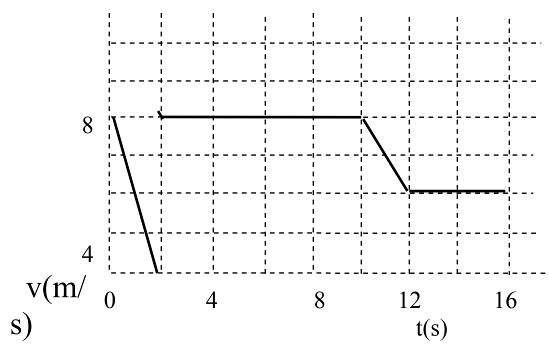
PHY1010 Tutorial Sheet 2 2019/20
Uniformly Accelerated Motion & Projectiles.

01. The velocity-time graph of a runner is shown in the figure.
(i) How far does the runner go in 16 s? (ii) What is the acceleration of the runner at t = 2 s? And at t = 11 s? [100 m] [$4\,m/s^2$,- $2\,m/s^2$]



02*. A truck travelling at 22.5 m/s decelerates at 2.27 m/s².

(a) How much time does it take for the truck to stop? [9.91s]

(b) How far does it travel while stopping? [112 m]

(c) How far does it travel during the third second after the brakes are applied? [16.8 m]

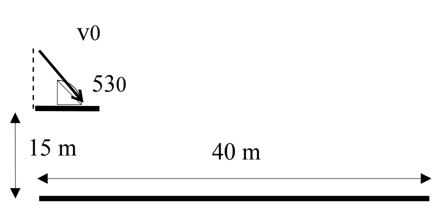
03**A ball is dropped off a high cliff, and 2 s later another ball is thrown vertically downward with an initial speed of 30 ms^{-1}. How long will it take the second ball to overtake the first? [$t = 3.88s$]

04*. You fire a projectile 35^0 above horizontal with an initial velocity of 200 m/s. It lands in a valley 300 m below the launch point. What is the time of flight of the projectile, and what is the range of projectile? [25.8 s, 4.23´ $10^3 m$].

05. A coin is projected at an angle of 53^0 above the horizontal from a point 15 m above the ground level. It reaches the ground at a horizontal distance of 40 m from the launch point.

Calculate (i) the time it was in air (ii) the velocity with which the coin was projected.

[$3.728s, 17.8 m/s$]



06**. A train approaching a station does two successive half-kilometres in 16 and 20 seconds respectively. Assuming the retardation to be uniform, find the further distance the train runs before stopping. [667.4 m]

07**. Suppose you have a car with a maximum acceleration of a = 6.0 m/s^2 and a maximum deceleration from braking of a = -8.0 m/s^2. We want to find the minimum time it would take you to start from rest, cover 500 m, and come to a stop at the 500 m mark. You do this by accelerating as much as possible for a part of the 500 m, followed by a period of maximum deceleration to the final stop.

Find the minimum time. $[17.1s]$

PHY1010 2019 Tutorial Sheet 2

01. Consider the different time periods:

0 – 2 sec: $v_{avg} = 4m/s,\ x_{0\to 2} = v_{avg}t = 4\frac{m}{s}\times 2s = 8\,m$

2 – 10 sec: $v_{avg} = v = 8m/s(cons\tan t),\ x_{2\to 10} = 8\frac{m}{s}\times(10-2)s = 64\,m$

10 – 12 sec: $v_{avg} = 6m/s,\ x_{10\to 12} = 6\frac{m}{s}\times(12-10)s = 12\,m$

12 – 16 sec: $v_{avg} = 4m/s,\ x_{12\to 16} = 4\frac{m}{s}\times(16-12)s = 16\,m$

Hence $x_{0-16} = x_{0-2} + x_{2-10} + x_{10-12} + x_{12-16} = 100\,m$

(ii) acceleration at $t = 1\sec$

Period 0 – 2 sec shows constant acceleration

$$a_1 = \frac{v_f - v_0}{t_f - t_0} = \frac{(8-0)m/s}{(2-0)\,\mathrm{s}} = 4m/s^2$$

Period 10 – 12 sec also shows constant acceleration. At $t = 11\sec$ we have

$$a_{11} = \frac{v_f - v_0}{t_f - t_0} = \frac{(4-8)m/s}{(12-10)\,\mathrm{s}} = -2m/s^2\,(deceleration)$$

02. (a) The truck decelerates from $v_0 = 22.5m/s$ to $v_f = 0$ with an acceleration of $a = -2.27m/s^2$, so the time taken to stop is $t = \frac{v_f - v_0}{a} = \frac{0-22.5}{-2.27} = 9.91\sec$

(b) The distance travelled is $x = v_{avg}t = 22.5\times 9.91/2 = 112m$

(c) The distance moved after 3 sec is $x_3 = v_0\times 3.0 + \frac{1}{2}a\times 3^2 = 57.3m,$ while the distance travelled after 2.0 sec is $x_2 = v_0\times 2.0 + \frac{1}{2}a\times 2^2 = 40.5m.$ Therefore, the distance moved during the third second is $x_3 - x_2 = 57.3 - 40.5 = 16.8\,m$

03. Write the equations of motion for the two balls:

First ball: $S_1 = ut + \frac{1}{2}gt^2\ with\ u = 0 \to S_1 = \frac{1}{2}gt^2 = 0.5\times 9.8t^2 = 4.9t^2$

Second ball:

$S_2 = u(t-2) + \frac{1}{2}g(t-2)^2 \; with \; u = 30m/s \rightarrow S_2 = 30(t-2) + 4.9(t^2 - 4t + 4)$

$= 30t - 60 + 4.9t^2 - 19.6t + 19.6$

Second ball catches up when

$S_1 = S_2 \rightarrow 4.9t^2 = 30t - 60 + 4.9t^2 - 19.6t + 19.6 \rightarrow 0 = 10.4t - 40.4$

Or $t = \frac{40.4}{10.4} = 3.88s$. Second ball will pass first ball after 3.88 seconds.

04. Take 'down' as the positive direction.

Vertical component of the initial velocity is $v_{0y} = -200\sin 35 = -115m/s.$

Find the projectile's time of flight $t.$ Vertical distance dropped by the projectile is

$y = 300 = v_{oy}t + \frac{1}{2}gt^2 = -115t + 4.9t^2$. Solving this we get $t = 25.8\sec.$

Thus the range of the projectile is $x = v_{0x}t = 200\cos 35 \times 25.8 = 4.23\times 10^3 m$

05. Take the origin of coordinates at the top of the ramp and take $+y$ to be upward. The object is displaced 40 m to the right when it is 15 m below the origin.

We have to find the time of flight, and the initial velocity. Write the equations for the horizontal and vertical displacements. Combine these two equations to eliminate one uncommon.

Y-component: $y - y_0 = -15m \quad a_y = -9.8m/s^2 \quad v_{0y} = v_0 \sin 53$

$y - y_0 = v_{oy}t + \frac{1}{2}a_y t^2 \rightarrow -15 = v_0 \sin 53 \times t - 4.9t^2$

X-component: $x - x_0 = 40m \quad a_x = 0 \quad v_{0x} = v_0 \cos 53$

$x - x_0 = v_{ox}t + \frac{1}{2}a_x t^2 \rightarrow 40 = v_0 \times t \times \cos 53$

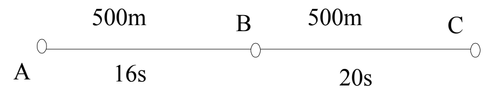
$40 = v_0 \times t \times \cos 53 \rightarrow v_0 \times t = \frac{40}{\cos 53} = 66.47m$

$-15 = v_0 \sin 53 \times t - 4.9t^2 = 66.47\sin 53 - 4.9t^2$

Hence $t = \sqrt{\frac{66.47\sin 53 + 15}{4.9}} = 3.728\sec$

Or $v_0 = \frac{40}{t \times \cos 53} = \frac{40}{3.728 \times \cos 53} = 17.8m/s$

06.



500 = v_A × 16 + ½ a × 16²

= 16 (v_A + 8a) or 500/16 =31.25

= v_A + 8a

1000 = v_A × 36 + ½ a × 36² = 36 (v_A + 18a) or 1000/36 = 27.78 = v_A + 18*a*

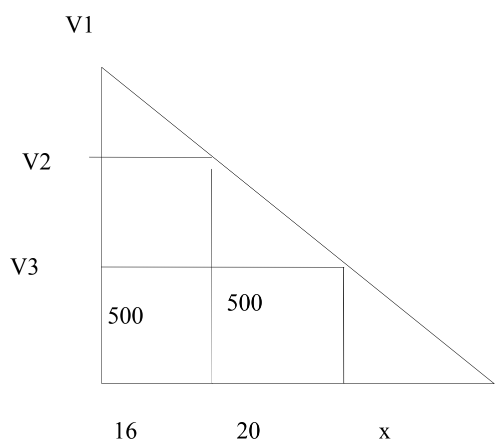
Hence 18*a* – 8*a* = 27.78 – 31.25 = -3.47 or 10 *a* = -3.47 hence *a* = - 0.347m/s²

Then v_A = 27.78 - 18 × (-0.347) = 34.03m/s and v_C = v_A + 36 × (-0.347)

= 34.03 – 12.49 = 21.54 m/s

S = $\dfrac{21.54^2}{2 \times 0.347} = 669m$

Q6.(2)



$$\frac{1}{2}(v_1 + v_2)16 = 500 \rightarrow 8v_1 + 8v_2 = 500 \rightarrow v_1 + v_2 = 62.5$$

$$\frac{1}{2}(v_2 + v_3)20 = 500 \rightarrow 10v_2 + 10v_3 = 500 \rightarrow v_2 + v_3 = 50$$

$$\frac{1}{2}(v_1 + v_3)36 = 1000 \rightarrow v_1 + v_3 = 500/9$$ giving $v_3 = 21.65m/s \ \ and \ \ x =$

07. Assume a period t_1 for acceleration from $v_0 = 0 \; to \; v_1$, reaching point x_1, and period t_2 decelerating from v_1 to $v_2 = 0$ to reach point x_2.

Total distance covered from $x_1 \; to \; x_2 = 500\,m.$

Use $v_f = v_0 + at$

For the first part: $v_1 = v_0 + a_1 t_1 \rightarrow v_1 = 6t_1$

For the second part $v_2 = v_1 + a_2 t_2 \rightarrow 0 = v_1 - 8t_2 \rightarrow v_1 = 8t_2$

Hence $6t_1 = 8t_2 \rightarrow t_2 = \frac{3}{4}t_1$

Now use $x = v_0 t + \frac{1}{2}at^2$

This gives for the first part: $x_1 = 0 + \frac{1}{2}a_1 t_1^2 = 3t_1^2$ (1)

For the second part: $500 - x_1 = v_1 t_2 + \frac{1}{2}a_2 t_2^2 = v_1 t_2 + 4t_2^2$

Substitute $v_1 = 6t_1 \; and \; t_3 = \frac{3}{4}t_1$ to obtain

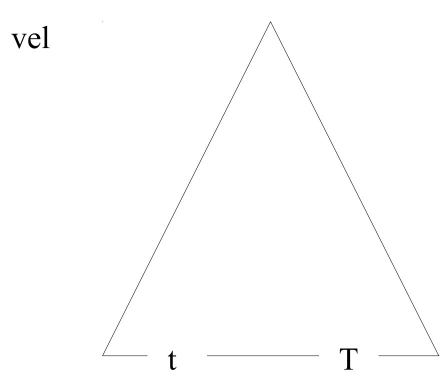
$$500 - x_1 = 6t_1 \times \frac{3}{4}t_1 - 4 \times \left(\frac{3}{4}t_1\right)^2 = \frac{18}{4}t_1^2 - \frac{9}{4}t_1^2 = \frac{9}{4}t_1^2 \quad (2)$$

(1)+(2) gives $500 = 3t_1^2 + \frac{9}{4}t_1^2 = \frac{21}{4}t_1^2 \rightarrow t_1 = \sqrt{\frac{2000}{21}} = 9.76\,\text{sec}$

And $t_2 = \frac{3}{4}t_1 = 7.32\,\text{sec}$

Minimum time $= t_1 + t_2 = 9.76 + 7.32 = 17.1\,\text{sec}$

Solution 2



$$\frac{1}{2}bh = 500$$ Also $$a = \frac{\Delta v - \Delta u}{t} \rightarrow 6 = \frac{\Delta v - 0}{t} \rightarrow 6t = \Delta v$$

$$a = \frac{\Delta v - \Delta u}{T - t} \rightarrow -8 = \frac{0 - 6}{T - t} \rightarrow T = \frac{7}{4}t$$

$$\frac{1}{2}bh = 500 \rightarrow \frac{1}{2} \times \frac{7}{4}t \times 6t = 500 \rightarrow 5.25t^2 = 500 \rightarrow t = 9.75s$$

$$T = \frac{7}{4} \times 9.75 = 17.1\text{sec}$$