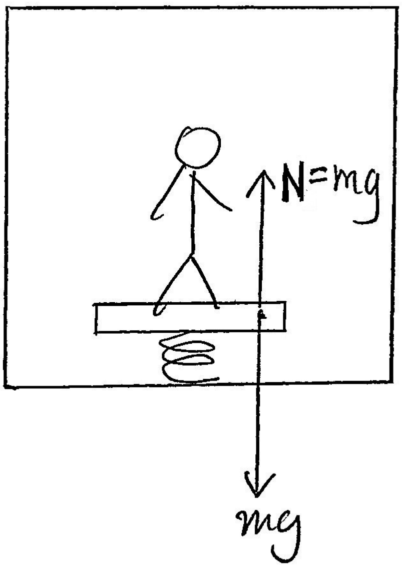
PHY 1010
TUTORIAL 3

Newton's laws of motion

Q1

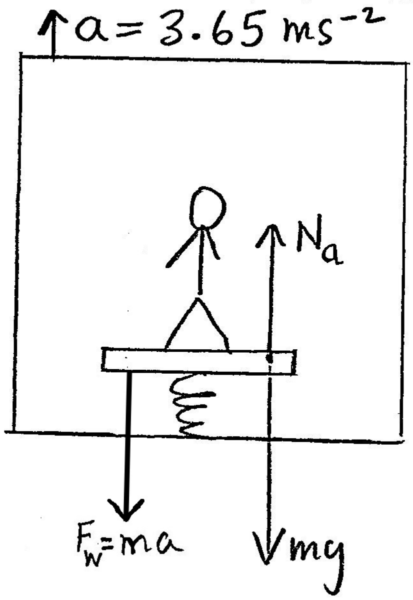


Data:
$m = 45\,kg$, $g = 9.81\,ms^{-2}$

By Newton's third law, at equilibrium, the woman's weight, mg, pushes on the scale, while the scale pushes back with the same force, the normal reaction N. THE NORMAL REACTION IS WHAT THE SCALE READS. Hence,

$$\text{Scale reading} = \text{normal reaction } N = mg = (45)(9.81) = 441.5\,N$$

(a)

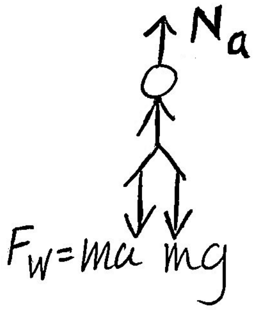


$F_W = ma$ = force producing acceleration a

The lift accelerates the scales, which in turn pushes the woman upward with a force ma (in addition to the normal force N). By Newton's third law at equilibrium, the woman pushes down on the scale with a force $F_W = ma$ in addition to her weight mg. Hence she appears HEAVIER. The normal reaction that the scale reads is thus increased

Q1(a) cont.

A free body diagram of the forces acting on the woman will help:



At equilibrium the net force $F_{net} = 0$, hence

$$F_{net} = N_a - ma - mg = 0$$

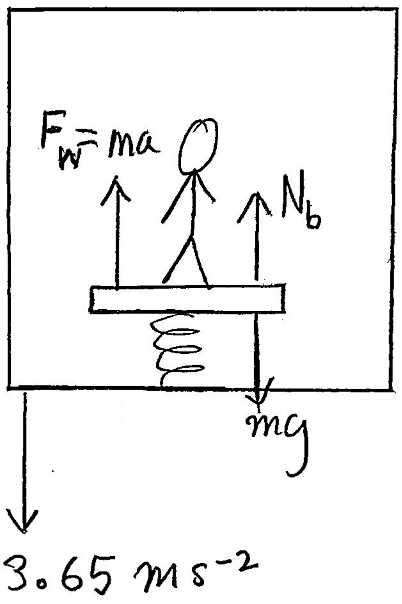
$$N_a = ma + mg = m(g+a)$$

$$N_a = 45(3.65 + 9.81) = 605.7\,N$$

$= $ Scale reading

Ans. Woman's apparant weight = normal reaction = 606 N (3sf)

(b)

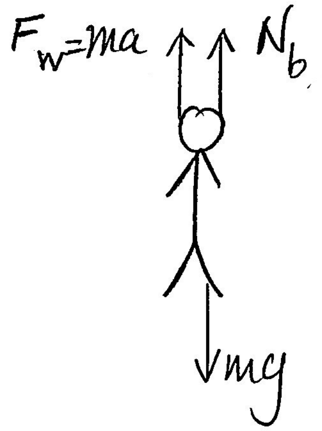


$F_W = ma = $ force producing acceleration a

The downward acceleration of the lift also accelerates the scales downward which results in the scales "pulling" the woman downward with a force ma. By Newton's third law the woman opposes the "pull" by a force of the same magnitude $F_W = ma$. This reduces the normal reaction N_b and she appears lighter.

→ A free-body diagram of the forces acting on the woman is helpful:

Q1. (b)



At equilibrium $F_{net} = 0$, hence

$$F_{net} = mg - ma - N_b = 0$$

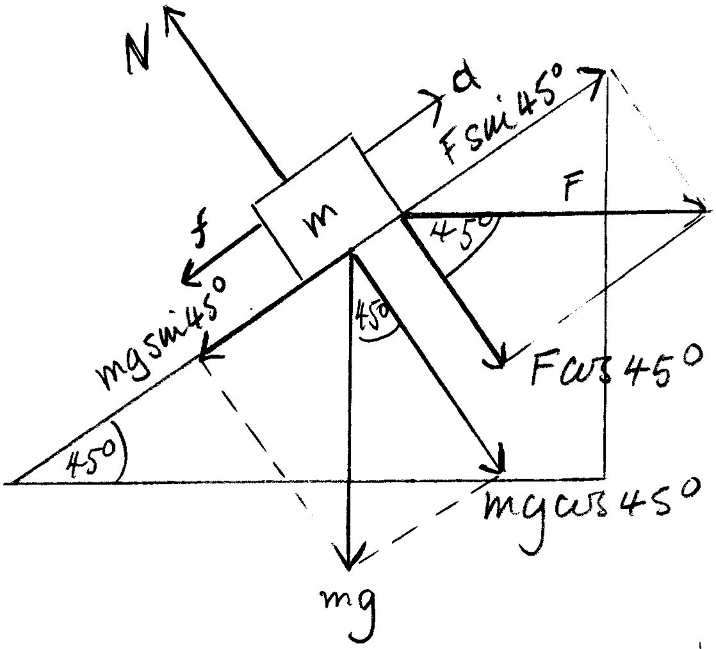
$$N_b = mg - ma = m(g-a) = 45(9.81 - 3.65)$$

$$N_b = 277.2\,N = \text{Scale reading}$$

Ans. Scale reading = normal reaction = 277 N (3sf) when the lift is accelerating downward.

Let me ask an additional question. What will be the apparent weight of the woman when the lift accelerates downwards with acceleration $a = g = 9.81\,ms^{-2}$?

Q2



Data:
$F = 450 N$
$m = 20 kg$
$g = 9.81 ms^{-2}$
$a = 0.7 ms^{-2}$
$mg\cos 45^\circ = (20)(9.81)\cos 45^\circ = 138.7 N$
$mg\sin 45^\circ = (20)(9.81)\sin 45^\circ = 138.7 N$

$F\cos 45^\circ = 450\cos 45^\circ = 318.2 N$

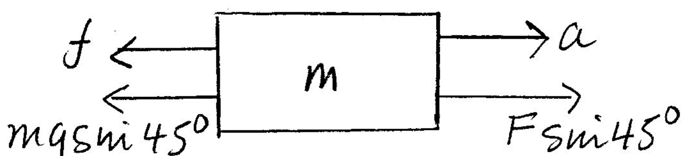
$F\sin 45^\circ = 450\sin 45^\circ = 318.2 N$

$f = \mu N$ (1)

N = normal reaction

$N = mg\cos 45 + F\cos 45$
$= 138.7 + 318.2 = 456.9 N$

Draw a free-body diagram:



$F_{net} = ma$

$F\sin 45^\circ - mg\sin 45^\circ - f = ma$

$f = F\sin 45^\circ - mg\sin 45^\circ - ma$

$f = 318.2 - 138.7 - (20)(0.7) = 165.5 N$

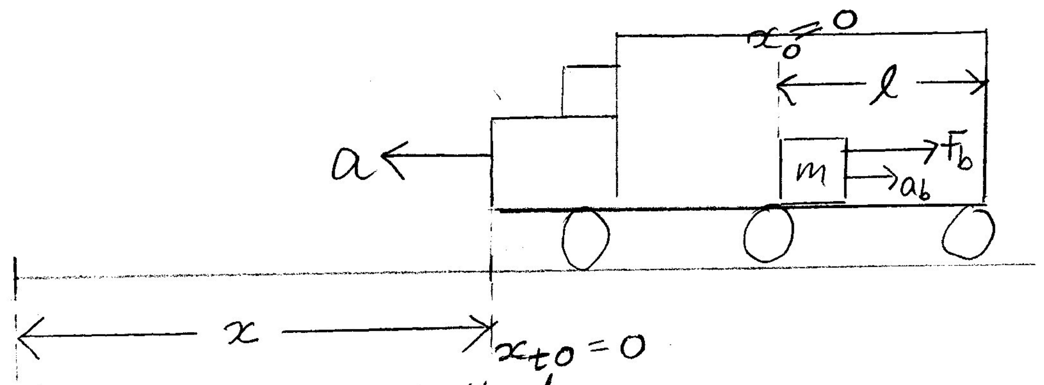
Find μ from Eq. (1):

$f = \mu N$

$\mu = \frac{f}{N} = \frac{165.5}{456.9} = 0.3622$

Ans $\mu = 0.362$ (3sf)

Q3.



Data:
$m = 20\,kg$
$l = 4\,m$
$a = 2\,ms^{-2}$
$\mu = 0.15$

Method
Step 1: Find the time t taken for box to fall.
Step 2: Find the distance x the truck will travel in time t

→ For the box not to slide, it must accelerate at the same acceleration as the truck. This requires a force F to accelerate the box given by

$$F = ma = (20)(2) = 40\,N$$

→ This force can only be provided from the frictional force between the box and the truck floor. The available friction is

$$f = \mu N = \mu mg = (0.15)(20)(9.81)$$
$$= 29.43\,N$$

→ We see that friction is not sufficient to accelerate the box forward with acceleration a. The shortfall F_b given by

$$F_b = F - f = 40 - 29.43 = 10.57\,N$$

Q3 cont.

→ The shortfall F_b appears as a force acting backwards (as shown in the diagram above), which produces an apparant acceleration a_b backwards.

It must be emphasized that both F_b and a_b are an apparant force and **an apparant** acceleration. There is no actual **force F_b** or **actual** acceleration a_b, it just **appears** this **way because there is not enough friction** for the box to keep up with the **truck**. The box therefore moves backwards relative to the **truck** floor, **but** forward relative to the **road**!

→ The apparant acceleration a_b is easily found as follows:

$$F_b = m a_b$$

$$a_b = \frac{F_b}{m} = \frac{10.57}{20} = a_b = 0.5285\ ms^{-2}$$

<u>Find t = time taken for box to fall of the truck</u>

$$l = \frac{1}{2}a_b t^2 + u_o t + x_o$$

u_o = initial velocity of **the box relative** to the truck **floor = 0**
x_o = initial position of the box relative to the truck floor = 0
Sub. we get $l = \frac{1}{2}a_b t^2 \Rightarrow t = \sqrt{\frac{2l}{a_b}} = \sqrt{\frac{(2)(4)}{0.5285}} = 3.891\,s$

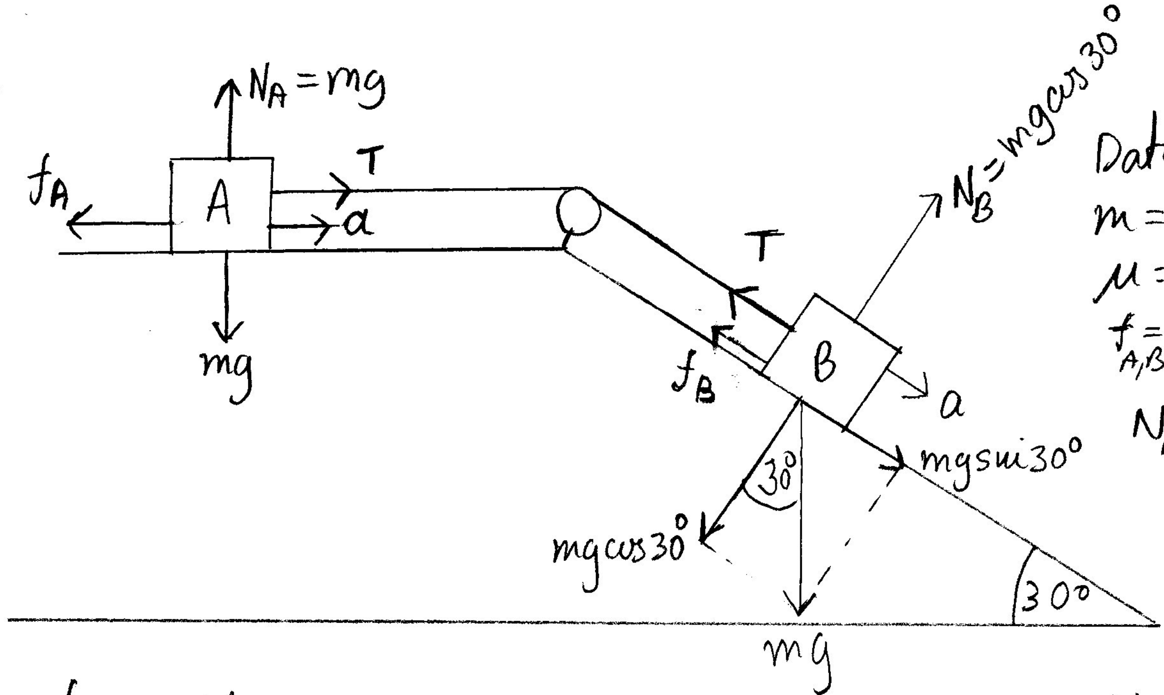
<u>Find x</u>
The distance travelled by the truck in time t is given by

$$x = \frac{1}{2}at^2 + u_t t + x_{to}$$

u_t = initial velocity of truck = 0, x_{to} = initial position of truck = 0
Sub. we get $x = \frac{1}{2}at^2 = \frac{1}{2}(2)(3.891)^2 = 15.14\ m$

<u>Ans. Distance travelled by truck in time t = x = 15.1 m (3sf)</u>

Q7.



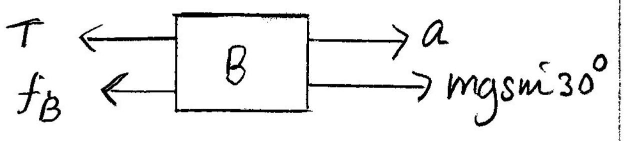
Data:
$m = 45$ Kg
$\mu = 0.15$
$f_{A,B}$ = frictional for on A and B
$N_{A,B}$ = Normal reaction on A and B

$$f_A = \mu N_A = \mu mg = (0.15)(45)(9.81) = 66.22\,N$$

$$f_B = \mu N_B = \mu mg\cos 30° = (0.15)(45)(9.81)\cos 30° = 57.35\,N$$

$$mg\sin 30° = (45)(9.81)\sin 30° = 220.7\,N$$

Free-body diagram for mass B | Free-body diagram for mass A



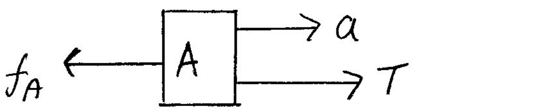
The forces ⊥ to the slope are equal and opposite, hence the net force in the ⊥ direction is zero, and so no need to consider them, except insofar as the normal force contributes to the friction:

$$F_{net(A)} = ma = 45a \quad (1)$$

$$F_{net(A)} = mg\sin 30° - T - f_B$$

$$F_{net(A)} = 220.7 - T - 57.35$$

$$F_{net(A)} = 163.4 - T \quad (2)$$



similarly vertical forces sum to zero, and can again be neglected.

IN A FREE BODY DIAGRAM, IT IS ONLY THE NET FORCE PRODUCING MOTION THAT SHOULD BE CONSIDERED

$$F_{net(B)} = ma = 45a. \quad (3)$$

$$F_{net(B)} = T - f_A$$

$$F_{net(B)} = T - 66.22 \quad (4)$$

Q7 cont

Sub. Eq.(2) into Eq.(1):	Sub Eq.(4) into Eq.(3)
$163.4 - T = 45a$ (5)	$T - 66.22 = 45a$ (6)

Equate Eqs (5) & (6):

$$163.4 - T = T - 66.22$$

$$163.4 + 66.22 = 2T$$

$$229.6 = 2T$$

$$T = 114.8\,N$$

Sub $T = 114.8N$ into Eq.(5) to find a:

$$163.4 - T = 45a$$

$$163.4 - 114.8 = 45a$$

$$a = \frac{163.4 - 114.8}{45}$$

$$a = 1.080\,ms^{-2}$$

Ans. Acceleration $a = 1.08\,ms^{-2}$, and tension $T = 115\,N$ (3sf)