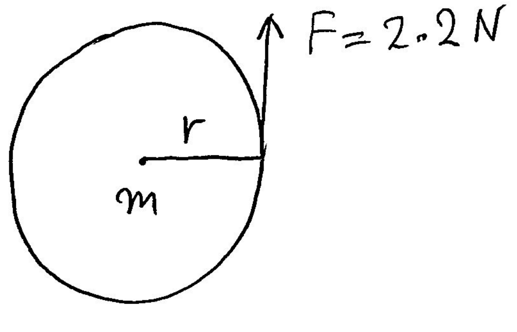
Rotational Work & Energy

Solutions

Q1.



Data: $1 \text{ rev} = 2\pi \text{ rad}$, $1 \text{ rad} = \frac{1 \text{ rev}}{2\pi}$

mass of solid disk $= m = 52 \text{ Kg}$

radius $= r = 32 \text{ cm} = 0.32 \text{ m}$

Moment of inertia of a solid disk $= I = \frac{1}{2} m r^2$

a) How long to accelerate the disk from rest, $\omega_i = 0$, to $\omega_f = 210 \text{ rev/min}$?

$$\omega_f = 210 \frac{\text{rev}}{\text{min}} = 210 \frac{(2\pi \text{ rad})}{(60 \text{ s})} = 21.99 \text{ rad}\cdot\text{s}^{-1}$$

Since the force F is constant the angular acceleration α is also constant

→ Use formula

$$\tau = I\alpha = I\left(\frac{\omega_f - \omega_i}{t}\right)$$

$$t = \frac{I(\omega_f - \omega_i)}{\tau} \quad (1)$$

Find τ:

$$\tau = F.r = (2.2)(0.32) = 0.704 \text{ Nm}$$

Find I:

$$I = \frac{1}{2} m r^2 = \frac{1}{2}(52)(0.32^2) = 2.662 \text{ Kg m}^2$$

Sub. into Eq. (1)

$$t = \frac{2.662(21.99 - 0)}{0.704}$$

$$t = 83.15 \text{ s}$$

Ans. Time taken to accelerate the disk $= t = 83.2 \text{ s}$ (3sf)

Q1 (b) Since the acceleration is constant, we can use the constant acceleration formula

$$\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0 \qquad (2)$$

Since $\omega_0 = 0$ and $\theta_0 = 0$ (i.e. choose $\theta_0 = 0$), we have

$$\theta = \frac{1}{2}\alpha t^2$$

From (a) $t = 83.15 s$

Find α:

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{21.99 - 0}{83.15} = 0.2645 \text{ rad.s}^{-2}$$

Sub. into (2)

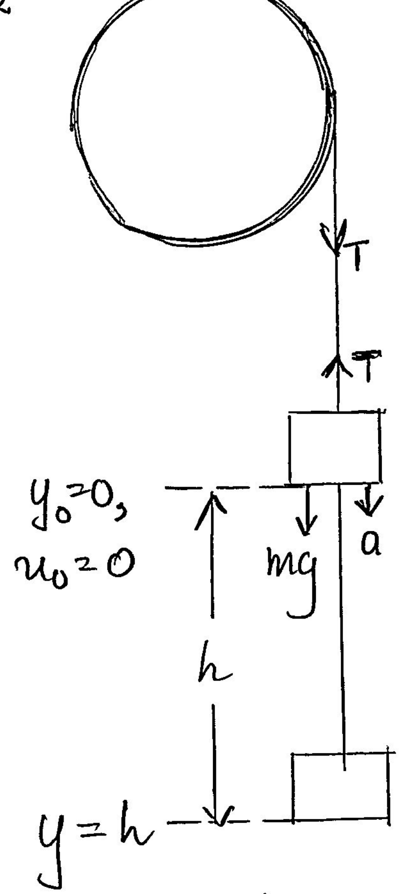
$$\theta = \frac{1}{2}(0.2645)(83.15^2) = 914.4 \text{ rad}$$

$$= 914.4 \left(\frac{\text{rev}}{2\pi}\right)$$

$$= 145.5 \text{ rev} = 146 \text{ rev}$$

Ans. No. of revolutions in time $t = 146$ rev (3sf)

Q2



Data:

$r = 24\text{ cm} = 0.24\text{ m}$

$m = 100\text{ g} = 0.1\text{ Kg}$

$h = 180\text{ cm} = 1.8\text{ m}$

$t = 1.5\text{ s}$

$I = ?$

$T = ?$

(a) $\tau = Tr$ (1)

$\tau = I\alpha$

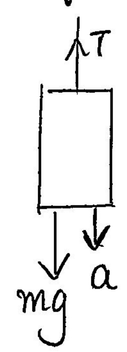
Sub $\alpha = \frac{a}{r}$ (2)

Equate Eqs (1), & (2)

$Tr = I\frac{a}{r}$

$T = \frac{Ia}{r^2}$ (3)

Find T from the free-body diagram of mass m



$F_{net} = ma$

$F_{net} = mg - T$

$mg - T = ma$

$T = mg - ma$

$T = m(g-a)$ (4)

Equate Eqs (3) & (4):

$$\frac{Ia}{r^2} = m(g-a)$$

$$I = \frac{mr^2}{a}(g-a) = mr^2\left(\frac{g}{a} - 1\right) \quad (5)$$

Q2 cont.

Find a:

$$\underset{h}{\overset{\shortparallel}{y}} = \frac{1}{2}at^2 + \underset{0}{\overset{\shortparallel}{u_0}}t + \underset{0}{\overset{\shortparallel}{y_0}}$$

$$h = \frac{1}{2}at^2$$

$$a = \frac{2h}{t^2} = \frac{2(1.8)}{1.5^2} = 1.6\ ms^{-2}$$

Find I from Eq.(5):

$$I = mr^2\left(\frac{g}{a} - 1\right) = (0.1)(0.24)^2\left(\frac{9.81}{1.6} - 1\right)$$

$$I = 0.02956\ Kg\,m^2$$

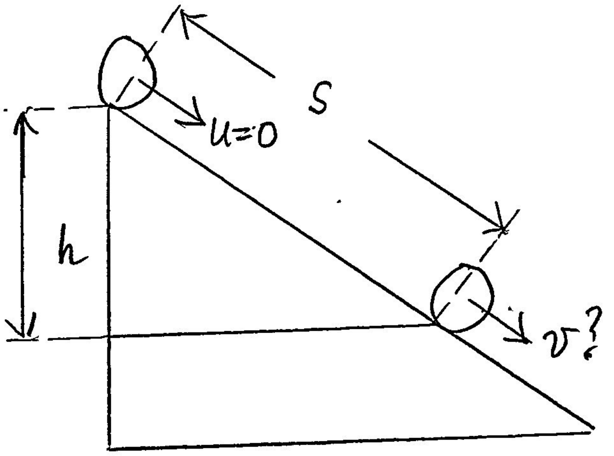
Find T:

Method 1	Method 2
From Eq. (3)	From Eq. (4)
$T = \frac{Ia}{r^2}$	$T = m(g-a)$
$T = \frac{(0.02956)(1.6)}{0.24^2}$	$T = 0.1(9.81 - 1.6)$
$T = 0.8211\ N$	$T = 0.8210\ N$

Ans. The moment of inertia of the cylinder is $I = 0.0296\ Kg\,m^2$ (3sf).

The tension is $T = 0.821\ N$ (3sf)

Q3



Data:

$g = 9.81\ ms^{-2}$

radius $= r = 6\ cm = 0.06\ m$

radius of gyration $= K = 5\ cm$
$= 0.05\ m$

$I = mK^2$

$h = 40\ cm = 0.4\ m$

(a) Find final linear speed?

Use conservation of energy

Energy lost = Energy gained

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Sub. $\omega = v/r$, and $I = mk^2$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(mK^2)\left(\frac{v^2}{r^2}\right)$$

Divide by m

$$gh = \frac{1}{2}v^2 + \frac{K^2v^2}{2r^2}$$

$$gh = \frac{v^2}{2}\left(1 + \frac{K^2}{r^2}\right)$$

$$\frac{2gh}{1 + \frac{K^2}{r^2}} = v^2$$

$$v = \sqrt{\frac{2gh}{1 + K^2/r^2}} = \sqrt{\frac{2(9.81)(0.4)}{\left(1 + \frac{0.05^2}{0.06^2}\right)}} = 2.152\ ms^{-1}$$

Ans. Velocity at the end point $= 2.15\ ms^{-1}$ (3sf)

Q3. b.

$$v = r\omega$$

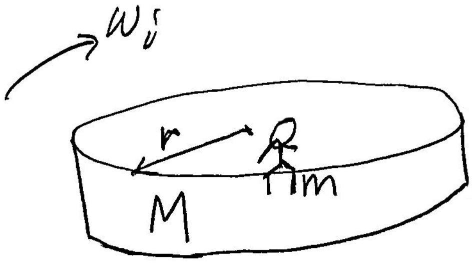
$$\omega = v/r = \frac{2.152}{0.06} = 35.87\ \frac{rad}{s}$$

$$= 35.87 \left(\frac{rev}{2\pi}\right)\frac{1}{s} = 5.709\ rev.s^{-1}$$

Ans. The wheel rotates with angular velocity

$\omega = 5.71\ rev.s^{-1}$ (3 sf)

Q4



Moment of inertia of a solid disk $= I_i = \frac{1}{2}Mr^2$ (1)

Moment of inertia of person on edge of the merry-go-round $= I_p = mr^2$ (2)

$M = 150\ \text{Kg}$, $m = 80\ \text{Kg}$, $r = 6\ \text{m}$

$\omega_i = 15\ \frac{\text{rev}}{\text{min}} = 15\left(\frac{2\pi f}{(60s)}\right) = 1.571\ \text{rad}\ s^{-1}$

$\omega_f = ?$

By conservation of angular momentum

$$L_i = L_f$$

$$I_i \omega_i = I_f\ \omega_f \qquad (3)$$

$$I_f = I_i + I_p$$

$$I_f = \frac{1}{2}Mr^2 + mr^2 \qquad (4)$$

Sub. Eq. (1) and Eq. (4) into Eq (3)

$$\frac{1}{2}Mr^2\omega_i = \left(\frac{M}{2} + m\right)r^2\omega_f$$

Divide by r^2 and rearrange

$$\omega_f = \frac{M\omega_i}{2\left(\frac{M}{2}+m\right)} = \frac{M\omega_i}{M+2m} = \frac{(150)(1.571)}{150+2(80)}$$

$$\omega_f = 0.7602\ \text{rad.s}^{-1} = 0.7602\left(\frac{\text{rev}}{2\pi}\right)\left(\frac{1}{s/60}\right) = 7.259\ \text{rev/min}$$

Ans. The new angular speed $\omega_f = 7.26$ rev/min (3sf)

Q 5(a)

Data: $I = 0.005\ Kgm^2$

$m_1 = 520g = 0.52Kg$

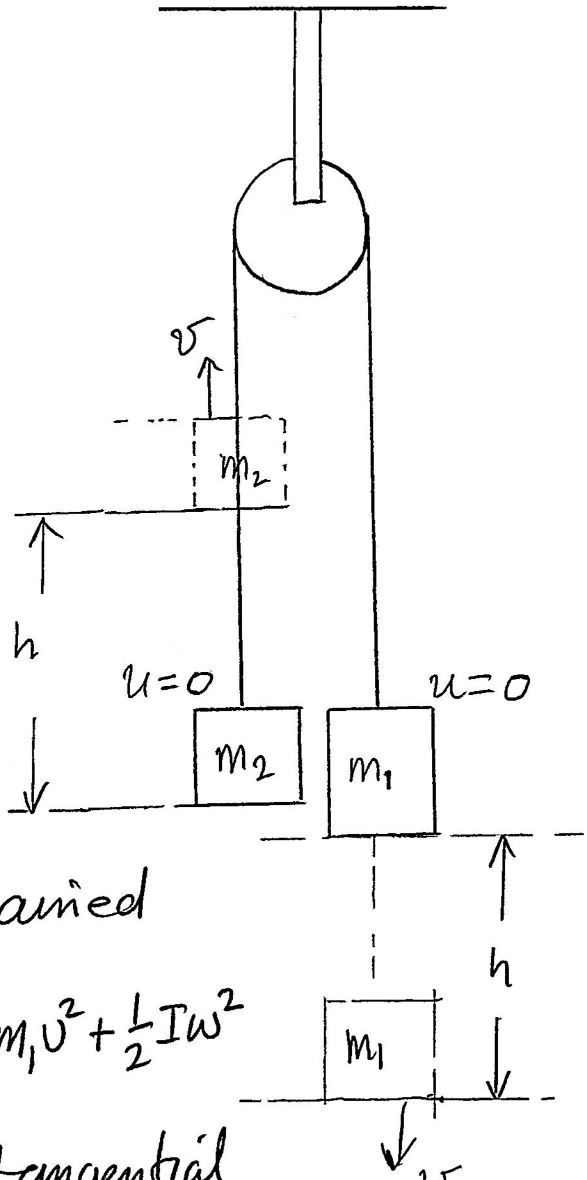
$m_2 = 500g = 0.5Kg$

$r = 7cm = 0.07m$

$h = 2m$

$g = 9.81ms^{-2}$

a) Use conservation of energy



Energy lost = Energy gained

(1) $m_1gh = m_2gh + \frac{1}{2}m_2v^2 + \frac{1}{2}m_1v^2 + \frac{1}{2}I\omega^2$

→ Note that v is also the tangential velocity so we can substitute

$v = r\omega$

into Eq. 1. to get

$$m_1gh = m_2gh + \frac{1}{2}m_2r^2\omega^2 + \frac{1}{2}m_1r^2\omega^2 + \frac{1}{2}I\omega^2$$

$$gh(m_1 - m_2) = \frac{\omega^2}{2}\left(m_2r^2 + m_1r^2 + I\right)$$

$$\omega = \sqrt{\frac{2gh(m_1 - m_2)}{m_2r^2 + m_1r^2 + I}}$$

Q5(a) cont.

$$\omega = \sqrt{\frac{2(9.81)2(0.52-0.5)}{(0.5)(0.07^2)+(0.52)(0.07)^2+0.005}}$$

$$\omega = \sqrt{\frac{0.7848}{0.009998}} = 8.860\ \text{rad s}^{-1}$$

Ans. The wheel is turning with angular velocity $\omega = 8.86\ \text{rad. s}^{-1}$ (3sf)

5.(b) Find time to drop.

Acceleration is constant since it is produced by gravity, so we can use constant acceleration formula.

$$h = \frac{1}{2}at^2 + ut + h_0$$

$u=0$ and choosing $h_0 = 0$, we have

$$h = \frac{1}{2}at^2, \qquad t = \sqrt{\frac{2h}{a}} \qquad (2)$$

Find a:

$$2ah = v^2 - u^2$$

since $u=0$, $\quad 2ah = v^2, \quad a = \frac{v^2}{2h}$

Sub. $v = r\omega$,

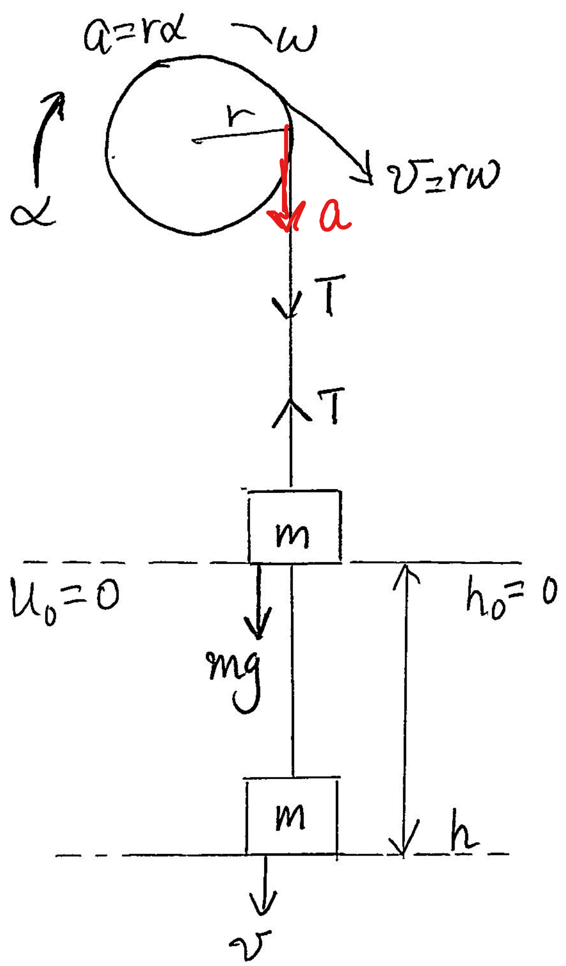
$$a = \frac{r^2\omega^2}{2h} = \frac{(0.07)^2(8.86^2)}{2(2)} = 0.09616\ \text{ms}^{-2}$$

Sub a into Eq. (2) we get

$$t = \sqrt{\frac{2(2)}{0.09616}} = 6.449\ s$$

Ans. Time taken to drop $= t = 6.45\ s$ (3sf)

Q6



$r = 6\,cm = 0.06\,m$

$m = 200g = 0.2\,Kg$

$h = 100\,cm = 1\,m$

$t = 5s$

$\alpha = ?$

$I = ?$

$T = ?$

Find a:

$$h = \frac{1}{2}at^2 + \underset{0}{u_0}t + \underset{0}{h_0}$$

$$h = \frac{1}{2}at^2$$

$$a = \frac{2h}{t} = \frac{2(1)}{5^2} = 0.08\,ms^{-2}$$

Find α:

The tangential acceleration of the wheel is equal to the acceleration of the mass m. Hence, the angular acceleration of the wheel α is found as follows:

$$a = r\alpha$$

Q6 cont.

$$\alpha = \frac{a}{r} = \frac{0.08}{0.06} = 1.333 \text{ rad. } s^{-2}$$

Find v after 5s:

$$v = at + u_0 = (0.08)(5) + 0 = 0.4 \, ms^{-1}$$

This tangential velocity is the same as the velocity v of the mass m.

Find I by conservation of energy:

Energy lost = Energy gained (after 5 sec)

$$mgh = KE + KE_{ROT}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Sub. $\omega = v/r$, since v of mass is the same as the tangential velocity

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$$

$$\frac{Iv^2}{2r^2} = mgh - \frac{1}{2}mv^2$$

$$I = \frac{2r^2m}{v^2}\left(gh - \frac{v^2}{2}\right)$$

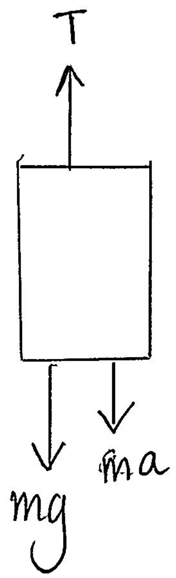
$$I = r^2m\left(\frac{2gh}{v^2} - 1\right)$$

$$I = (0.06^2)(0.2)\left[\frac{2(9.81)(1)}{0.4^2} - 1\right]$$

$$I = 0.08757 \text{ Kg} m^2$$

Q6. cont

Find T from a free body diagram:



$$F_{net} = ma$$

$$F_{net} = mg - T$$

$$mg - T = ma$$

$$T = mg - ma = m(g-a)$$

$$T = 0.2(9.81 - 0.08)$$

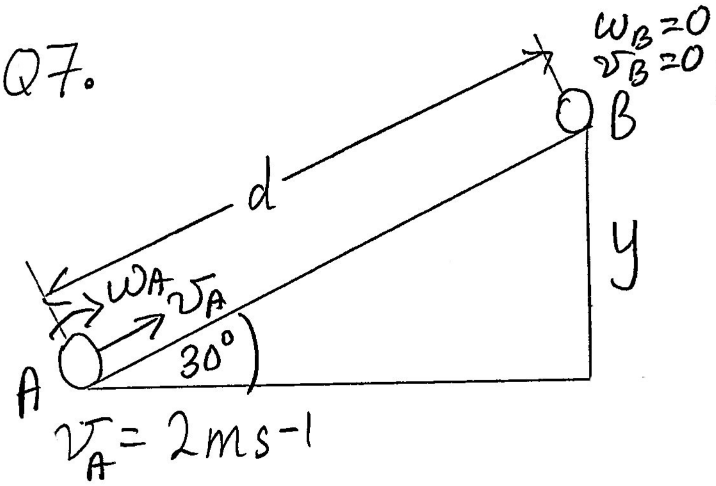
$$T = 1.946\,N$$

Ans. Angular acceleration of the wheel $\alpha = 1.33\,rad.s^{-2}$ (3sf)

Moment of inertia of wheel $I = 0.0876\,Kgm^2$ (3sf)

Tension in the string $T = 1.95\,N$ (3sf)

Q7.



For a disk the moment of inertia is

$$I = \frac{1}{2} m r^2$$

$$y = d \sin 30^\circ$$

$$\omega = v/r \quad , \quad g = 9.81\, ms^{-2}$$

At A: $PE_A = 0, \quad KE_A = \frac{1}{2} m v_A^2, \quad KE_{A,ROT} = \frac{1}{2} I \omega_A^2$

At B:

$$PE_B = mgy, \quad KE_B = 0, \quad KE_{B,ROT} = 0$$

By conservation of energy:

Energy lost = Energy gained

$$KE_A + KE_{A,ROT} = PE_B$$

$$\frac{1}{2} m v_A^2 + \frac{1}{2} I \omega_A^2 = mgy$$

$$\frac{1}{2} m v_A^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\frac{v_A^2}{r^2} = mgd\sin 30^\circ$$

Divide by m and simplify:

$$\frac{2v_A^2}{4} + \frac{v_A^2}{4} = gd\sin 30^\circ$$

$$\frac{3v_A^2}{4} = gd\sin 30^\circ$$

$$d = \frac{3v_A^2}{4g\sin 30^\circ} = \frac{3(2^2)}{4(9.81\sin 30^\circ)} = 0.6116\,m$$

Ans. The disk rolls a distance up the incline

$d = 0.612\ m$ (3sf)

— END —