PHY-1010 TUTORIAL SHEET 12 (2017)

Waves and Vibrations

01. What fraction of the total energy of a body undergoing SHM is kinetic and what fraction is potential when the displacement is one half of the amplitude? At what displacement are the kinetic and potential energies equal to each other?

02. A string with a mass per unit length of 0.006kg/m is stretched by the application of a force of 250N between two fixed points. (i) What length of the string will be required to produce a fundamental frequency of 400Hz? (ii) What tension is required to give this string a fundamental frequency of 700 Hz?

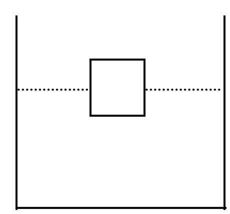
03. A steel strip is clamped at one end, while the free end vibrates with a frequency of 50Hz and an amplitude of 8mm. Find (a) the velocity of the free end when passing through the zero position (b) the acceleration at the maximum displacement.

04. The bob of a pendulum is drawn aside to a certain angle and released. When the ball passes the low point of the arc, the tension in the string is twice the weight of the bob. Show that the original displacement angle is 60 degrees.

05. A string held at both ends vibrates in 5 segments to a frequency of 460Hz. (a) What is the fundamental frequency? (b) What frequency will cause it to vibrate in three segments ? Draw appropriate figures.

06. The displacement along a wave is given by $y = 0.08 \sin [\, 2\pi(x - 0.1t)]$ where the distances are in metres and the times are in seconds. ($k = 2\pi/\lambda$). Find (a) the amplitude (b) the frequency (c) the wavelength, and (d) the speed of the wave motion.

07. A wooden cube of density ρ and length L on each edge floats in a liquid of density ρ' in such a way that the upper and lower faces are horizontal. The cube is pushed down and released. Show that the cube will oscillate with SHM with a frequency $f = (1/2\pi)\sqrt{(g\rho'/L\rho)}$.



08. A particle executes simple harmonic motion of period 8 seconds and amplitude 4cm. Find the velocity and acceleration when the particle is 2cm from the central position and also their maximum values.

09. A particle is moving with s.h.m. in a straight line. When the distance of the particle from the equilibrium position has the values x_1 and x_2, the corresponding values of the velocities are u_1 and u_2. Show that the period is
$2\pi\sqrt{[(x_2^2 - x_1^2)]/(u_1^2 - u_2^2)}$.

10. Two particles *A* and *B* start vibrating together simple harmonically along the same straight line. If their periods are 40s and 60s respectively, calculate their phase difference (a) after 20s from the start, and (b) when particle A is at the end of its path, while the other is at the middle of its path.

Vibration and Waves (Sheet 12) 2017

01. Total energy E = ½ $m v_{max}^2$ = ½ $m\omega^2 x_0^2$ [$v = \omega x$; x_0 = amplitude and x = displacement]. Kinetic energy (at x = $\frac{x_0}{2}$) ½ mv² = ½ $m\omega^2(x_0^2 - x^2)$ = ½ $m\omega^2(x_0^2 - \frac{x_0^2}{4})$ = $\frac{1}{2}m\omega^2 \frac{3}{4}x_0^2$. Potential energy = ½ $m\omega^2 x^2$; [$\omega = \sqrt{\frac{k}{m}}$]

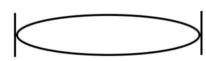
Hence $\frac{kinetic energy}{total energy} = \frac{\frac{1}{2}m\omega^2\frac{3}{4}x_0^2}{\frac{1}{2}m\omega^2 x_0^2} = \frac{3}{4}$ and $\frac{potential energy}{total energy} = \frac{\frac{1}{2}m\omega^2 x^2}{\frac{1}{2}m\omega^2 x_0^2} = \frac{1}{4}$

as $x = \frac{x_0}{2}$. Hence of the total energy, kinetic energy is three-fourth and pot energy is one-fourth. When K.E. = P.E. ½ $m\omega^2(x_0^2 - x^2)$ = ½ $m\omega^2 x^2$ or $x^2 = x_0^2 - x^2$

or $2x^2 = x_0^2$ *hence displacement* $x = \frac{x_0}{\sqrt{2}}$

02. Tension T = 250N; m/L = 0.006kg/m. Hence v = $\sqrt{\frac{T}{m/L}} = \sqrt{\frac{250}{0.006}} = 204 m/s.$

We have $\lambda_0 = \frac{v}{f_0} = \frac{204}{400}$ = 0.51m. Length L = $\frac{\lambda_0}{2} = \frac{0.51}{2} = 0.255m$



(ii) f_0 = 700Hz; $\lambda_0 = \frac{v}{f_0} = \frac{1}{f_0}\sqrt{\frac{T}{m/L}}$

Hence T = $\lambda_0^2 f_0^2 \frac{m}{L} = (2L)^2 f_0^2 \frac{m}{L}$ = (0.51)² (700)² x 0.006 = 764.7N

03. We have f = 50 Hz = $\frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow \sqrt{\frac{k}{m}}$ = 100π.

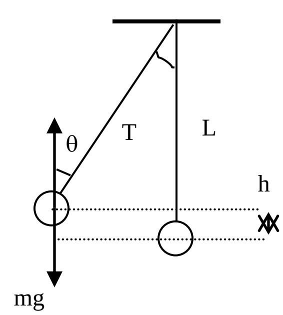
Also acceleration a = + $\frac{k}{m}x_0 = 100^2\pi^2 x_0 = 100^2\pi^2 \times 8 \times 10^{-3} = 788 m/s^2$.

And $v_m = \sqrt{\frac{k}{m}}.x_0 = 8 \times 10^{-3} \times 100\pi = 2.5 m/s$

04. mgh = ½ mv² hence v² = 2gh = 2g (L – L cos θ).

Maximum tension T_m = 2mg.

Also mg = T cos θ = 2mg cos θ.



Hence cos θ = 0.5 and θ = 60⁰. **(Not ok)**

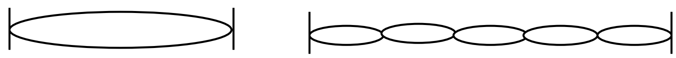
Another (OK): $\frac{mv^2}{L} + mg = 2mg.$

Hence g = $\frac{v^2}{L} = \frac{2gL(1-\cos\theta)}{L}$ or 1-cos θ = 0.5 .

Hence cos θ = 0.5 and θ = 60^0.

05. One segment: L= λ/2. For n segments length L = $\frac{n\lambda}{2}$. Wavelength $\lambda = \frac{v}{f_n}$

giving L = $\frac{n}{2}\frac{v}{f_n}$ or $f_n = n.\frac{v}{2L}$. Given f_5 = 460Hz or 460 = $5.\frac{v}{2L}$



Hence $\frac{v}{2L}$ = 92Hz = f_1 And f_3 = 3 x 92 = 276 Hz.

06. We have y = 0.08 sin [2π(x – 0.1t)] = A sin(kx-ωt).

(i) Amplitude A = 0.08m (ii) frequency ω= 0.2π = 2πf giving f = 0.2/2 = 0.1Hz.

(iii) wavelength k = $\frac{2\pi}{\lambda}$ = 2π(given) giving λ = 1m.

(iv) speed v = fλ = 1x 0.1 = 0.1m/s.

07. Force required to push the cube down thru' a height *y* = weight of the displaced liquid. = volume x density = ($y.L^2.g$).ρ**'**

Hence restoring force (in opposite direction) = - ($y.L^2.g$).ρ**'**

From Hooke's law F = -kx hence k = L^2.g.ρ**'** or frequency f = $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ **=** $\frac{1}{2\pi}\sqrt{\frac{l^2 g\rho'}{m}}$

Now mass m = vol x density = l^3 x ρ Hence f = $\frac{1}{2\pi}\sqrt{\frac{l^2 g\rho'}{l^3\rho}} = \frac{1}{2\pi}\sqrt{\frac{g\rho'}{l\rho}}$ Q.E.D.

08. Period T = $\frac{2\pi}{\omega}$ $giving\ \omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$. Velocity at displacement y is given by v $= \omega\sqrt{y_0^2 - y^2}$. Given *y* = 2cm and *y₀* = 4cm, hence v = $\frac{\pi}{4}\sqrt{4^2 - 2^2} = \frac{\pi}{4}\sqrt{12}$ = 2.73cm/s. At y = 0, velocity is maximum : $v_{\max} = y_0\omega = \frac{\pi}{4}.4 = \pi = 3.14cm/s$

Acceleration at y is given by f = $-\omega^2 y = -\frac{\pi^2}{16}\times 2 = -1.23cm/s^2$. Acceleration is maximum when y = y_0 : Hence $f_{\max} = -\omega^2 y_0 = -\frac{\pi^2}{16}\times 4 = -2.46cm/s^2$

09. The velocity *u* of a particle executing s.h.m., at a distance x from the equilibrium position is *u* = $\omega\sqrt{x_0^2 - x^2}$ where x_0 is the amplitude. Given $u = u_1$ when $x = x_1$ *and* $u = u_2$ when $x = x_2$. Hence $u_1^2 = \omega^2(x_0^2 - x_1^2)\ and\ u_2^2 = \omega^2(x_0^2 - x_2^2)$

Or $u_1^2 - u_2^2 = \omega^2(x_2^2 - x_1^2)$ giving $\omega = \sqrt{\frac{u_1^2 - u_2^2}{x_2^2 - x_1^2}}$.

Time period T is T = $\frac{2\pi}{\omega}$ = $2\pi\sqrt{\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}}$ Q.E.D.

10.(a) After 20s from start, phase at A = $\frac{t}{T_1} = \frac{20}{40} = \frac{1}{2}$, phase at B = $\frac{t}{T_2} = \frac{20}{60} = \frac{1}{3}$

Hence phase difference = 1/2 - 1/3 = 1/6. This corresponds to an angle $\frac{2\pi}{6}$ =60^0.

(b) Let A be at the end of its path, and B be at the middle of its path. Then phase of A at the end of its path (maximum displacement) is either $\frac{\pi}{2} or \frac{3\pi}{2}$ depending on if it is at maximum positive or maximum negative displacement.

The phase of B is either 0 or π depending on the movement being in the positive or negative direction. Phase difference builds up with time. Hence phase difference is

$\frac{\pi}{2} - 0 = \frac{\pi}{2}$ $(\frac{3\pi}{2} - \pi = \frac{\pi}{2})$ or $\frac{3\pi}{2} - 0 = \frac{3\pi}{2}$; measured as a fraction of period it is $\frac{T}{4}$, or $\frac{3T}{4.}$