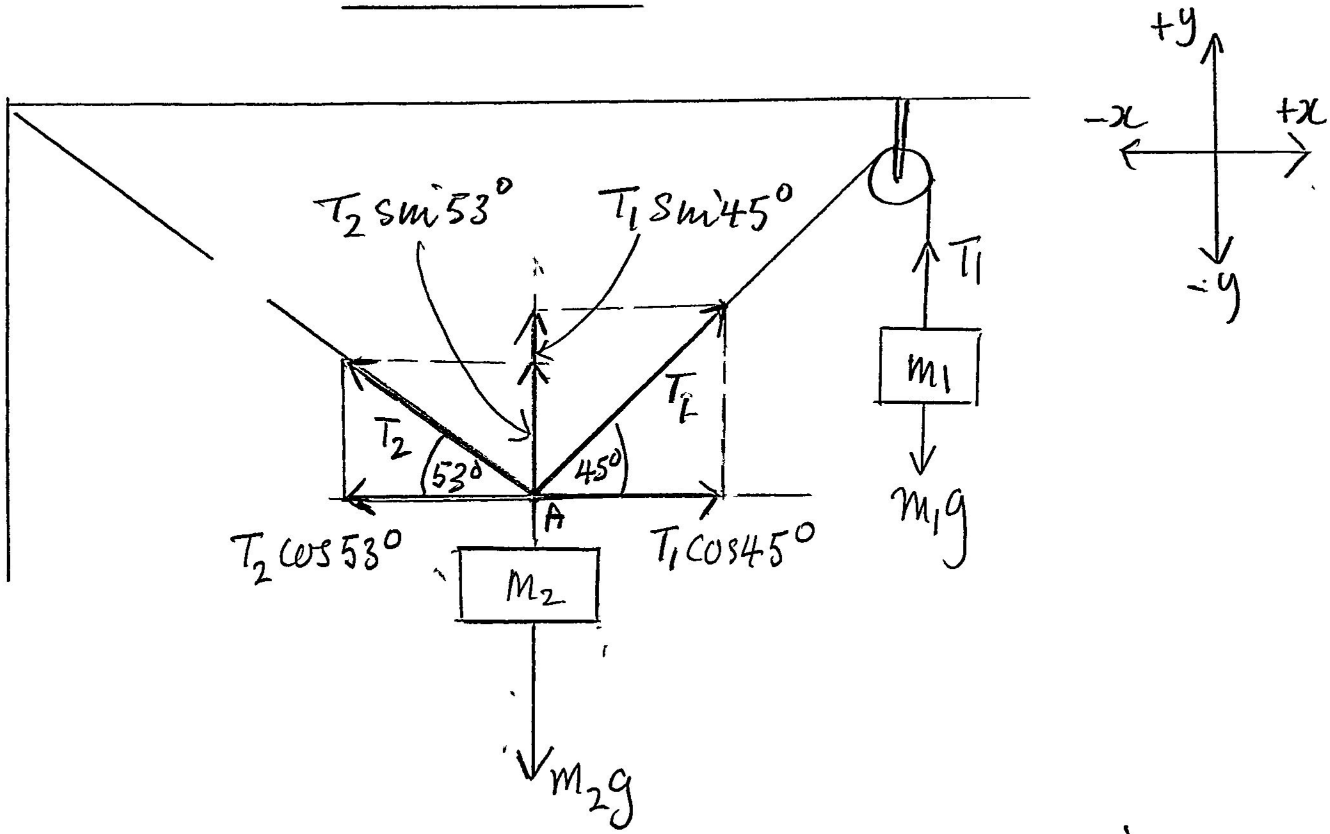
PHY 1010
TUTORIAL 8.
STATIC EQUILIBRIUM
SOLUTIONS

Q1.



→ Since the chord is uniform and the pulley is frictionless

$$T_1 = m_1 g = (10)(9.81) = 98.10 \text{ N}$$

→ At node A:

$\sum F_x = 0$: $\quad T_1 \cos 45^\circ - T_2 \cos 53^\circ = 0$

$$T_2 \cos 53^\circ = T_1 \cos 45^\circ$$

$$T_2 = \frac{T_1 \cos 45^\circ}{\cos 53^\circ} = \frac{98.1 \cos 45^\circ}{\cos 53^\circ} = 115.2 \text{ N}$$

Q1 cont.

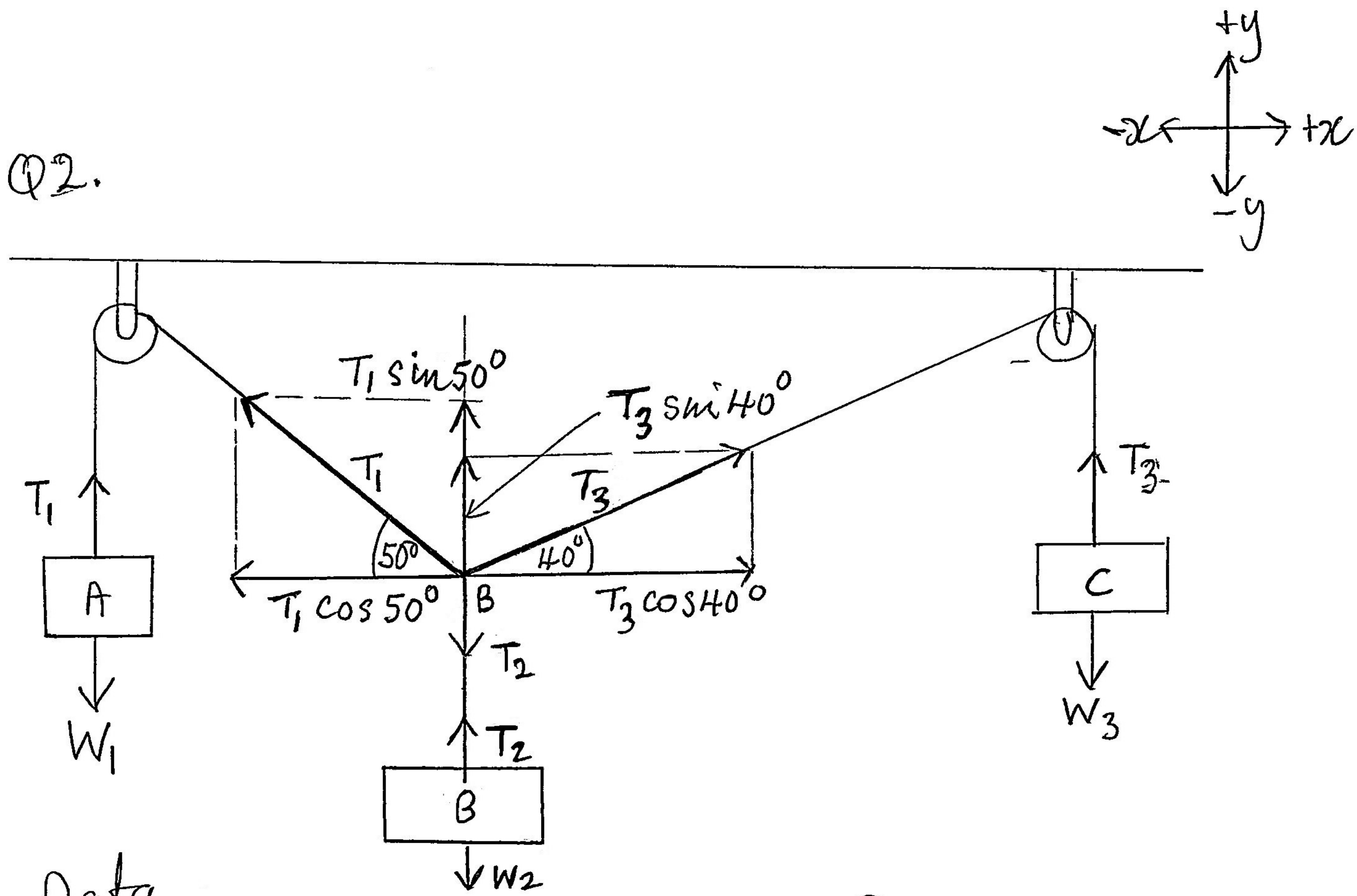
$$\sum F_y = 0: \quad T_2 \sin 51^\circ + T_1 \sin 45^\circ = m_2 g$$

$$m_2 = \frac{T_2 \sin 53 + T_1 \sin 45^\circ}{g}$$

$$m_2 = \frac{115.2 \sin 53^\circ + 98.1 \sin 45^\circ}{9.81} = 16.44\,\text{Kg}$$

Ans. Tensions $T_1 = 98.1\,\text{N}$ and $T_2 = 115\,\text{N}$,
with mass $m_2 = 16.4\,\text{Kg}$ (3sf)

Q2.



<u>Data</u>

$W_1 = 8.00\,N$, $W_2 = ?$, $W_3 = ?$

→ For a uniform (homogeneous), frictionless string the tension has the same magnitude throughout the string.

→ Hence, the string acting W_1 with tension T_1. also acts on W_2 with tension T_1

→ And, the string acting on W_3 with tension T_3 also acts on W_2 with tension T_3.

Q2 cont

Apply first condition for static equilibrium

Forces on block A:

$\Sigma F_x = 0, \quad \Sigma F_y = T_1 - W_1 = 0 \Rightarrow T_1 = W_1 = 800N$

Forces on block B:

$\Sigma F_x = 0, \quad \Sigma F_y = T_2 - W_2 = 0 \Rightarrow W_2 = T_2$

Forces at point B:

$$\Sigma F_x = -T_1 \cos 50^\circ + T_3 \cos 40^\circ = 0$$

$$T_1 \cos 50^\circ = T_3 \cos 40^\circ$$

$$T_3 = \frac{T_1 \cos 50}{\cos 40^\circ} = \frac{800 \cos 50^\circ}{\cos 40^\circ} = 671.3N$$

$$\Sigma F_y = T_1 \sin 50^\circ + T_3 \sin 40^\circ - T_2 = 0$$

$$T_2 = 800 \sin 50^\circ + 671.3 \sin 40^\circ$$

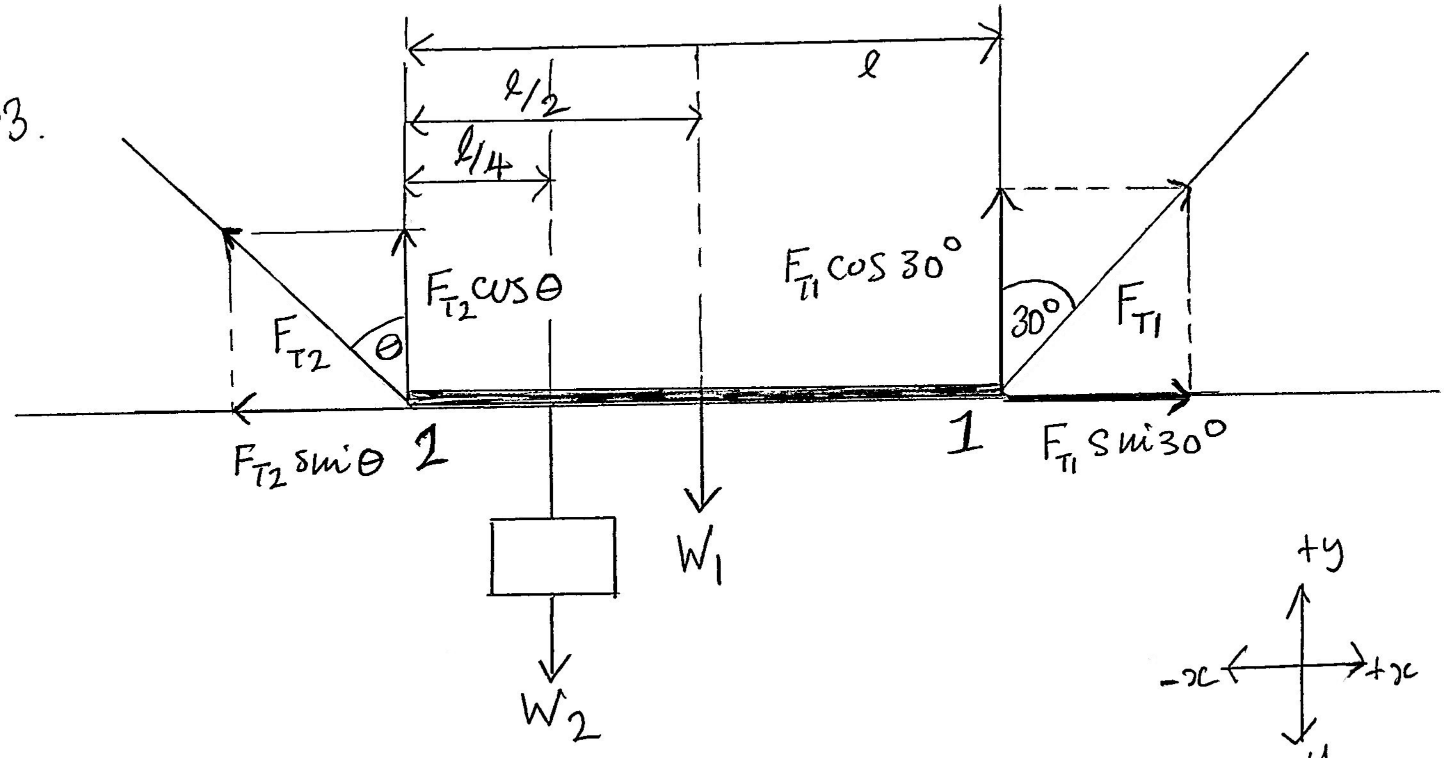
$T_2 = 1044\,N$, hence $W_2 = T_2 = 1044\,N$

Forces on block C:

$\Sigma F_x = 0, \quad \Sigma F_y = T_2 - W_3 = 0 \Rightarrow W_3 = T_3 = 671.3N$

Ans $W_2 = 1040\,N$, $W_3 = 671\,N$ (3sf)

Q3.



<u>Data</u>

Weight of board $= W_1 = 120N$

Weight of block $= W_2 = 400N$

$F_{T1} = ?$, $F_{T2} = ?$, $\theta = ?$

Apply 1st condition for static equilibrium to whole system:

$\Sigma F_x = 0$: $\quad -F_{T2}\sin\theta + F_{T1}\sin 30° = 0$

$$F_{T2}\sin\theta = F_{T1}\sin 30°$$

$$F_{T2} = F_{T1}\frac{\sin 30°}{\sin\theta} \qquad (1)$$

$\Sigma F_y = 0$: $\quad F_{T2}\cos\theta + F_{T1}\cos 30° - W_1 - W_2 = 0$

$$F_{T2}\cos\theta = W_1 + W_2 - F_{T1}\cos 30°$$

$$F_{T2} = \frac{W_1 + W_2 - F_{T1}\cos 30°}{\cos\theta} \qquad (2)$$

Equate Eq (1) & (2)

$$F_{T1}\frac{\sin 30°}{\sin\theta} = \frac{W_1 + W_2 - F_{T1}\cos 30°}{\cos\theta}$$

Q3 cont

$$\frac{F_{T1}\sin 30^\circ}{W_1+W_2-F_{T1}\cos 30^\circ} = \frac{\sin\theta}{\cos\theta} = \tan\theta \qquad (3)$$

To proceed, we must find F_{T1} using the 2^{nd} condition for static equilibrium.

Apply 2^{nd} condition for static equilibrium

→ Choose axis of rotation at point 2 since we have two unknown quantities at point 2: θ and F_{T2}.

$$\sum \tau_c = W_1\frac{l}{2} + W_2\, l/4$$

$$\sum \tau_{AC} = l F_{T1}\cos 30^\circ$$

$$\sum \tau_{AC} - \sum \tau_c = l F_{T1}\cos 30^\circ - W_1\frac{l}{2} - W_2\, l/4 = 0$$

$$l F_{T1}\cos 30^\circ = W_1\frac{l}{2} + W_2\, l/4$$

Divide by l and make F_{T1} the subject

$$F_{T1} = \frac{W_1/2 + W_2/4}{\cos 30^\circ} = \frac{120/2 + \frac{400}{4}}{\cos 30^\circ} = 184.8\,N$$

Sub. $F_{T1} = 184.8\,N$ into Eq. (3)

$$\tan\theta = \frac{F_{T1}\sin 30^\circ}{W_1+W_2-F_{T1}\cos 30^\circ} = \frac{184.8\sin 30^\circ}{120+400-184.8\cos 30^\circ}$$

$$\tan\theta = 0.2567$$

$$\theta = \tan^{-1} 0.2567$$

$$\theta = 14.39^\circ$$

Q3. cont

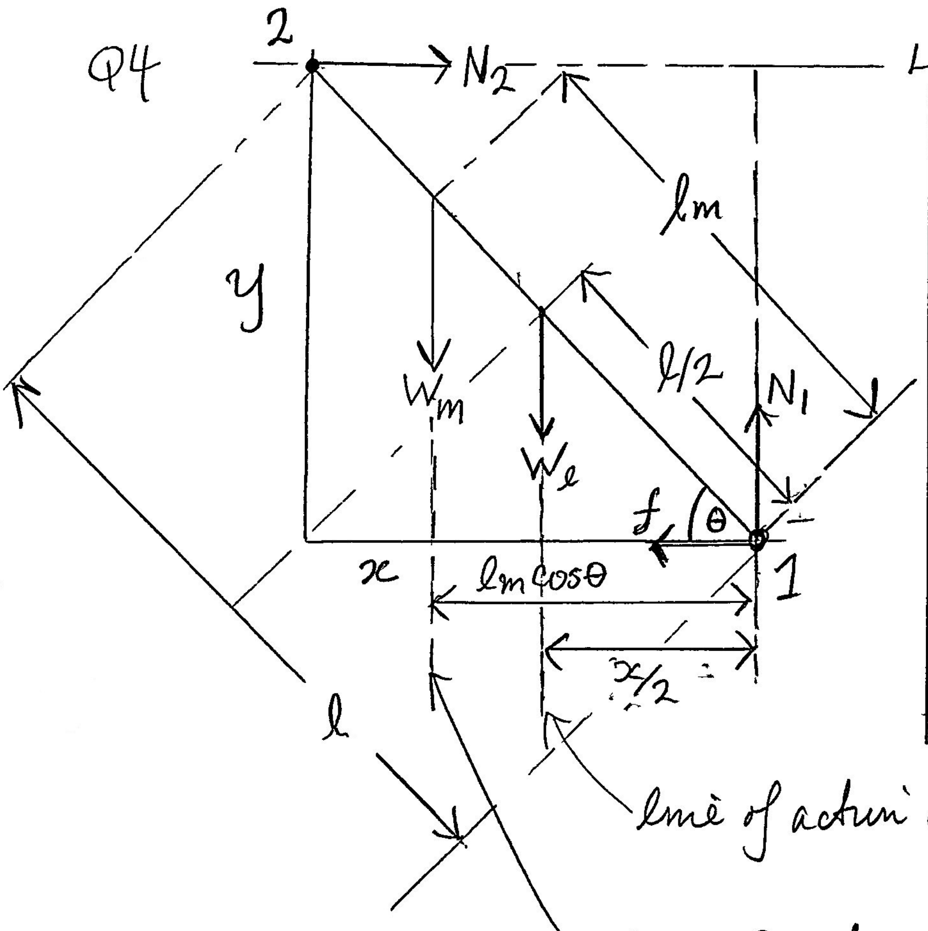
Find F_{T2} from Eq. (1)

$$F_{T2} = F_{T1}\,\frac{\sin 30^\circ}{\sin\theta} = \frac{184.8 \sin 30^\circ}{\sin 14.39^\circ}$$

$$F_{T2} = 371.5\ \text{N}$$

Ans. $F_{T1} = 185\ \text{N}$, $F_{T2} = 372\ \text{N}$ and $\theta = 14.4^\circ$ (3sf)

Q4

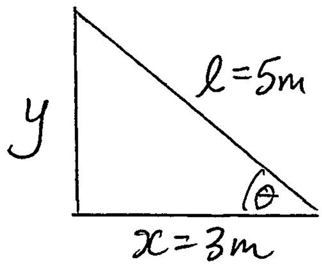


NOTE THE FOLLOWING

1. By the geometry of a triangle, the lines drawn through the center of the hypotenuse ⊥ to the sides bisect the sides.
2. Since the wall is frictionless, there are no frictional forces at the wall.
3. CHOOSE POINT 1 AS THE AXIS OF ROTATION since f, N_1 & θ are not known.
4. $f = \mu N_1$

Data

$l = 5m$, $x = 3m$, $W_l = 200N$; $W_m = 600N$, $\mu = 0.45$



$$l^2 = x^2 + y^2$$
$$y = \sqrt{l^2 - x^2} = \sqrt{5^2 - 9^2}$$
$$y = 4\ m$$

$$\tan\theta = \frac{y}{x} = \frac{4}{3}$$
$$\theta = \tan^{-1} 4/3$$
$$\theta = 53.13^\circ$$

Apply 1st condition for static equilibrium

$$\Sigma F_x = 0: \quad N_2 - f = 0 \Rightarrow N_2 = f \Rightarrow N_2 = \mu N_1 \quad (1)$$

$$\Sigma F_y = 0: \quad N_1 - W_l - W_m = 0 \Rightarrow N_1 = W_l + W_m = 200 + 600$$
$$N_1 = 800N \quad (2)$$

From (1) & (2)

$$N_2 = \mu N_1 = (0.45)(800) = 360\ N$$

Q4 cont.

Apply 2nd condition for static equilibrium

$$\Sigma \tau_{AC} = W_m l_m \cos\theta + W_e \frac{x}{2}$$

$$\Sigma \tau_C = y N_2$$

$$\Sigma \tau_{AC} - \Sigma \tau_C = W_m l_m \cos\theta + W_e \frac{x}{2} - y N_2 = 0$$

Make l_m the subject:

$$W_m l_m \cos\theta = y N_2 - W_e \frac{x}{2}$$

$$l_m = \frac{y N_2 - W_e x/2}{W_m \cos\theta}$$

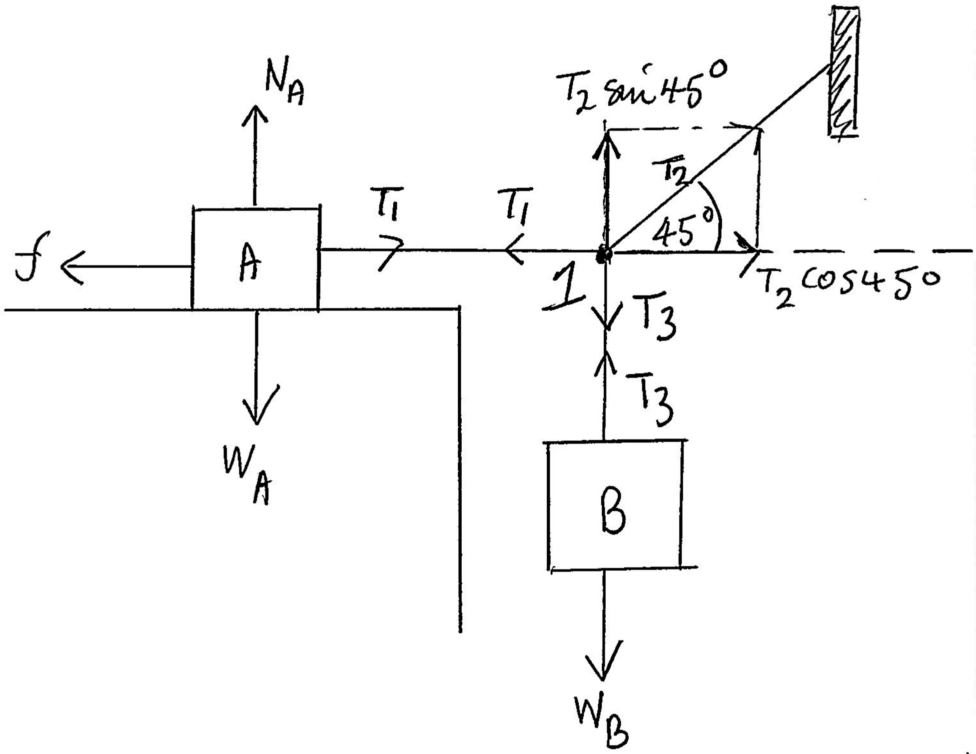
$$l_m = \frac{(4)(360) - (200)(3)(\frac{1}{2})}{600 \cos 53.13}$$

$$l_m = 3.167 \text{m}$$

→ Note that this l_m is calculated for the maximum allowed friction, $f = f_{max} = \mu N_1$ (we left out the subscript "max" to save writing). Thus, if the man climbs beyond l_m the ladder will slip; if he climbs to $l' < l_m$ there is more than enough frictional force to prevent slipping. We conclude that $l_m = 3.167$ m is the maximum length the man can climb without the ladder slipping.

Ans. Maximum l_m without slipping $= l_m = 3.17$ m (3sf)

Q5



Data:

$W_A = 90N$

$W_B = 15N$

$T_3 = W_B = 15N$

$\mu = 0.30$

Apply 1^{st} condition of static equilibrium to point 1.

$$\Sigma F_x = 0: \quad T_2 \cos 45^\circ - T_1 = 0 \Rightarrow T_1 = T_2 \cos 45^\circ \quad (1)$$

$$\Sigma F_y = T_2 \sin 45^\circ - T_3 \Rightarrow T_2 \sin 45^\circ = T_3$$

$$T_2 = \frac{T_3}{\sin 45^\circ} = \frac{15}{\sin 45^\circ} = 21.21\,N$$

Sub. $T_2 = 21.21$ into Eq. (1)

$$T_1 = T_2 \cos 45^\circ = 21.21 \cos 45^\circ = 15.00\,N$$

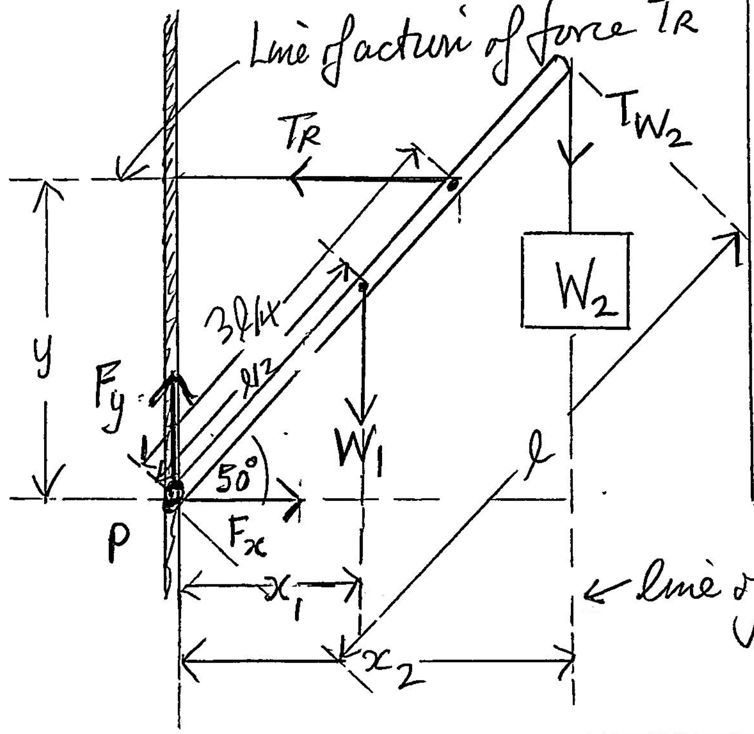
Available frictional force f

$$f = \mu N_A = (0.3)(90) = 27N$$

Ans For the system to remain in equilibrium there must be enough frictional force f to balance the tension T_1, i.e., we must have $f \geq T_1$

Indeed, this is satisfied since $(f = 27N) > (T_1 = 15.0N)$

Q6



- Consider only forces acting on the boom.
- Take axis of rotation at the hinge P
- $x_2 = l\cos 50°$
- $y = 3/4\, l \sin 50°$
- $x_1 = l/2 \cos 50°$
- $W_1 = 400N$, $W_2 = T_{W_2} = 2000N$

Apply 1st condition for statitic equilibrium to whole system (1)

$\sum_x F_x' = 0$: $F_x - T_R = 0 \Rightarrow F_x = T_R$

$\sum_y F_y' = 0$: $F_y - T_{W_2} - W_1 = 0 \Rightarrow F_y = T_{W_2} + W_1 = 2000 + 400$

$F_y = 2400N$

Apply 2nd condition for static equilibrium

$\sum \tau_{AC} = y\, T_R = T_R 3/4\, l \sin 50° = 0.5745\, l\, T_R$

$\sum \tau_C = x_1 W_1 + T_{W_2} x_2 = \left(\frac{l}{2}\cos 50°\right) W_1 + \left(l \cos 50°\right) T_{W_2}$

$= l\,(128.6 + 1286) = 1415\, l$

$\sum \tau_{AC} - \sum \tau_C = 0.5745\, l\, T_R - 1415\, l = 0$

Divide by l and make T_R the subject

$0.5745\, T_R = 1415$

$T_R = \frac{1415}{0.5745} = 2463N$

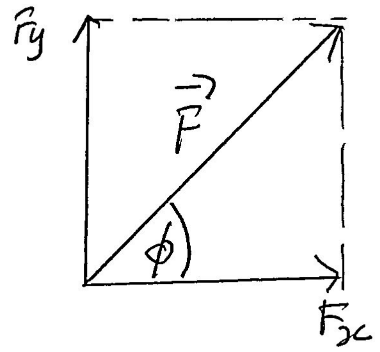
Q.6 Cont.

From Eq (1)

$$F_x = T_R = 2463\,N$$

Ans $F_x = 2460\,N$, $F_y = 2400\,N$ (3sf)

AN EXTRA BIT

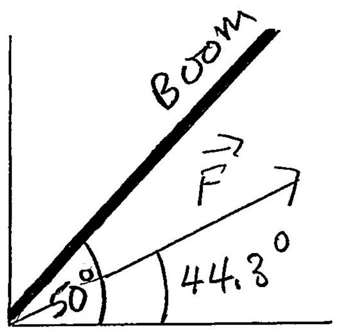


$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{2463^2 + 2400^2}$$

$$= 3439\,N = 3440\,N \text{ (3sf)}$$

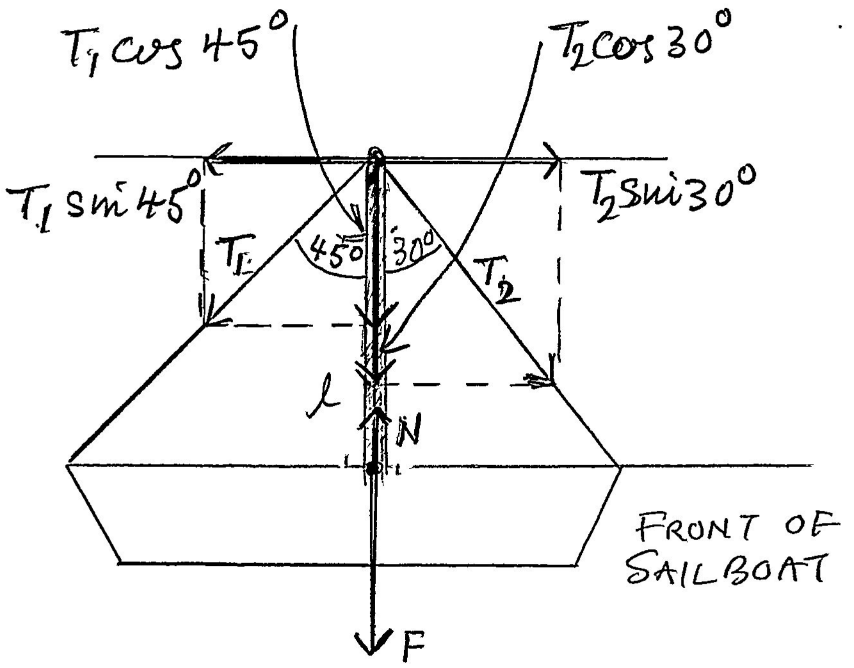
$$\phi = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{2400}{2463}\right)$$

$$\phi = 44.26° = 44.3° \text{ (3sf)}$$



This examplifies the point in the revision notes that the resultant force in such problems will not, in general, lie along the boom (or beam).

Q7.



Data: $l = 10\,m$

$T_2 = 5000\,N$

$T_1 = ?$

F = force mast exerts on sailboat

N = Normal reaction of sailboat on mast.

By Newton's third law

$$F = N \quad (1)$$

vectors N and F not drawn to scale since $N = F$.

→ since the mast is in static equilibrium

$\Sigma F_x = 0$: $\quad T_1 \sin 45^\circ - T_2 \sin 30^\circ = 0$

$$T_1 \sin 45^\circ = T_2 \sin 30^\circ$$

$$T_1 = \frac{T_2 \sin 30^\circ}{\sin 45^\circ} = \frac{5000 \sin 30^\circ}{\sin 45^\circ} = 3535\,N$$

and

$\Sigma F_y = 0$: $\quad N - T_1 \cos 45^\circ - T_2 \cos 30^\circ = 0$

$$N = T_1 \cos 45^\circ + T_2 \cos 30^\circ$$

$$N = 3535 \cos 45^\circ + 5000 \cos 30^\circ = 6829\,N$$

From Eq.(1), $N = F = 6829\,N$

Ans. Tension in the rear cable $T_1 = 3540\,N$ (3sf)

The force the mast exerts on the sailboat

$F = 6830\,N$ (3sf)

— END —