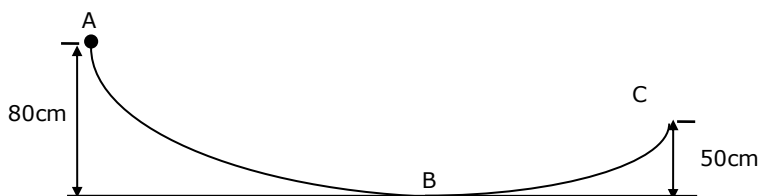


PHY1010 Introductory Physics 2018
Tutorial Sheet 4 Work and Energy.

01. A pump is needed to lift water from a well through a height of 3.0 m at a rate of 0.6kg/min. What must be the minimum power of the pump in watts, and in horsepower? [Power = 0.294 watt or = 3.94×10^{-4} hp]

02. A bead of mass 15g is sliding on a wire. It has a speed of 2.0m/s at A, and it stops as it reaches the point C. The length of the wire from A to C is 250cm.

How large an average friction force opposed the motion of the bead? [$f = 0.0296\text{N}$]



03. A steel block weighing 12N is pulled up an inclined plane 20° above the horizontal by a constant force of 7.35 N which makes an angle of 10° above the inclined plane.

The block starts from rest and is pulled 2.0 m along the inclined plane.

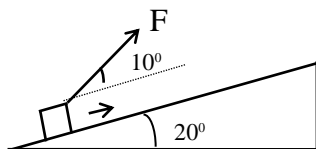
The coefficient of friction between the block and the inclined plane is 0.20.

Determine: (a) the work done on the block by the force, [$W = 14.48\text{J}$]

(b) increase in potential energy of the block [$\Delta PE = 8.21\text{J}$]

(c) increase in kinetic energy of the block, and [$KE_{final} = 2.27\text{J}$]

(d) amount of work required to overcome the frictional force. [$W_f = W - E = 4\text{J}$]

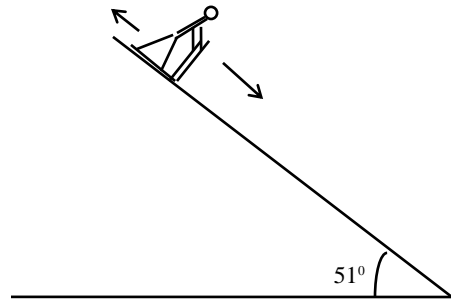


04**. A record skiing speed of 203.1km/h was achieved on a mountain slope inclined downward at 51° . At this speed, the force of friction on the skier (air and sliding friction) balances the pull of gravity along the slope, so that the motion proceeds at a constant velocity.

(a) What is the rate at which gravity does work on the skier? Assume that the mass of the skier is 75kg. [32227 J/s]

(b) What is the rate at which sliding friction does work? Assume that the coefficient of sliding friction $\mu_k = 0.03$ [783 J/s]

(c) What is the rate at which air friction does work? [31444 J/s]



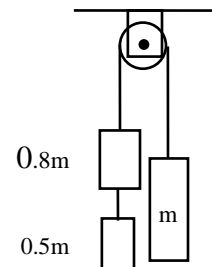
05. A running man has one-half the kinetic energy that a running boy of half his mass has. The man speeds up by 1.0 m/s and then has the same kinetic energy as the boy.

What were the original speeds of the man and the boy?

$$[v_{man-orig} = 2.42\text{ m/s}, v_{boy-orig} = 4.83\text{ m/s}]$$

06*. The frictionless system shown is released from rest. After the right-hand mass has risen 75 cm, the object of mass 0.50 m falls loose from the system.

What is the speed of the right-hand mass when it returns to its original position? [$v_{down} = 1.88\text{ m/s}$]



PHY1010 2018 Tutorial Sheet 4 Work and Energy.

01. Energy supplied by the pump in 60 seconds is $mgh = 0.600 \times g \times 3 = 17.64 J$

$$\text{Power} = \text{energy/time} = 17.64 J / 60 \text{sec} = 0.294 \text{ watt or } \frac{0.294}{746} = 3.94 \times 10^{-4} \text{ hp}$$

02. When the bead moves from A to C, it experiences a change in total energy.

Both K.E. and G.P.E. are lost, which equals the work done on the bead by the friction force.

$$\Delta PE + \Delta KE = \text{work done by friction}$$

$$mg(h_C - h_A) + \frac{1}{2}m(v_C^2 - v_A^2) = f \cdot s \cdot \cos \theta$$

$$\cos \theta = -1, v_C = 0, v_A = 2 \text{ m/s}, h_C - h_A = -0.30 \text{ m}$$

$$0.015 \times 9.8(-0.3) + \frac{1}{2} \times 0.015(0 - 2^2) = f \times 2.5 \times (-1)$$

$$-0.0441 - 0.03 = -2.50f \rightarrow 0.0741 = 2.50f \rightarrow f = \frac{0.0741}{2.50} = 0.0296 \text{ N}$$

03. A steel block weighing 12N is pulled up an inclined plane 20° above the horizontal by a constant force of 7.35 N which makes an angle of 10° above the inclined plane.

The block starts from rest and is pulled 2.0 m along the inclined plane.

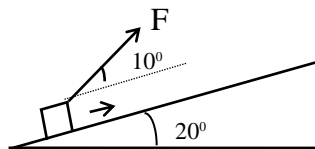
The coefficient of friction between the block and the inclined plane is 0.20.

Determine: (a) the work done on the block by the force,

(b) increase in potential energy of the block

(c) increase in kinetic energy of the block, and

(d) amount of work required to overcome the frictional force.



(a) Force is constant, displacement $s = 2 \text{ m}$.

$$\text{Work } W = F \cdot s = Fs \cos \theta = 7.35 \times 2 \times \cos 10 = 14.48 J$$

(b) To get PE, first calculate the vertical rise h .

$$\sin 20 = \frac{h}{s} \rightarrow h = s \sin 20 = 2 \times \sin 20 = 0.684 \text{ m}$$

$$\text{Hence } \Delta PE = mgh = wh = 12 \times 0.684 = 8.21 J$$

(c) KE depends on the final speed of the block. We can calculate the net force on the block in the direction of motion, and the acceleration it acquires.

$$\begin{aligned}\sum F_y &= F \sin 10 - w \cos 20 + F_N \rightarrow F_N = w \cos 20 - F \sin 10 \\ &= 12 \times 0.9396 - 7.35 \times 0.1736 = 10N\end{aligned}$$

Net force in the x -direction:

$$\sum F_x = F \cos 10 - w \sin 20 - \mu_k F_N = 7.35 \times 0.985 - 12 \times 0.342 - 0.2 \times 10 = 1.134N$$

$$\text{Hence } a = \frac{\sum F_x}{m} = \frac{\sum F_x}{w/g} = \frac{1.134 \times 9.8}{12} = 0.926m/s^2$$

Using $v^2 = u^2 + 2as$ to get the final velocity, $v^2 = 0^2 + 2 \times 0.926 \times 2 = 3.705m^2/s^2$

$$KE_{final} = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{w}{g}\right)v^2 = \frac{1}{2}\left(\frac{12}{9.8}\right)3.705^2 = 2.27J$$

(d) Total mechanical energy acquired by the block is

$$E = PE + KE = 8.21 + 2.27 = 10.48J$$

Hence work done to overcome friction is $W_f = W - E = 14.48 - 10.48 = 4J$

04. $203.1 \text{ km/h} = 56.42 \text{ m/s}$. $mg \sin \theta = 75 \times 9.8 \times \sin 51 = 571.2 \text{ N}$.

Friction force $f = \mu_k mg \cos \theta + \text{air friction}$.

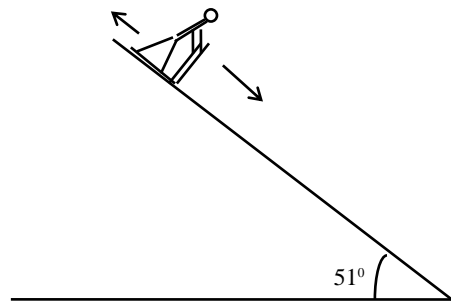
$$\mu_k mg \cos \theta = 0.03 \times 75 \times 9.8 \times \cos 51 = 13.88 \text{ N}$$

Work = force \times distance. Rate of work = work/time = Force \times velocity.

(a) Rate of work (gravity) = $571.2 \times 56.42 = 32227 \text{ J/s}$.

(b) Hence rate of work (sliding friction) = $13.88 \times 56.42 = 783 \text{ J/s}$.

(c) Hence rate of work (air friction) = $(571.2 - 13.88) \times 56.42 = 31444 \text{ J/s}$.



05. (a) A running man has one-half the kinetic energy that a running boy of half his mass has. The man speeds up by 1.0 m/s and then has the same kinetic energy as the boy.

What were the original speeds of the man and the boy?

Let M be the mass of the man and m that of the boy. Then $m = \frac{M}{2}$.

Let v_1 be the original speed of the man, and v_2 be the original speed of the boy.

Kinetic energy of the man = $\frac{1}{2}Mv_1^2$, and kinetic energy of the boy $\frac{1}{2}mv_2^2 = \frac{1}{2}\left(\frac{M}{2}\right)v_2^2$.

Kinetic energy of man = $\frac{1}{2}$ kinetic energy of the boy.

$$\frac{1}{2}Mv_1^2 = \frac{1}{2} \times \frac{1}{2} \left(\frac{M}{2}\right)v_2^2 \rightarrow v_1^2 = \frac{v_2^2}{4} \rightarrow v_1 = \frac{v_2}{2}$$

The man speeds up by 1 m/s, or new speed of the man = $(v_1 + 1)$.

New kinetic energy of the man $\frac{1}{2}M(v_1 + 1)^2$.

Now kinetic energy of man = kinetic energy of boy giving,

$$\frac{1}{2}mv_2^2 = \frac{1}{2}\left(\frac{M}{2}\right)v_2^2.$$

$$\text{Hence } \frac{1}{2}(v_1 + 1)^2 = \frac{v_2^2}{2} \rightarrow v_1 + 1 = \frac{v_2}{\sqrt{2}} = 0.707v_2$$

$$\text{Or } v_1 = 0.707v_2 - 1 = \frac{v_2}{2} \rightarrow v_2 = 1.414v_2 - 2 \rightarrow 0.414v_2 = 2 \rightarrow v_2 = 4.83\text{m/s}$$

$$\text{Hence } v_1 = 2.42\text{m/s}$$

06. First calculate the velocity of m when the small mass falls off using conservation of energy: GPE is positive as the mass goes up, KE is positive in both directions.

From energy conservation $\Delta PE + \Delta KE = 0$

$$\left[(1.0 - 1.3)mg(0.75 - 0)\right] + \left(\frac{1}{2} \times 2.3mv_{up}^2 - 0\right) = 0 \rightarrow v_{up} = 1.38\text{m/s}$$

Now solve the new problem using conservation of energy:

$$\left[0 - (1.0 - 0.8)mg(0.75)\right] + \left(\frac{1}{2} \times 1.8mv_{down}^2 - \frac{1}{2} \times 1.8m(1.38)^2\right) = 0$$

$$\rightarrow -1.47 + 0.9v_{down}^2 - 1.71 = 0 \text{ or}$$

$$0.9v_{down}^2 = 1.47 + 1.71 = 3.18 \rightarrow v_{down}^2 = 3.53 \rightarrow v_{down} = 1.88\text{m/s}$$