1}$ (3sf)

Q2. Data:

mass of bullet $= m_b = 10g = 0.01\ kg$

mass of block $= M = 2kg$

$l = 5m$, height block rises $= h = 10\ cm = 0.1\ m$

initial velocity of bullet $= u_b = 500\ ms^{-1}$

initial velocity of block $= u = 0$

velocity of bullet after collision $= v_b = ?$

velocity of block the instant after collision $= v$

$g = 9.81\ ms^{-2}$

Before collision

Instant after collision

→ Collision is INELASTIC since energy is lost through friction as the bullet passes through the block. Momentum is conserved, but energy is not.

→ By conservation of momentum:

MOMENTUM BEFORE COLLISION = MOMENTUM AFTER COLLISION

$$m_b u_b + Mu = m_b v_b + Mv$$

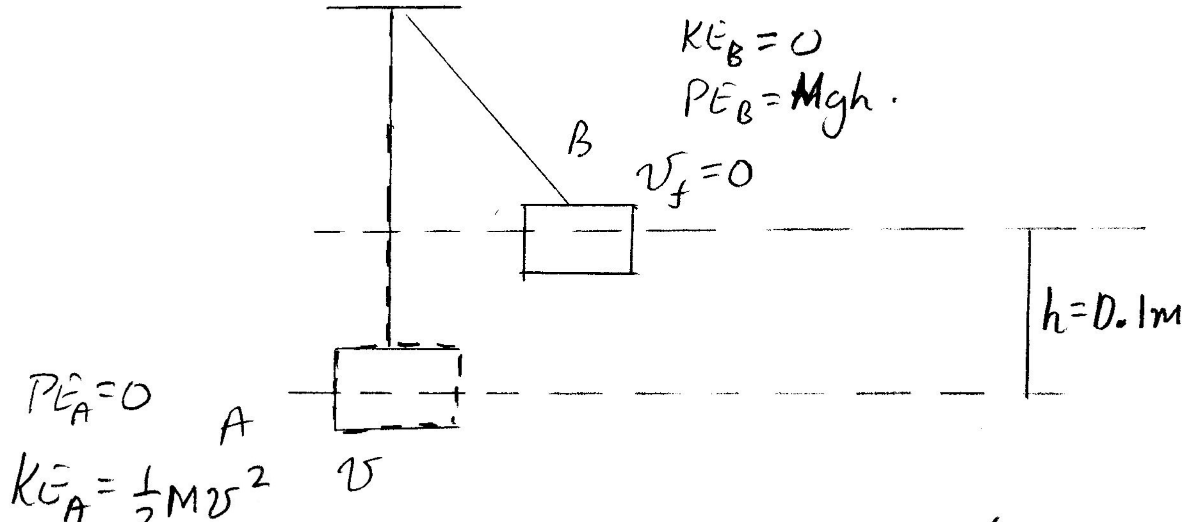
Q2. cont

since $u = 0$

$$m_b u_b = m_b v_b + Mv \qquad (1)$$

To find v_b from Eq (1) we first have to find v.

Find v



In the block moving from A to B energy is conserved:

ENERGY LOST = ENERGY GAINED

$$\Delta KE = \Delta PE$$

$$\Delta KE = KE_A - KE_B = \frac{1}{2}Mv^2 - 0 = \frac{1}{2}Mv^2$$

$$\Delta PE = PE_B - PE_A = Mgh - 0 = Mgh.$$

$$\frac{1}{2}Mv^2 = Mgh$$

$$v = \sqrt{2gh} = \sqrt{2(9.81)0.1} = 1.401\ ms^{-1}$$

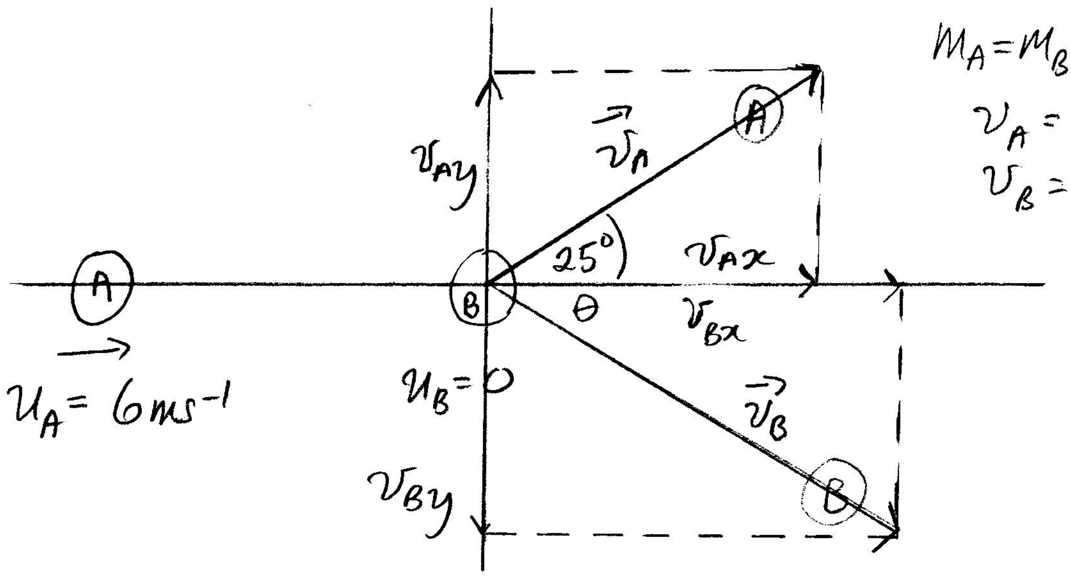
Find v_b from Eq. (1)

$$m_b u_b = m_b v_b + Mv \Rightarrow m_b v_b = m_b u_b - Mv$$

$$\Rightarrow v_b = u_b - \frac{Mv}{m_b} = 500 - \frac{2}{0.01}(1.401) = 219.8\ ms^{-1}$$

Ans. velocity of bullet as it emerges $= v_b = 220\ ms^{-1}$ (3sf)

Q3



$m_A = m_B = m$
$v_A = |\vec{v}_A|$
$v_B = |\vec{v}_B|$

$v_{Ax} = v_A \cos 25$, $v_{Ay} = v_A \sin 25$

$v_{Bx} = v_B \cos\theta$, $v_{By} = v_B \sin\theta$

→ Collision is elastic, so both energy & momentum are conserved.

→ By conservation of momentum

x-direction	y-direction
$mu_A + mu_B = mv_{Ax} + mv_{Bx}$	$0 = mv_{Ay} + mv_{By}$
sub $u_B = 0$, $u_A = 6\,ms^{-1}$ and divide by m	Divide by m
$6 = v_A \cos 25 + v_B \cos\theta$	$0 = v_{Ay} + v_{By}$
$v_B \cos\theta = 6 - v_A \cos 25$	$v_{Ay} = -v_{By}$
$v_B^2 \cos^2\theta = (6 - v_A \cos 25)^2$	$v_A \sin 25^\circ = -v_B \sin\theta$ (2a)
$v_B^2 \cos^2\theta = 36 - 12 v_A \cos 25 + v_A^2 \cos^2 25^\circ$ (1)	$v_B^2 \sin^2\theta = v_A^2 \sin^2 25$ (2)

→ By conservation of energy:

Energy before collision = Energy after collision

$$\frac{1}{2} m u_A^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2$$

Q3 cont

Sub $u_A = 6\,ms^{-1}$ and divide by $\frac{1}{2}m$

$$36 = v_A^2 + v_B^2$$

$$v_B^2 = 36 - v_A^2 \qquad (3)$$

Eq.(1) + Eq.(2)

$$v_B^2\cos^2\theta + v_B^2\sin^2\theta = 36 - 12v_A\cos 25 + v_A^2\cos^2 25 + v_A^2\sin^2 25$$

$$v_B^2\underbrace{(\cos^2\theta + \sin^2\theta)}_{=1} = 36 - 12v_A\cos 25 + v_A^2\underbrace{(\cos^2 25 + \sin^2 25)}_{=1}$$

$$v_B^2 = 36 - 12v_A\cos 25 + v_A^2 \qquad (4)$$

Sub Eq.(3) and Eq.(4)

$$36 - v_A^2 = 36 - 12v_A\cos 25 + v_A^2$$

$$12v_A\cos 25 = 2v_A^2$$

$$6\cos 25 = v_A$$

$$v_A = 5.438\,ms^{-1}$$

From Eq.(3): $v_B^2 = 36 - v_A^2$

$$v_B = \sqrt{36 - v_A^2} = \sqrt{36 - 5.438^2} = 2.535\,ms^{-1}$$

From Eq.(2.a) $v_A\sin 25 = -v_B\sin\theta$

$$\sin\theta = -\frac{v_A\sin 25}{v_B} = -\frac{5.438\sin 25}{2.535}$$

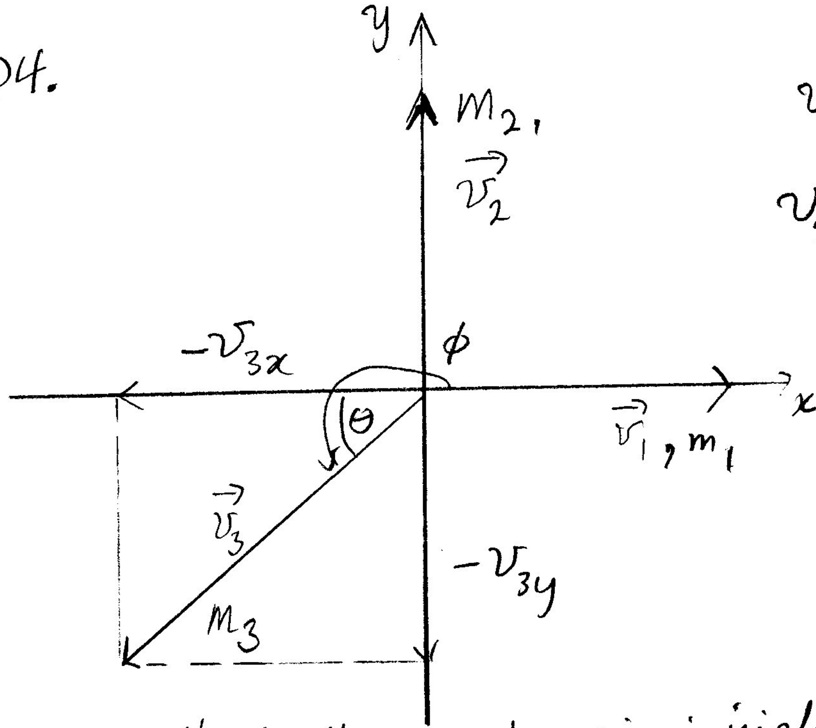
$$\sin\theta = -0.9065$$

$$\theta = \sin^{-1}(-0.9065) = -65.03^\circ$$

Since θ is clearly indicated in the diagram we can neglect the minus sign:

Ans $v_A = 5.44\,ms^{-1}$, $v_B = 2.54\,ms^{-1}$, $\theta = 65.0^\circ$ (3sf)

Q4.



$v_{1x} = |\vec{v}_1| = v = 30\,ms^{-1}$, $v_{1y} = 0$

$v_{2x} = 0$, $v_{2y} = |\vec{v}_2| = v = 30\,ms^{-1}$

$m_1 = 2m_2$

$m_3 = 3m_2$

$m_3 = 3\left(\frac{m_1}{2}\right) = \frac{3}{2} m_1$

→ Note that the explosion is inelastic. This can be seen by noting that the kinetic energy before the explosion is zero, while after the explosion it is $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 \neq 0$.

→ By conservation of momentum:

x-direction

mom. before explosion = mom. after explosion

$0 = m_1 v + (-m_3 v_{3x})$

$m_3 v_{3x} = m_1 v$

$\frac{3m_1}{2} v_{3x} = m_1 v$

$v_{3x} = \frac{2}{3} v = \frac{2}{3}(30)$

$v_{3x} = 20\,ms^{-1}$

y-direction

mom. before explosion = momentum after explosion

$0 = m_2 v + (-m_3 v_{3y})$

$m_3 v_{3y} = m_2 v$

$3m_2 v_{3y} = m_2 v$

$v_{3y} = \frac{v}{3} = \frac{30}{3}$

$v_{3y} = 10\,ms^{-1}$

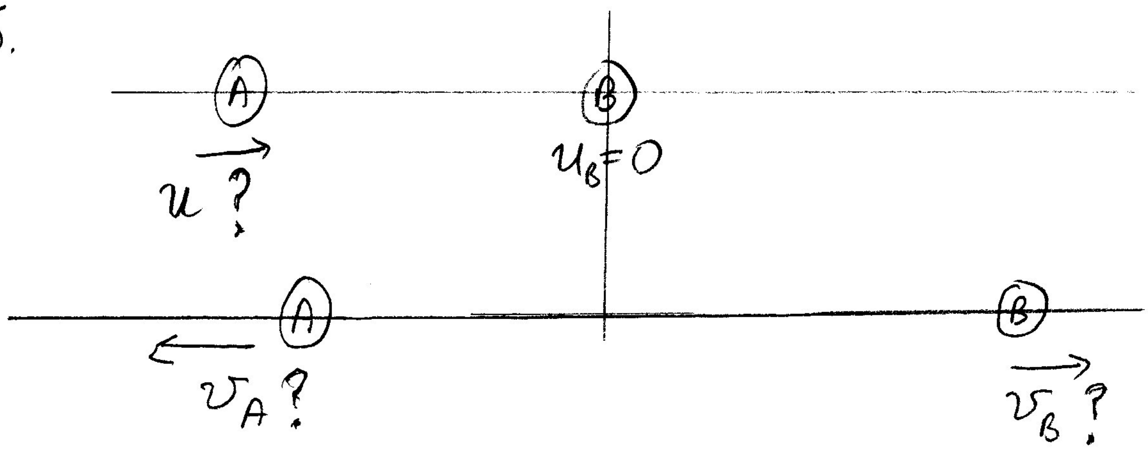
$v_3 = \sqrt{v_{3x}^2 + v_{3y}^2} = \sqrt{20^2 + 10^2} = 10.36\,ms^{-1}$

$\theta = \tan^{-1} \frac{-v_{3y}}{-v_{3x}} = \tan^{-1}\left(\frac{10}{20}\right) = 26.56°$

$\phi = 180 + 26.56 = 206.6°$

Ans $|\vec{v}_3| = 10.4\,ms^{-1}$, $\phi = 207°$ (3sf)

Q5.



→ Show that mass m losses 8/9 of its KE

→ Collision is elastic so energy & momentum are conserved.

→ Since collision is head-on, both masses continue moving horizontally.

→ By conservation of momentum:

$$mu + 2m u_B = m v_A + 2m v_B$$

Sub. $u_B = 0$ and divide by m

$$u = v_A + 2 v_B$$

$$v_B = \frac{u - v_A}{2} \qquad (1)$$

→ By conservation of energy:

$$\frac{1}{2} m u^2 + \frac{1}{2}(2m) u_B^2 = \frac{1}{2} m v_A^2 + \frac{1}{2}(2m) v_B^2$$

Sub. $u_B = 0$, and divide by $\frac{1}{2} m$

$$u^2 = v_A^2 + 2 v_B^2$$

$$2 v_B^2 = u^2 - v_A^2 = (u - v_A)(u + v_A) \qquad (2)$$

Sub. (1) into (2):

$$2 \left(\frac{u - v_A}{2} \right)^2 = (u - v_A)(u + v_A)$$

$$(u - v_A)^2 = 2(u - v_A)(u + v_A)$$

$$u - v_A = 2(u + v_A)$$

Q5

$$u - v_A = 2u + 2v_A$$
$$-u = 3v_A$$
$$v_A = -u/3 \quad (3)$$

KE of mass m before collision = $KE_{A(before)} = \frac{1}{2}mu^2$

KE of mass m after collision = $KE_{A(after)} = \frac{1}{2}mv_A^2$

$$KE_{A(lost)} = \frac{1}{2}mu^2 - \frac{1}{2}mv_A^2 = \frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{-u}{3}\right)^2$$
$$KE_{A(lost)} = \frac{1}{2}mu^2\left(1 - \frac{1}{9}\right) = \frac{1}{2}mu^2\left(\frac{8}{9}\right) = 8/9\, KE_{A(before)}$$

$$KE_{A(lost)} = 8/9\, KE_{A(before)}$$

Ans1 This shows that after the collision mass m loses $\frac{8}{9}$ of its original KE.

We can also express the answer as the fractional loss of $KE_{A(before)}$

$$\text{Fractional loss of KE of mass m} = \frac{KE_{A(lost)}}{KE_{A(before)}} = \frac{8/9\left(\frac{1}{2}mu^2\right)}{\frac{1}{2}mu^2} = 8/9$$

Ans2 The fractional loss of KE = 8/9

— END —