THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATITICS

MAT 1100 FOUNDATION MATHEMATICS TUTORIAL SHEET 9-2021

1. Sketch the graph of the function and state its domain, range, y-intercept and the equation of the horizontal asymptote: (a) $y = 2^x$+3 (b) $y = -2^x - 1$
   (c) $y = 2^{-x} - 1$ (d) $y = 2^{x+1}$ (e) $y = 2^{x-3}$ (f) $y = 4^{-x}$ (g) $y = \left(\frac{3}{2}\right)^{-x}$
   (h) $y = -\left(\frac{3}{2}\right)^{x} + 2$ (i) $y = 2^{-x^2}$ (j) $y = 3(3^{-x}) - 3$ (k) $y = 2^{-|x-2|} + 1$.
2. The population of a city triples every 25 years. At time $t = 0$, the population is 100,000. Write a function for the population $P(t)$ as a function of t. What is the population after (a) 50 years (b) 70 years (c) 100 years?
3. An experiment involves a colony of bacteria in an solution. It is determined that the number of bacteria doubles approximately every 3 minutes and the initial number of bacteria at the start of the experiment is 10^4. Write a function for the number of bacteria $N(t)$ as a function of t (in minutes). Approximately how many bacteria are there after: (a) 3 minutes (b) 9 minutes (c) 27 minutes (d) one hour?
4. Strontium-90 has a half-life of 29.1 years. The amount S of 100 kilograms of strontium-90 present after t years is given by $S = 100e^{-0.0238t}$. How much of the 100 kilograms will remain after 50 years?
5. After t years, the balance A in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas:
   (i) For n compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
   (ii) For continuous compounding: $A = Pe^{rt}$.
   (a) A bank offers two types of interest-bearing accounts. The first account pays 5% interest compounded quarterly. The second account pays 3% interest compounded continuously. Which account earns more money? Why?
   (b) If K10,000 is invested at annual interest rate of 11% for a period of five years, find the value of the investment for the following compounding periods:
   (i) Annually (ii) Monthly (c) daily (d) hourly.

(6) The present value of a sum of money is the amount of money that must be invested now (Principal), at a given rate of interest, to produce the desired sum at a later date.

Find the present value of K10, 000 if interest is paid at a rate of 9% per year, compounded semiannually, for three years.

(7) Express each of the following logarithmic equations in exponential form:

(a) $log_4 16 = 2$ (b) $log_2 \frac{1}{16} = -4$ (c) $log_4 2 = \frac{1}{2}$ (d) $log_5 0.2 = -1$.

(8) Without using a calculator, evaluate the following expressions:

(a) $log_3 9$ (b) $log_2 \frac{1}{16}$ (c) $log_8 2$ (d) $log_6 \frac{1}{36}$ (e) $log_{10}(0.0001)$ (f) $log_{12} 1$

(g) $\ln e^{10}$ (h) $\ln \frac{1}{e^3}$ (i) $\ln 1$ (j) $log_{49} 7$ (k) $log_9 \sqrt{3}$ (l) $3^{log_3 8}$ (m) $10^{log 87}$.

(9) Solve the following equations for real x: (a) $log_2 x = 5$ (b) $log_2 16 = x$

(c) $log_x 1000 = 4$ (d) $log_{10} 0.1 = x$ (e) $log_x 8 = \frac{3}{2}$ (f) $\ln(x + 2) = \ln 6$.

(10) In each case, find the domain, range, vertical asymptote and x-intercept of the logarithmic function. Then sketch its graph:

(a) $f(x) = log_2 x$ (b) $f(x) = log_2(x+4)$ (c) $f(x) = -log_4(x - 1)$

(d) $f(x) = \ln(x + 2)$ (e) $f(x) = log_3(x - 1) - 2$ (f) $f(x) = 1 - log_{10} x$

(g) $f(x) = 1 + \ln(-x)$ (h) $f(x) = |\ln x|$ (i) $f(x) = \ln|x|$ (j) $f(x) = -\ln(x - 3)$.

(11) Find the domain of the function: (a) $f(x) = log_2(10 - 3x)$ (b) $f(x) = log_3(x^2 - 1)$

(c) $f(x) = \ln(x - x^2)$ (d) $f(x) = \ln x + \ln(2 - x)$ (e) $f(x) = \ln(10 - x) - \sqrt{x - 2}$.

(12) A googol is 10^{100}, and a googolplex is 10^{googol}. Find $\log(\log(googol))$ and $\log(\log(\log(googolplex)))$.

(13) Use the laws of logarithms to rewrite the expression in a form with no logarithm of a product, quotient, or power: (a) $\log_2(x(x - 1))$ (b) $\log_5 \frac{x}{2}$ (c) $\log 7^{33}$ (d) $\ln(\pi x)$

(e) $\log_2(AB^2)$ (f) $\log_6 \sqrt[4]{17}$ (g) $\log_5 \sqrt[3]{x^2 + 1}$ (h) $\ln \frac{3x^2}{(x+1)^{10}}$ (i) $\ln\left(\frac{10^x}{x(x^2+1)(x^4+2)}\right)$.

(14) Evaluate the expression: (a) $\log_5 \sqrt{125}$ (b) $\log_2 \frac{1}{8}$ (c) $\log 0.001$ (d) $e^{\ln \sqrt{2}}$

(e) $\log_4 2$ (f) $\log_3(81^{22})$ (g) $\log 2 + \log 5$ (h) $\log_4 192 - \log_4 3$ (i) $e^{3 \ln 5}$.

(15) Rewrite the expression as a single logarithm: (a) $\log_3 5 + 5 \log_3 2$

(b) $\log 12 + \frac{1}{2}\log 7 - \log 2$ (c) $\log_5(x^2 - 1) - \log_5(x - 1)$ (d) $\ln 5 + 2 \ln x + 1$.

(e) $2\left(\log_5 x + 2\log_5 y - 3\log_5 z\right)$ (f) $4\log x - \frac{1}{3}\log(x^2+1) + 2\log(x-1)$.

(16) Show that $\log e = \frac{1}{\ln 10}$.

(17) Solve the following exponential equations: (a) $2(3^x) = 16$ (b) $2^{x-3} = 31$

(c) $2^{x+1} = 3^{x-1}$ (d) $4^x - 2^{x+1} = 48$ (e) $2^{2x+1} - 2^{x+1} + 1 = 2^x$ (f) $6^{\frac{x}{2}} = 5^{1-x}$

(g) $6^{2x+1} - 17(6^x) + 12 = 0$ (h) $\frac{50}{1+e^{-x}} = 4$ (i) $x^2 2^x - 2^x = 0$ (j) $e^x - 12e^{-x} = 1$

(k) $e^{4x} + 4e^{2x} - 21 = 0$ (l) $x^2 10^x - x10^x = 2(10^x)$.

(18) Solve the following logarithmic equations: (a) $\ln(x+2) = 1$ (b) $\log_3(3-x) = 3$

(c) $\log_2(x^2 - x - 2) = 2$ (d) $\log_5 x + \log_5(x+1) = \log_5 20$ (e) $\log_2(\log_3 x) = 4$.

(19) If $\log_2 4\sqrt{2} = x$, $\log_z y = 4$, and $y = 4x^2 - 2x - 6 + z$, find y.

(20) For what values of x is it true that (a) $\log(x+3) = \log x + \log 3$ (b) $\left(\log x\right)^3 = 3\log x$?

(21) Radioactive decay is modeled by the equation $m(t) = m_0 e^{-rt}$ where r is the rate of decay expressed as a proportion of the mass and m_0 is the initial mass. The half-life, h, is the time take for half the mass to decay. Show that $r = \frac{\ln 2}{h}$.

Polonium-210 (210_{Po}) has a half-life of 140 days. Suppose a sample of this substance has a mass of 300mg.

(a) Find a formula for the amount of sample remaining at time t.

(b) Find the mass remaining after one year.

(c) How long will it take for the sample to decay to a mass of 200mg?

(d) Draw a graph of the sample mass as a function of time.

[Here you may use a calculator]

(22) Newton's Law of Cooling: If D_0 is the initial temperature difference between an object and its surroundings, and if its surroundings have temperature T_s, then the temperature of the object at time t is given by $T(t) = T_s + D_0 e^{-kt}$ where k is a positive constant that depends on the type of object.

(i) A hot bowl of soup is served at a dinner party. It starts to cool according to Newton's Law of cooling so that its temperature at time t is given by $T(t) = 65 + 145e^{-0.05t}$ where t is measured in minutes and $T$is measured in °F.

(a) What is the initial temperature of the soup?

(b) What is the temperature after 10 min?

(c) |After how long will the temperature be 100°F?

(ii) Newton's Law of cooling is used in homicide investigations to determine the time of death. The normal body temperature is 98.6°F. Immediately following death the body begins to cool. It has been determined experimentally that the constant in Newton's Law of cooling is approximately $k = 0.1947$. If the temperature of the surroundings is 60°F and the temperature of the body is now 72°F, how long ago was the time of death?

(iii) A kettle full of water is brought to a boil in a room with temperature 20°C. After 15 min the temperature of the water has decreased from 100°C to 75°C. Find the temperature after another 10 min. Illustrate by sketching the graph of the temperature function.