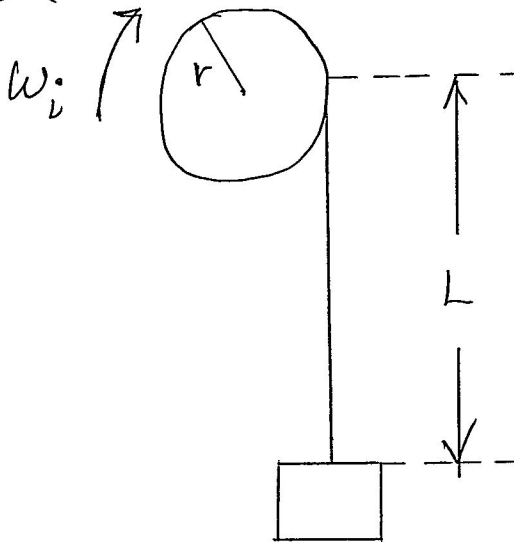


Tutorial 6

Motion in a circle

Solutions

Q1



Data:

$$r = 44 \text{ cm} = 0.44 \text{ m}$$

$$L = 29.5 \text{ cm}$$

$$C = \text{circumference} = 2\pi r$$

$$= 2\pi(0.44) = 2.765 \text{ m}$$

$$\omega_i = 1.6 \frac{\text{rev}}{\text{s}} = 1.6(2\pi \text{ rad}) \cdot \text{s}^{-1}$$

$$\omega_i = 10.05 \text{ rad} \cdot \text{s}^{-1}$$

$$\omega_f = 0 \text{ rad} \cdot \text{s}^{-1}$$

$$\text{Number of revolutions} = \theta = \frac{29.5}{2.765} = 10.67 \text{ rev}$$

$$= 10.67(2\pi \text{ rad})$$

$$= 67.04 \text{ rad}$$

Find the acceleration:

$$2\alpha\theta = \omega_f^2 - \omega_i^2 \Rightarrow \alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta} = \frac{0 - 10.05^2}{2(67.04)}$$

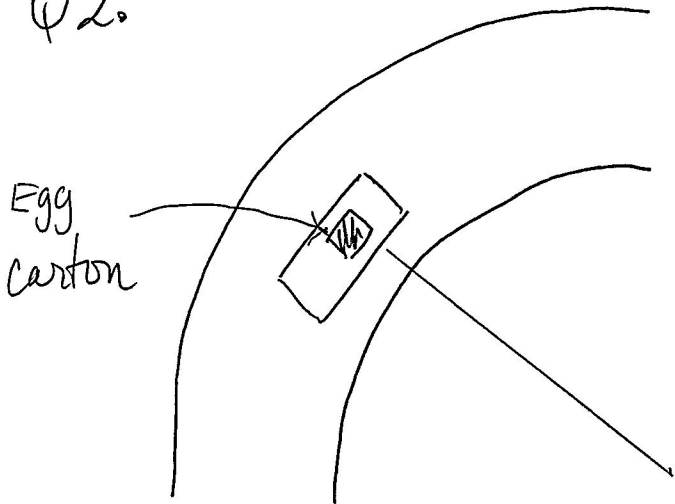
$$\alpha = 0.7533 \text{ rad} \cdot \text{s}^{-2} = 0.7533 \left(\frac{\text{rev}}{2\pi} \right) \cdot \text{s}^{-2}$$

$$\alpha = 0.1199 \text{ rev} \cdot \text{s}^{-2}$$

Ans. Number of revolutions = 10.7 rev = 67.0 rad (3sf),

Acceleration $\alpha = 0.120 \text{ rev} \cdot \text{s}^{-2}$

Q2.



Data:

$$g = 9.81 \text{ ms}^{-2}$$

$$r = 26 \text{ m}$$

$$v = 16.5 \text{ ms}^{-1}$$

$$\mu = ?$$

$$f_{\text{min}} = \mu N = \mu mg \quad (1)$$

$$F_c = \frac{mv^2}{r} \quad (2)$$

→ The only force stopping the carton from slipping is the frictional force between the carton and the car seat. Hence, this is the only force that can provide the centripetal force needed to make the carton turn with the car.

→ Thus, for the carton not to slip, the minimum frictional force f_{min} needed must equal the centripetal force F_c

$$f_{\text{min}} = F_c$$

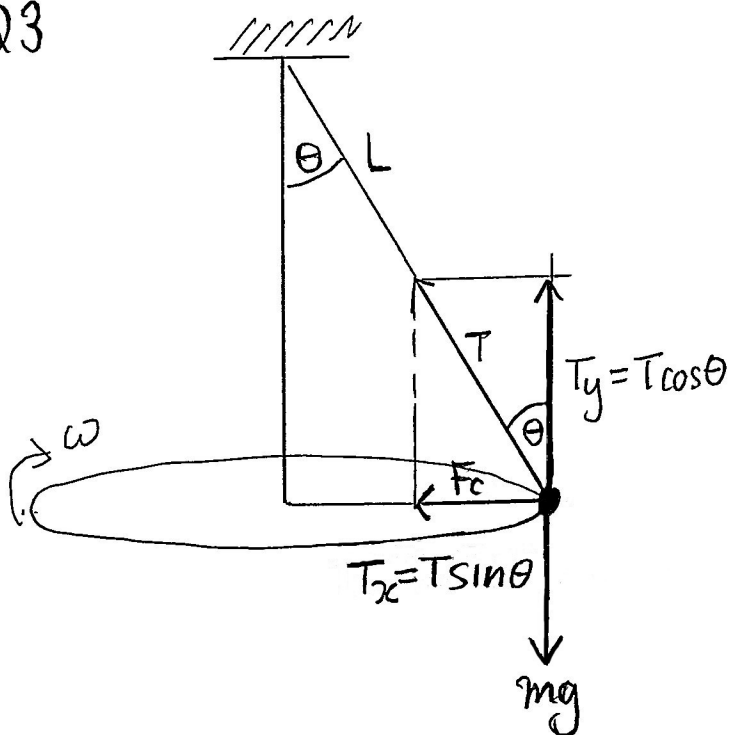
$$\mu mg = \frac{mv^2}{r}$$

[from Eq. (1) and Eq. (2)]

$$\mu = \frac{v^2}{gr} = \frac{16.5^2}{(9.81)(26)} = 1.067$$

Aus. Minimum coefficient of static friction $\mu = 1.07$ (3sf)

Q3



Data:

$L = 30 \text{ cm} = 0.3 \text{ m} = \text{length of string}$

$m = 100 \text{ g} = 100(10^{-3} \text{ kg}) = 0.1 \text{ kg}$

$\omega = 80 \frac{\text{rev}}{\text{min}} = 80 \frac{(2\pi \text{ rad})}{(60 \text{ s})}$

$\omega = 8.378 \text{ rad.s}^{-1}$

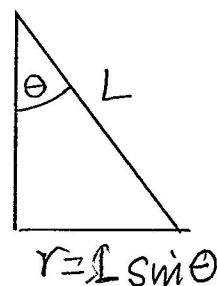
$r = L \sin \theta$ (1)

T_y balances mg ,
hence

$T_y = mg$ (2)

T_x provides the centripetal force, hence

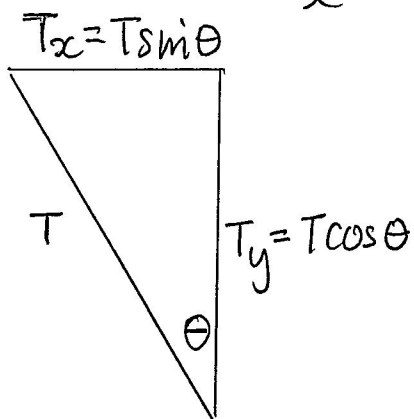
$T_x = F_c = m r \omega^2$ (3)



(ii) Find T

Sub. Eq. (1) into Eq. (3)

$T_x = m \omega^2 L \sin \theta$



$T \sin \theta = m \omega^2 L \sin \theta$

$T = m \omega^2 L = (0.1)(8.378^2)(0.3)$

$T = 2.106 \text{ N}$

Ans. The tension in the string is $T = 2.11 \text{ N}$ (3sf)

Q3(i) Find θ

$$T_y = T \cos \theta$$

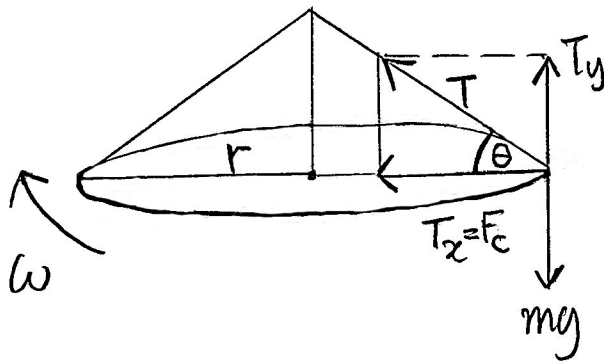
$$\cos \theta = \frac{T_y}{T} = \frac{mg}{T} = \frac{(0.1)(9.81)}{2.106} = 0.4658$$

$$\theta = \cos^{-1} 0.4658$$

$$\theta = 62.24^\circ$$

Ans. The angle of inclination of the string
to the vertical $\theta = 62.2^\circ$ (3sf)

Q4



Data:

$$g = 9.81 \text{ ms}^{-2}$$

$$m = 450 \text{ g} = 0.45 \text{ Kg}$$

$$\text{radius} = r = 1.25 \text{ m}$$

$$v = 8.5 \text{ ms}^{-1}$$

$$T_x = F_c = \frac{mv^2}{r} = \frac{(0.45)(8.5^2)}{1.25} = 26.01 \text{ N}$$

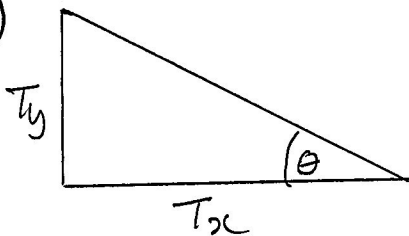
$$T_y = mg = (0.45)(9.81) = 4.415 \text{ N}$$

a) Find the tension

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{26.01^2 + 4.415^2} = 26.38 \text{ N}$$

Ans. The tension in the string is $T = 26.4 \text{ N}$ (3sf)

b)

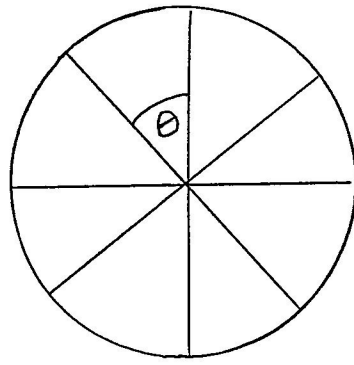
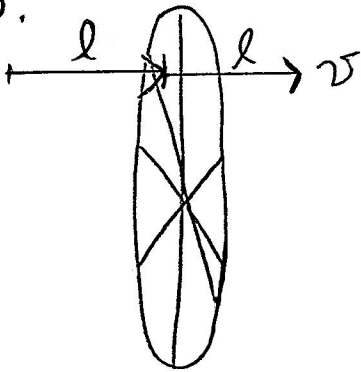


$$\tan \theta = \frac{T_y}{T_x} = \frac{4.415}{26.01} = 0.1697$$

$$\theta = \tan^{-1} 0.1697 = 9.631^\circ$$

Ans. The angle the chord makes with the horizontal $\theta = 9.63^\circ$ (3sf)

Q5.



$$r = 25 \text{ cm} = 0.25 \text{ m}$$

$$\theta = \frac{360}{8} = 45^\circ$$

$$\text{or } \theta = \frac{2\pi}{8} = 0.7854 \text{ rad}$$

$$v = 6 \text{ m s}^{-1}$$

$$l = 20 \text{ cm} = 0.2 \text{ m}$$

→ Time taken for arrow to travel a distance equal to its length is

$$t = \frac{l}{v} = \frac{0.2}{6} = 0.03333 \text{ s}$$

→ For the arrow not to be hit by a spoke, the gap between spokes must remain open for at least the time it takes the arrow to travel a distance equal to its length, i.e., $t = 0.03333 \text{ s}$.

→ This minimum time $t = 0.03333 \text{ s}$ corresponds to the maximum angular velocity ω_{max} that the wheel can rotate without the spokes hitting the arrow.

→ This minimum time $t = 0.03333 \text{ s}$ is the time for the spokes to cover an angle of $\theta = 45^\circ = 0.7854 \text{ rad}$, hence

$$\omega_{\text{max}} = \theta / t = \frac{0.7854}{0.03333} = 23.56 \text{ rad s}^{-1}$$

$$\omega_{\text{max}} = 23.56 \text{ rad s}^{-1} = 23.56 \left(\frac{\text{rev}}{2\pi} \right) \text{ s}^{-1}$$

$$\omega_{\text{max}} = 3.750 \text{ rev s}^{-1}$$

Q5 cont.

Because angular velocity is the same for any radius inside the wheel's rim it doesn't matter how far from the center the arrow passes, so "no" it doesn't matter where between the axle and the rim the arrow is aimed.

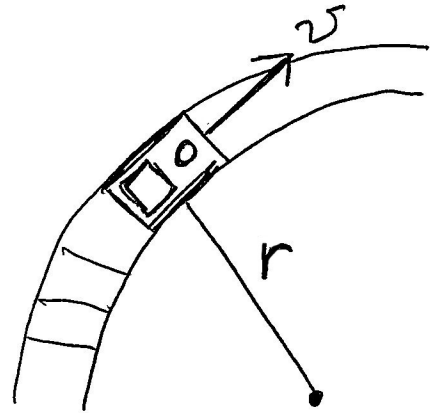
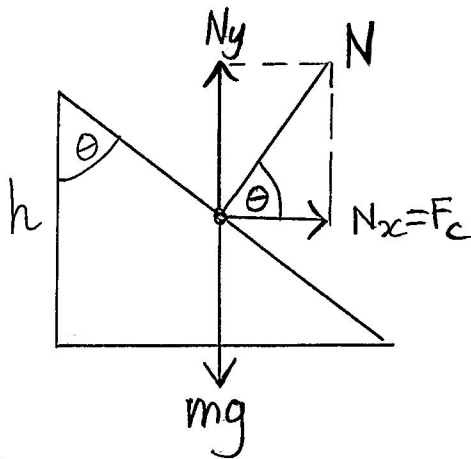
Ans. The maximum angular velocity the wheel can rotate without the spokes hitting the arrow is

$$\omega_{\max} = 23.6 \text{ rad}\cdot\text{s}^{-1} = 3.75 \text{ rev}\cdot\text{s}^{-1} \text{ (3sf)}$$

and

No, it doesn't matter where in the gap between spokes the arrow is aimed.

Q6.



Data:

Radius of curve = $r = 500\text{m}$ Velocity of train = $v = 50 \frac{\text{km}}{\text{h}} = 50 \left(\frac{1000\text{m}}{3600\text{s}} \right) = 13.89 \text{ms}^{-1}$ Distance between rails = $d = 1.5\text{m}$ Centripetal force = $F_c = \frac{mv^2}{r}$ (1) $F_c = N_x = N \cos \theta$ (2) $mg = N_y = N \sin \theta$ (3) $h =$ height outer rail must be raised.

→ For the train to go round the curve without lateral thrust (i.e. no forces such as friction along the radial direction), there must be enough centripetal force F_c provided by the x -component N_x of the normal reaction N and the y -component N_y of N must balance mg .

Q6. cont.

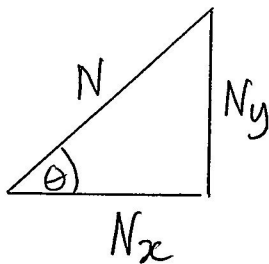
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→ For a given tangential (linear) velocity and for a given radius of the curve, there is only one angle θ , hence one height h , for which N_x provides enough centripetal force and for which N_y balances mg .

→ The condition for the correct θ and hence h for the train to round the bend is

$$N_x = N_c = \frac{mv^2}{r}$$

Find θ :

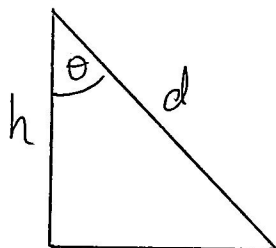


$$\tan\theta = \frac{N_y}{N_x} = \frac{mg}{mv^2/r} = \frac{gr}{v^2}$$

$$\tan\theta = \frac{(9.81)(500)}{(13.89)^2} = 25.42$$

$$\theta = \tan^{-1}(25.42) = 87.75^\circ$$

Find h

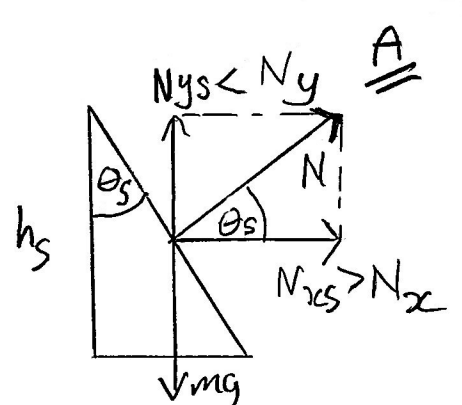


$$h = d \cos\theta = 1.5 \cos 87.75$$

$$h = 0.05889 \text{ m} = 5.889 \text{ cm}$$

Ans. Outer rail must be raised to a height of at least $h = 5.89 \text{ cm}$ (3sf)

Q6. For a given velocity of the train and a given radius of the curve, only the angle θ of B will balance mg and provide the required centripetal force.



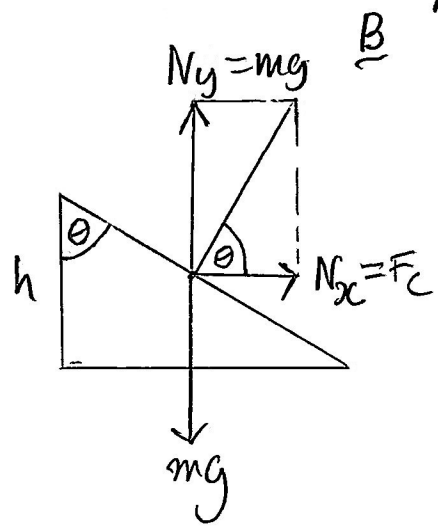
$\theta_s < \theta$, hence $h_s > h$

$N_{xs} > F_c$ (OK)

but

$N_{ys} < mg$, (NOT OK)

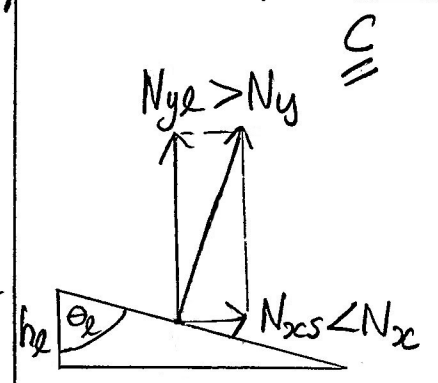
Hence, without friction, the train will slide down the slope, i.e., N_{ys} is too small to balance mg .



$N_x = F_c$

$N_y = mg$

The correct θ for a given velocity v and orbital radius r .
 N_y balances mg , and N_x provides the needed centripetal force.



$\theta_2 > \theta$, hence

$h_2 < h$

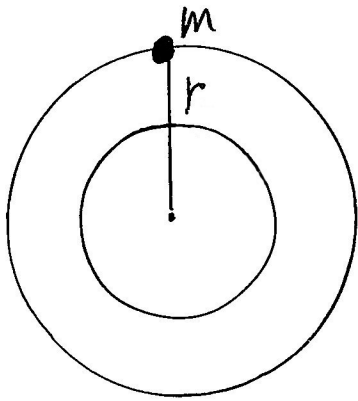
$N_{y2} > mg$ (OK)

but

$N_{x2} < F_c$ (NOT OK)

Hence, without friction, the train will slide up the slope and fall off the rail, i.e. N_{x2} is too small to provide enough centripetal force.

Q.7.



$$T = 87 \text{ min} = 87 (60 \text{ s}) = 5220 \text{ s}$$

$$r = 6500 \text{ km} = 6500 (10^3 \text{ m}) \\ = 6.5 \times 10^6 \text{ m}$$

$$G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$

mass of satellite = m

→ The centripetal force is provided by the attraction of the earth, hence

$$F_c = F_{\text{earth}}$$

$$mr\omega^2 = \frac{GmM}{r^2}$$

$$M = \frac{r^3\omega^2}{G} \quad (1)$$

Find ω :

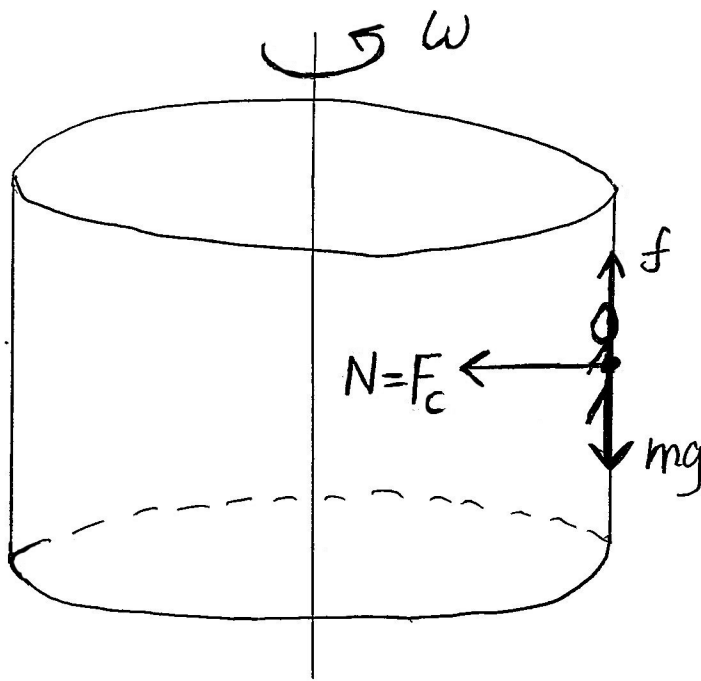
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{5220} = 1.204 \times 10^{-3} \text{ rad s}^{-1}$$

Sub ω into Eq. (1)

$$M = \frac{(6.5 \times 10^6)^3 (1.204 \times 10^{-3})^2}{6.67 \times 10^{-11}} = 5.969 \times 10^{24} \text{ kg}$$

Ans. Mass of the earth $M = 5.97 \times 10^{24} \text{ kg}$

Q8



Data:

$$r = 2.5 \text{ m}$$

$$\mu = ?$$

$$\omega = 0.6 \frac{\text{rev}}{\text{s}}$$

$$= 0.6 \frac{(2\pi \text{ rad})}{\text{s}}$$

$$= 3.770 \text{ rad}\cdot\text{s}^{-1}$$

$$g = 9.81 \text{ m}\cdot\text{s}^{-2}$$

$N = \text{normal reaction}$

$$\text{frictional force } f = \mu N \quad (1)$$

$$\text{Centripetal force } F_c = mr\omega^2 \quad (2)$$

→ The normal reaction provides the centripetal force, hence

$$N = F_c = mr\omega^2 \quad (3)$$

Sub. Eq. (3) into Eq. (1):

$$f = \mu mr\omega^2 \quad (4)$$

→ For the person not to fall, the frictional force must balance the person's weight, hence

$$f = mg \quad (5)$$

→ Sub. Eq. (5) into Eq. (4):

$$mg = \mu mr\omega^2$$

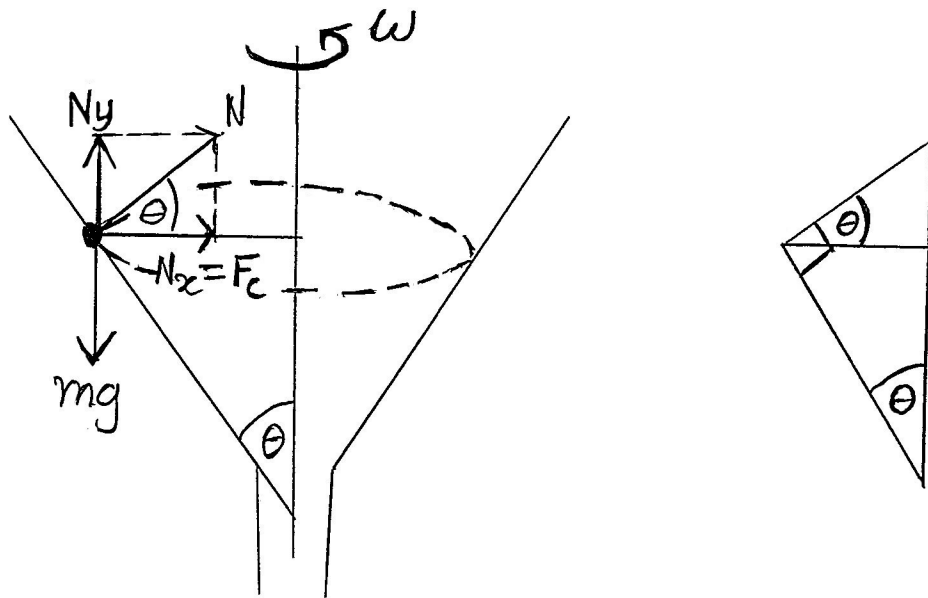
$$g = \mu r\omega^2$$

$$\mu = \frac{g}{r\omega^2} = \frac{9.81}{(2.5)(3.77^2)} = 0.2760$$

Ans. the minimum coefficient of friction is

$$\underline{\underline{\mu = 0.276 \text{ (3sf)}}}$$

Q9



$$N_x = N \cos \theta \quad (1)$$

$$F_c = \frac{mv^2}{r} \quad (2)$$

$$N_x = F_c \quad (3)$$

Sub. Eq. (1) and (2) into Eq. (3)

$$N \cos \theta = \frac{mv^2}{r}$$

$$N = \frac{mv^2}{r \cos \theta} \quad (4)$$

$$N_y = N \sin \theta \quad (5)$$

$$N_y = mg \quad (6)$$

Equate Eq. (5) and Eq. (6):

$$mg = N \sin \theta$$

$$N = \frac{mg}{\sin \theta} \quad (7)$$

Equate Eq. (4) and Eq. (7)

$$\frac{mv^2}{r \cos \theta} = \frac{mg}{\sin \theta} \Rightarrow \frac{v^2}{r \cos \theta} = \frac{g}{\sin \theta} \Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{rg}{v^2}$$

$$\tan \theta = \frac{rg}{v^2}$$

$$v = \sqrt{\frac{rg}{\tan \theta}}$$

Ans. $v = \sqrt{\frac{rg}{\tan \theta}}$ is the required speed for particle

to maintain its motion in a circle.

-END-