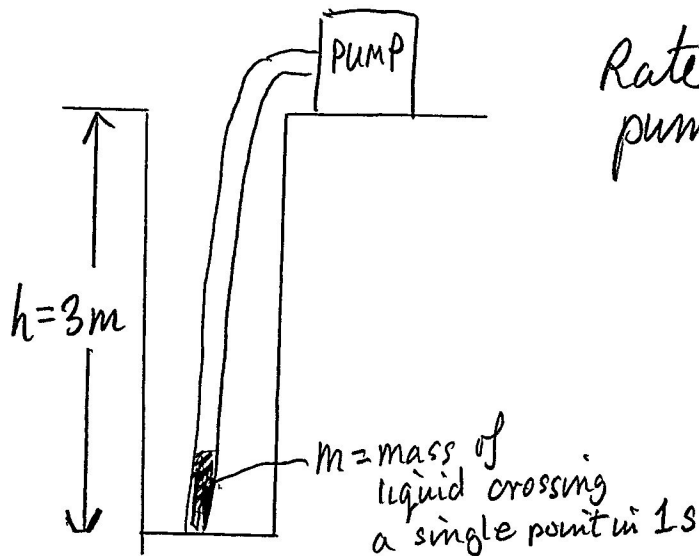


Tutorial 5

Work & Power:

$$1 \text{ hp} = 746 \text{ W}$$

Q1



Rate of pumping

$$= 0.6 \frac{\text{Kg}}{\text{min}}$$

$$= 0.6 \frac{\text{Kg}}{(60\text{s})}$$

$$= 0.01 \text{ Kg} \cdot \text{s}^{-1}$$

$$h = 3\text{m}$$

→ Since the rate of pumping is $0.01 \text{ Kg} \cdot \text{s}^{-1}$,
 ∴ the energy needed each second to raise 0.01 Kg by 3m is

$$E_{1\text{s}} = mgh = (0.01)(9.81)(3) \\ = 0.2943 \text{ J} \cdot \text{s}^{-1}$$

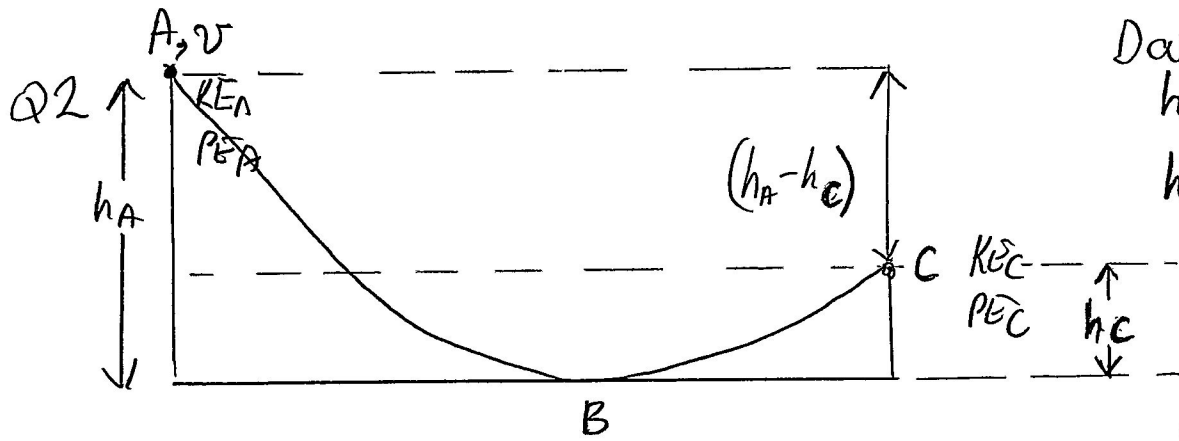
→ But, energy per second is power, hence

$$P = \frac{E}{t} = \frac{0.2943 \text{ J}}{1 \text{ s}} = 0.2943 \text{ W}$$

→ Now $1 \text{ hp} = 746 \text{ W}$ so that

$$P = 0.2943 \text{ W} = 0.2943 \left(\frac{1}{746} \text{ hp} \right) = 3.945 \times 10^{-4} \text{ hp}$$

Ans. Minimum power of pump = $0.294 \text{ W} = 3.95 \times 10^{-4} \text{ hp}$



Data:

$$h_A = 80\text{cm} = 0.8\text{m}$$

$$h_C = 50\text{cm} = 0.5\text{m}$$

$$v = 2\text{ms}^{-1}$$

$$m = 15\text{g} = 0.015\text{kg}$$

$$s = 250\text{cm} = 2.5\text{m}$$

$$g = 9.81\text{ms}^{-1}$$

By conservation of energy:

$$\text{ENERGY LOST} = \text{ENERGY GAINED} +$$

WORK
DONE BY
FRICTION

$$KE_A = \frac{1}{2}mv^2, \quad KE_C = 0$$

$$PE_A = mgh_A, \quad PE_C = mgh_C$$

The ΔKE lost is

$$\begin{aligned} \Delta KE &= KE_A - KE_C = \left(\frac{1}{2}mv^2 - 0\right) \\ &= \frac{1}{2}(0.015)2^2 = 0.03\text{J} \end{aligned}$$

The ΔPE lost is

$$\begin{aligned} \Delta PE &= PE_A - PE_C = mg(h_A - h_C) \\ &= (0.015)(9.81)(0.8 - 0.5) \\ &= 0.04415\text{J} \end{aligned}$$

$$\text{work done by friction} = f \cdot s$$

→ Sub. we get

$$(\Delta KE + \Delta PE)_{\text{LOST}} = (0)_{\text{GAINED}} + f \cdot s$$

$$f = \frac{(\Delta KE + \Delta PE)_{\text{LOST}}}{s} = \frac{0.03 + 0.04415}{2.5} = 0.02966\text{N}$$

$$\text{Ans. Frictional force } f = 0.0297\text{N (3sf)}$$

DATA: $g = 9.81 \text{ ms}^{-2}$
 Friction is zero
 $h = 75 \text{ cm} = 0.75 \text{ m}$

$v_c = \text{velocity at } C = 0$
 $u_A = \text{initial velocity at } A = 0$
 $v_A = \text{final velocity at } A$
 $v_B = \text{velocity at } B$

FIG 1

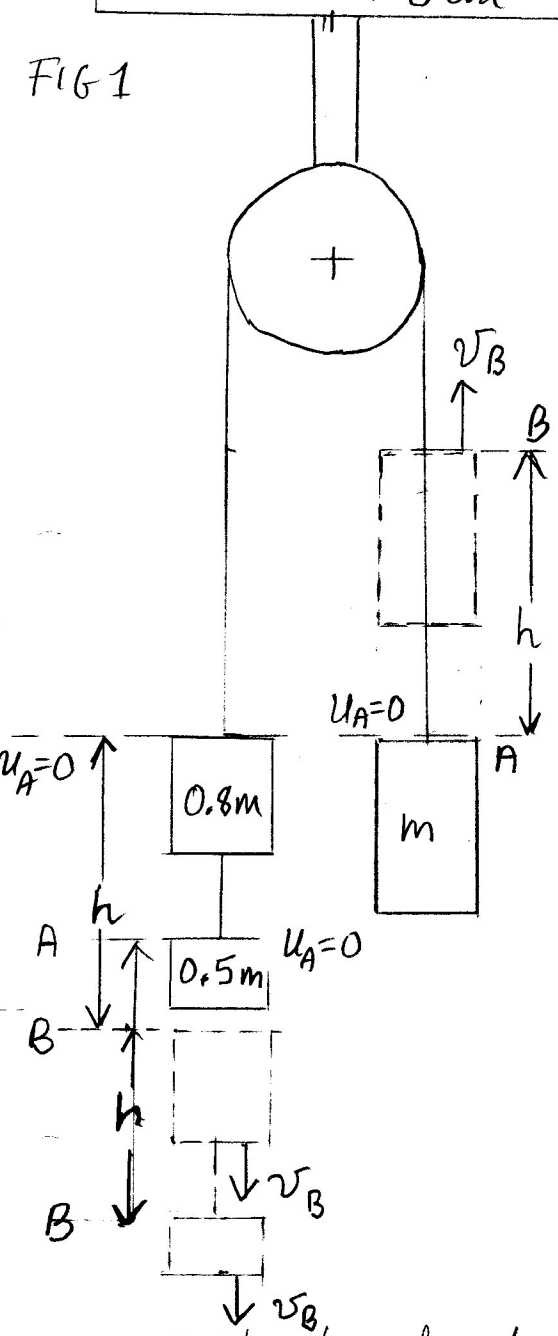


FIG 2

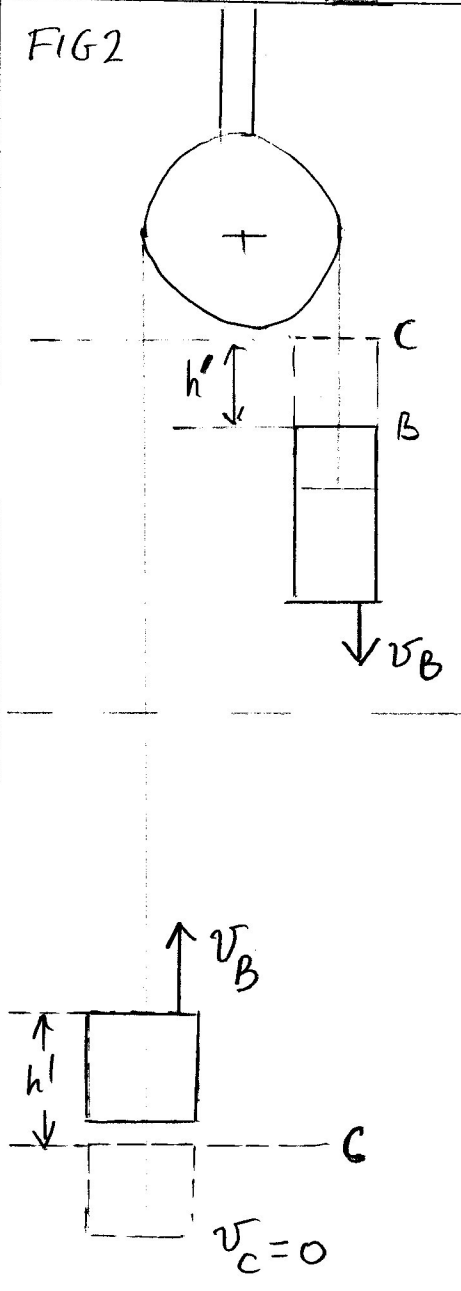
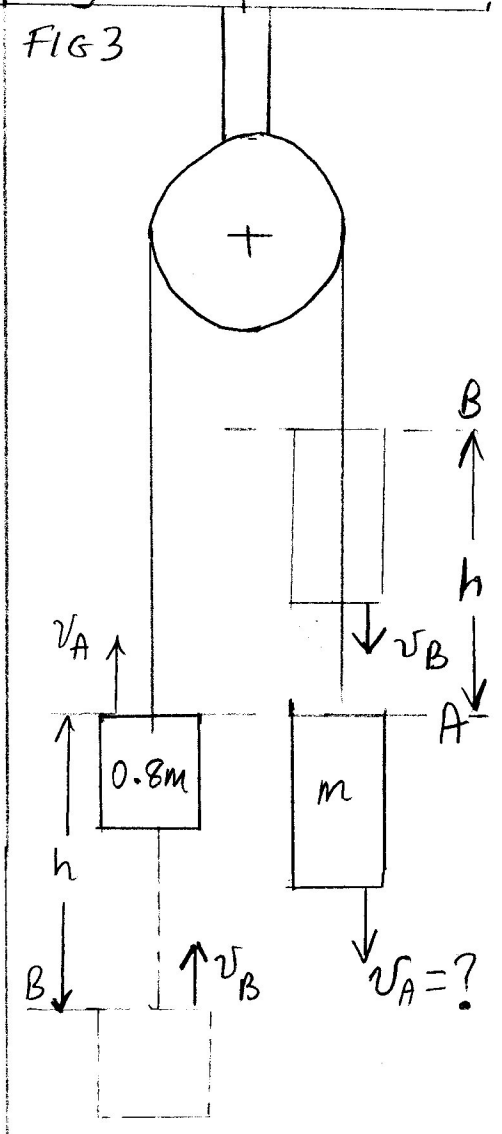


FIG 3



NOTE 1. The top of each mass is chosen as the reference point for the distance moved by each mass.

NOTE 2. Each mass moves from A to B a distance h

NOTE 1. After mass m is cut mass $0.8m$ continues moving downward a further distance h' where it stops and reverses direction. Similarly mass m reaches a further height h' , stops and reverses direction.

NOTE 2: MASS $0.8m$ AND m DO NOT STOP AND REVERSE DIRECTION INSTANTANEOUSLY, THE WOULD REQUIRE AN INFINITE ACCELERATION (IMPOSSIBLE)

NOTE 3. By conservation of energy, when $0.8m$ and m return to position B, their velocity is still v_B , but in the opposite direction.

Q6 cont

→ First look at the motion in Fig. 1. and apply conservation of energy:

ENERGY LOST = ENERGY GAINED

$$(0.8m)gh + (0.5m)gh = mgh + \frac{1}{2}m(v_B^2 - u_A^2) + \frac{1}{2}(0.8m)(v_B^2 - u_A^2) + \frac{1}{2}(0.5m)(v_B^2 - u_A^2)$$

Since $u_A = 0$ and dividing by m , we get

$$0.8gh + 0.5gh = gh + \frac{1}{2}v_B^2 + 0.4v_B^2 + 0.25v_B^2$$

collecting terms, we get

$$0.3gh = 1.15v_B^2$$

$$v_B = \sqrt{\frac{0.3gh}{1.15}} = \sqrt{\frac{0.3(9.81)(0.75)}{1.15}}$$

$$= 1.385 \text{ ms}^{-1}$$

Fig 2 shows that after mass 0.5m is cut, masses 0.8m and 0.5m continue moving in the same direction. But, because $m > 0.8m$, both mass begin to slow down, stop and reverse direction. Because energy is conserved (no friction) the mass have velocity v_B when they return to position B, but pointing in the opposite direction.

Q6 cont.

→ Now apply conservation of energy to Fig. 3.

Energy lost = Energy gained

$$mgh = (0.8m)gh + \frac{1}{2}m(v_A^2 - v_B^2) + \frac{1}{2}0.8m(v_A^2 - v_B^2)$$

Divide through by m and rearrange,

$$gh - 0.8gh = \frac{1}{2}v_A^2 - \frac{1}{2}v_B^2 + 0.4v_A^2 - 0.4v_B^2$$

$$0.2gh = 0.9v_A^2 - 0.9v_B^2$$

$$0.9v_A^2 = 0.2gh + 0.9v_B^2$$

$$v_A = \sqrt{\frac{0.2gh + 0.9v_B^2}{0.9}}$$

$$v_A = \sqrt{\frac{0.2(9.81)(0.75) + 0.9(1.385)^2}{0.9}}$$

$$v_A = 1.885 \text{ ms}^{-1}$$

Ans. The final velocity of mass $m = v_A = 1.89 \text{ ms}^{-1}$ (3sf)