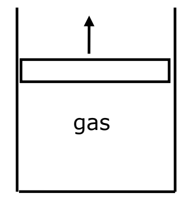
PHY1010 TUTORIAL SHEET 11 (2017)

THERMODYNAMICS

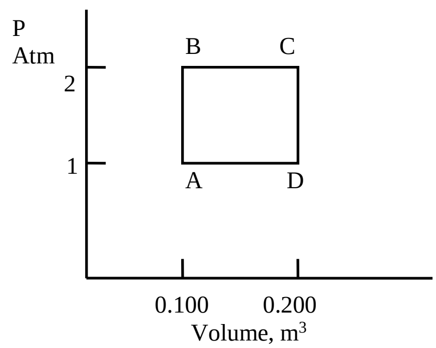
1. An ideal gas in the cylinder is initially at conditions p_1, V_1, and T_1. It is slowly expanded at constant temperature by allowing the piston to rise. Its final conditions are p_2, V_2, and T_2, where $V_2 = 3V_1$. Find the change in entropy of the gas during this expansion. The mass of the gas is 2 g, and M = 28 kg/kmol for it.



2. A reversible engine takes in heat from a high temperature reservoir at 527 ^{0}C and gives out heat to the sink at 127 ^{0}C.

How many joules per second must it take from the hot reservoir to produce useful mechanical work at the rate of 750 watts?

3. A heat engine contains helium originally in a state where V = 0.100 m^3, P = 1 atm, and T = 300 K. The gas is taken through the cycle ABCDA. (a) Calculate the heat inputs and outputs for the four parts of the cycle. Observe the sign of Q in each case. (b) Calculate the work inputs and outputs for the four parts of the cycle. Observe the sign of W in each case. (c) Calculate the efficiency W_{out}/W_{in} for this engine.



4. A sample of air (γ =1.40) is *slowly* compressed isothermally from 1.0 atm pressure to 2.0 atm. The original volume and temperature are V_1 = 20 litres and T_1 = 290 K. Next the air is suddenly expanded back to its original pressure of 1 atm.

(a) Sketch a *P-V* diagram of these processes,

(b) Find the final volume and temperature, and

(c) Find ΔU, ΔQ , and ΔW for each process.

5. *A dish of hot food is placed in a refrigerator maintained at 5^{0}C. To cool to this temperature, the food must lose 220,000 J.

(a) How much electric energy is needed to operate the compressor if we assume the room temperature is 23^{0}C and the refrigerator runs at half its theoretical maximum *COP*?

(b) If electricity sells for 200 Kwacha/kWh, how much money does this cooling cost?

6. A 1200 kg car is to accelerate from rest to a velocity of 8.0 m/s in 7 seconds.

(a) What is the minimum horsepower the engine must deliver if we ignore all friction losses?

(b) Assuming the car uses fuel with an efficiency of 20%, determine the amount of gasoline consumed in the 7 second time period.

Gasoline delivers about 50,000J per gram burned. 1h.p. = 736 watts.

7. *A Carnot engine having an ideal gas as the working substance is driven backwards and is used to freeze water already at 0^0C. If the engine is driven by a 500 watt electric motor having an efficiency of 60%, how long will it take to freeze 15kg of water? Take 15^0C and 0^0C as the working temperatures of the engine, and assume that there is no heat loss in the refrigerating system. Latent heat of fusion of ice = 336,000 J/kg.

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1. For an isothermal expansion of an ideal gas $\Delta W = \Delta Q = p_1V_1 \ln\left(\frac{V_2}{V_1}\right)$.

Hence $\Delta S = \frac{\Delta Q}{T} = [(p_1V_1)/T_1]\ln\left(\frac{V_2}{V_1}\right) = \frac{m}{M}R\ln\left(\frac{V_2}{V_1}\right)$, using the ideal gas law.

Or, $\Delta S = \frac{2\times10^{-3}kg}{28kg/kmol}\times\frac{8314J}{kmol.K}\times\ln 3$ = 0.65 J/K

2. Efficiency $\eta = 1-\frac{T_2}{T_1}$. T_2 = 527 ^{0}C = 800K.

T_1 = 127 ^{0}C = 400K. Hence $\eta = 1-\frac{400}{800} = 1-0.5 = 0.5 = 50\%$.

Useful mechanical work produced is W = 750watts = 750 J/s.

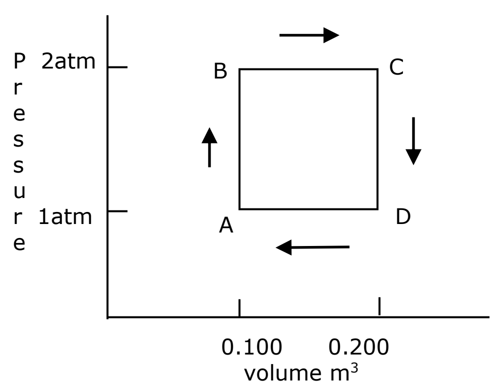
Efficiency = $\frac{work\ done}{heat\,input\,at\,higher\ temp} = \frac{W}{Q} \rightarrow Q = \frac{W}{\eta} = \frac{750}{0.5} = 1500J/s$

3. Use the ideal gas law to calculate the temps. at B, C, and D.

$T_B = \frac{2.013\times0.1\times300}{1.013\times0.1} = 600K$ Similarly,

T_C = 1200K and T_D = 600K. The number of mols of the gas, n (from $PV = nRT$) =

$$\frac{P_AV_A}{RT_A} = \frac{1.013\times10^5\times0.1}{8.314\times300} = 4.06$$



For A to B, volume is constant: heat $Q_{AB} = nC_V\,\Delta T$ = 4.06 x 1.50 x 8.314 x 300 = 15,190J [C_V = (3/2)R]

For B to C, pressure is constant: heat $Q_{BC} = nC_P\,\Delta T$ = 4.06 x 2.50 x 8.314 x 600 = 50,632J [$C_P = C_V + R$ = (5/2)R] Also $Q_{BC} = \Delta U + p\Delta V = \frac{3}{2}nR\Delta T + p\Delta V$

Similarly, $Q_{CD} = nC_V\,\Delta T$ = 4.06 x 1.50 x 8.314 x (600-1200) = -30,379J and

$Q_{DA} = nC_P\,\Delta T$ = 4.06 x 2.50 x 8.314 x (300-600) = -25316J

Total heat input $Q_{in} = Q_{AB} + Q_{BC}$ = 15190 + 50632 = 65822J and heat output

Q_{out} = Q_{CD} + Q_{DA} = -(30379+25316) = -55695 J

(b) Work done for constant-volume processes *A* to *B* and *C* to *D*: $W_{AB} = 0 = W_{CD}$

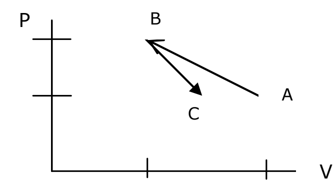
Work for *B* to *C* is $W_{BC} = P_B.\Delta V$= 2 x 1.013x10^5 (0.2-0.1) = 20260J and

$W_{DA} = P.\Delta V$= 1.013x10^5 (0.1-0.2) = -10130J.

Total work output $W_{out} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$ = 20260 +(-10130)J =10130J

(c) $\frac{W_{out}}{Q_{in}} = \frac{10130}{65822} = 0.154$

4. (a) The *P-V* diagram for the processes is as shown:



Along *AB* the process is isothermal, along *BC* it is adiabatic.

(b) Using the isothermal part the volume at *B* is $V_B = V_A(P_A/P_B)$ = 20(1.0/2.0=10.0 litre. The final volume V_C is obtained by using the adiabatic relation $P_C V_C^{\gamma} = P_B V_B^{\gamma}$, giving V_C = $V_B(P_B/P_C)^{1/\gamma} = 10(2/1)^{1/1.40} = 16.4\,litre$. Now using the ideal gas law, the final temperature is T_C = $T_B(P_C/P_B)(V_C/V_B)$ = 290 x (1/2)(16.4/10) = 238K.

(c) For the isothermal process along *AB*: ΔT = 0, so ΔU = 0. The work done is: W_{AB} = $nRT\ ln(V_B/V_A) = P_A\ V_A\ ln(V_B/V_A)$ = 1.01 x 10^5 x 20 x 10^{-3} *ln* (10/20) = -1400J. From the first law $Q_{AB} = \Delta U + W_{AB}$ = -1400J = -335 cal.

For the adiabatic process along *BC:* Q_{BC} = 0 so W_{BC} = - ΔU_{BC} and $\Delta U_{BC} = C_V\ n\Delta T$ = 2.5 $R.n.\Delta T$. From the initial pressure, volume, and temperature the number of mols is $n = P_A V_A / RT_A$ = 1.01 x 10^5 x 20 x 10^{-3}/(8.314 x 290) = 0.838 mol, so ΔU_{BC} = 2.5 x 8.314 x 0.838(238-290) = -906J and W_{BC} = 906 J (work done by decrease in internal energy).

5. Maximum COP = $\frac{T_c}{T_h - T_c} = \frac{278}{296-278} = 15.4$.

Half of maximum efficiency: electrical energy required is

ΔW = $\frac{\Delta Q_c}{COP} = \frac{220{,}000}{15.4/2} = 28{,}489J$.

Energy consumed is ΔW = $\frac{28489J \times 1kW.h}{3.6 \times 10^6 J} = 7.91 \times 10^{-3} kW.h$

Hence cost = 7.91x 10^{-3} x 200 = 1.58 Kwacha.

6. (a)Average power delivered to the car is = kinetic energy/time or

$$\frac{mv^2/2}{t} = \frac{1200\times 8^2\times 0.5}{7} = 5485.7 watts = 7.45 hp$$

(b) Input power is 5485.7/0.2= 27428.5 watts. Energy supplied by the gasoline(petrol)

E = power x time =27428.5 x 7 =191999.5 joules.

Hence mass of gasoline to be burnt = energy/50000 J/g = 3.84 grams

07. The effective power of the motor used to drive the engine is

500 x $\frac{60}{100} = 300 watts = 300 J/s$.

The freezing of 15kg of water at 0 ^{0}C requires an extraction of 15 x 336000 J of energy. This is done by a Carnot engine working in reverse as a refrigerator.

COP η = $\frac{T_2}{T_1 - T_2} = \frac{273}{15}$. T_1 (=273+15K) and T_2 (273K) are the working temperatures.

Input energy required by the engine = $\frac{15\times 336000}{\eta} = \frac{15\times 336000\times 15}{273} = 2.769\times 10^5 J$

Hence time = $\frac{2.769\times 10^5 J}{300 J/s} = 9.23\times 10^2 s = 15.4 \text{minutes}$.