THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATITICS

MAT 1100 FOUNDATION MATHEMATICS TUTORIAL SHEET 6-2021

1. Compute explicitly the given quantity, where possible: (a) $\frac{5!}{3!0!}$ (b) $\binom{12}{7}$ (c) $\binom{12}{\binom{5}{3}}$

   (d) $\binom{-2}{3}$ (e) $\binom{-3}{\binom{2}{4}}$ (f) $\binom{0}{2}$ (g) $\binom{0}{0}$ (h) $\binom{-3}{-1}$.

2. The coefficient of x^3 in the expansion of $(3+bx)^5$ is -720. Find the value of the constant b.

3. Show that (a) $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\cdots+\binom{n}{n}=2^n$ (b) $\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\cdots\cdots-\binom{n}{n}=0$.

4. Find the first three terms in the expansion of each of the following:

   (a) $(2-x)^5$ (b) $(1-4y)(1-5y)^8$ (c) $(1+2x)^6$ (d) $\left(x-2y^2\right)^4$ (e) $(\frac{1}{x}-x^2)^{12}$.

5. Given that $(1+2y)^7 = 1 + Ax + Bx^2 + Cx^3 + \cdots$, A, B find and C.

6. Write the term indicated in the expansion of each of the following expansions:

   (a) fourth term of $(1+2y)^7$;(b) fifth term of $\left(x^2-\frac{1}{x}\right)^8$;(c) seventh term $(x^{\frac{1}{3}}-2x^{\frac{2}{3}})^{10}$

   (d) constant term in $(x^{1/4}-\frac{3}{x^{1/2}})^8$ (e) term involving x^{14} in $(\frac{2}{x}-x^2)^{10}$.

7. Find the term independent of *x* in the expression of $\left(x^2-\frac{3}{x}\right)^n$ when

   (a) $n=3$ (b) $n=12$.

8. The first three terms in the expansion of $(1+kx)^n$ are 1, $14x$ and $84x^2$ respectively. Find the values of the constants *k* and *n* and the coefficients of and terms.

9. Expand the following functions as a series of ascending powers of *x* up to and including the term in x^3. In each case give the range of values of *x* for which the expansion is valid:

   (a) $(1-2x)^{\frac{1}{2}}$ (b) $(4+x)^{-1}$ (c) $\frac{2x+1}{(x-1)(x+1)}$ (d) $\frac{1}{\sqrt{x+1}}$ (e) $(x^2+\frac{2}{x})^{-2}$

10. By substituting 0.08 for *x* in $(1+x)^{\frac{1}{2}}$ and its expansion find $\sqrt{3}$ correct to four significant figures.

11. Use a suitable binomial expansion to find $\frac{1}{\sqrt{0.99}}$ correct to five decimal places.

12. Find the expansion of $(1+y)^6$.
(a) By writing $y = x + x^2$, find the first four terms, in the expansion of $(1+x+x^2)^6$.

(b) By putting $x = 0.01$ in your first four terms, find an approximation for $(1.0101)^6$.

13. Find the first four terms in descending powers of x in the following expansions, and indicate the range of values of x for which the expansion is valid: (a) $(\frac{x}{a} + \frac{a^3}{x^2})^{-3}$

(b) $(x^2 - \frac{2y^2}{x})^{3/2}$ (c) $(1 - \frac{3y^2}{x})^{1/3}$ (d) $(x^2 - \frac{2}{x^4})^{1/2}$ (e) $(1-x^3)^{-2/3}$.

14. Sketch the graphs of the following and determine whether or not the limit exists at the given point for each of the following:

(a) $f(x) = \begin{cases} x^2+1, x<0 \\ 1-x^2, 0 \le x \le 1 \\ x, x>1 \end{cases}$, at c=0, 1, 3 (b) $f(x) = \begin{cases} x+2, & x \le -1 \\ x^2, & -1 < x < 1 \\ 3x+1, & x \ge 1 \end{cases}$ at $c = -1, 1$.

15. Evaluate each of the following limits:(a) $\lim_{x\to -5} \frac{x^2}{2x-3}$ (b) $\lim_{x\to 2} \frac{x^3-8}{x-2}$ (c) $\lim_{x\to -2} \frac{x^3+8}{x+2}$

(d) $\lim_{x\to 2} \sqrt{\frac{x^2-4}{x^2-3x+2}}$ (e) $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{x^2}\right)$ (e) $\lim_{x\to +\infty} \frac{5x-1}{-x+4}$ (f) $\lim_{x\to +\infty} \frac{x^2+3x+7}{3x^2-2x-1}$

(g) $\lim_{x\to -\infty} \frac{\sqrt{x^2+6}}{x+6}$ (h) $\lim_{x\to 2} \sqrt{\frac{x^2-4}{x^2-3x+2}}$ (i) $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x^2}$ (j) $\lim_{x\to 0} \frac{2x^2+x}{|x|}$ (k) $\lim_{x\to 1} |\frac{2x+1}{x-1}|$.

16. Sketch the graph of each of the following functions defined and determine if each is continuous at the given point c: (a) $f(x) = \sqrt{3x-1}$ at $c = \frac{1}{3}$

(b) $f(x) = \begin{cases} |x-2|, & x \ne 2 \\ 1, & x = 2 \end{cases}$ at $c = 2$ (c) $f(x) = \begin{cases} 2x+1, & x \le 1 \\ -x+4, & x > 1 \end{cases}$ at $c = 1$

17. Define $f(c)$ so that the function is continuous:

(a) $f(x) = \frac{(x-1)(x+2)}{x-1}, \quad x \ne 1, c = 1$ (b) $f(x) = \frac{x^2-3x+2}{x^2-4}, \quad x \ne \pm 2, c = 2$

(c) $f(x) = \frac{x^2-1}{x+1}, \quad x \ne -1, c = -1$ (d) $f(x) = \frac{x^3+8}{x+2}, \quad x \ne -2, c = -2$.