THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATITICS

MAT 1100 FOUNDATION MATHEMATICS TUTORIAL SHEET 7-2021

1. Find the derivative of each of the following functions from first principle:

   (a) $f(x) = \sqrt{x+1}$ (b) $f(x) = 3x^2 + 4x - 8$ (c) $f(x) = \frac{1}{\sqrt{1-x}}$ (d) $f(x) = \frac{1}{x-3}$

2. Differentiate with respect to x:

   (a) y= $x^4 - 9x^3 + 6$ (b) y= $x^3\sqrt{x-2}$ (c) y= $\frac{x^2 - 7x + 4}{x^3 + 2}$ (d) y= $(3x^3 - x^2)^6$

   (e) y= $\frac{(x^2+3)^2}{x^{3/2}}$ (f) $y = \frac{8x}{(x^3+2)^2}$ (g) $y = \left(\frac{x+1}{x-1}\right)^{15}$ (h) $y = \sqrt{\frac{2x-1}{x+1}}$ (i) $y = \sqrt{x + \sqrt{x}}$

3. Use implicit differentiation to find $\frac{dy}{dx}$ of each of the following functions:

   (a) $x^2 + y^2 + 8x - 2y - 8 = 0$ (b) $2x^2 + 2y^2 - 3x + 2y + 1 = 0$

   (c) $x^3 + xy - x^2y^2 = 7$ (d) $(x - y)^5 = x + y + 1$ (e) $\sqrt{xy} + x + y^2 = 2.$

4. Find the equation of the tangent to each of the following curves at the given point:

   (a) $2y^2 - x^2 = 1$ at $(-1, -1)$ (b) $x^2 + 2x + y^2 - 4y - 24 = 0$ at $(4, 0)$

   (c) $(x + y)^3 - x^3 - y^3 = 0$ at $(1, -1)$ (d) $x = \frac{2y}{x^2-y}$ at $(-1, -1).$

5. Evaluate the following indefinite integrals:

   (a) $\int(3x^2 + 2x + 1)dx$ (b) $\int\left(x^{5/2} - \frac{5}{x^4} - \sqrt{x}\right)dx$ (c) $\int(2x\sqrt{x} - \frac{1}{\sqrt{x}})dx$

   (d) $\int\frac{(3x+4)^2}{\sqrt{x}}$dx (e) $\int\frac{2x^4-3x^2+5}{7x^2}dx$ (f) $\int\left(1 - \frac{1}{\sqrt[3]{x^2}}\right)dx$ (g) $\int\left(\frac{2x}{t^5} - \sqrt{tx} + 4x\right)dt.$

6. Use a suitable substitution to evaluate each of the following integrals:

   (a) $\int x(2x^2 + 1)^{13}\,dx$ (b) $\int\frac{x^2}{(x^3-1)^2}dx$ (c) $\int\frac{x^2+3}{\sqrt[3]{x^3+9x}}dx$ (d) $\int\frac{(3+2\sqrt{x})^5}{\sqrt{x}}dx$

   (e) $\int\left(2 + \frac{3}{x}\right)^5\left(\frac{1}{x^2}\right)dx$ (f) $\int\frac{dx}{(\sqrt{x}+2)\sqrt{x}}$ (g) $\int\frac{k^3x^3}{\sqrt{a^2-a^4x^4}}dx$ (h) $\int t^2\sqrt{1-t}\,dt$

   (i) $\int 3x\sqrt{x-1}\,dx$ (j) $\int\frac{t^2-1}{\sqrt{2t-1}}dt$ (k) $\int\frac{r^2+2}{\sqrt{r-5}}dr$ (l) $\int\frac{x^3-2}{\sqrt[5]{x^4-8x+13}}dx.$

7. A particle moves along a line so that its speed v at time t is given by

$v(t) = \begin{cases} 5t, & 0 \le t < 1 \\ 6\sqrt{t} - \frac{1}{t}, & t \ge 1 \end{cases}$ where t is in seconds and v is in cm/s. Estimate the time(s) at which the particle is $4cm$ from its starting position.

8. Find the equation of the function f whose graph passes through the point $\left(0, \frac{7}{3}\right)$ and whose derivative if $f'(x) = x\sqrt{1 - x^2}$.

9. The marginal cost of a product is modelled by $\frac{dC}{dx} = \frac{12}{\sqrt[3]{12x+1}}$, where x is the number of units. When $x = 13, C = 100$.

(a) Find the cost function.

(b) Find the cost of producing 30 units.

10. Evaluate the following definite integrals:

(a) $\int_{-1}^{1}(3x^2 - 4x^3)dx$ (b) $\int_{1}^{7} \frac{2x^2-3x+5}{\sqrt{x}} dx$ (c) $\int_{0}^{2} |3x|dx$ (d) $\int_{-2}^{0} |3x|dx$ (e) $\int_{-2}^{2} |3x|dx$

(f) $\int_{0}^{4} \frac{x^3}{\sqrt{1+x^2}} dx$ (g) $\int_{-2}^{2} \left(\frac{1}{2}t^4 + 1\right) dt$ (h) $\int_{1}^{4}\left(\sqrt{x} + x\right)dx$ (i) $\int_{0}^{4} \frac{1}{\sqrt{2x+1}}$dx.

11. A company purchases a new machine for which the rate of depreciation can be modelled by $\frac{dV}{dt} = 10,000(t - 6)$, $0 \le t \le 5$, where V is the value of the machine after t years. Set up and evaluate the definite integral that yields the total loss of value of the machine over the first 3 years.

12. An evergreen nursery usually sells a type of shrub after 5 years of growth and shaping. The growth rate during those 5 years is approximated by $\frac{dh}{dt} = \frac{17.6t}{\sqrt{17.6t^2+1}}$ where t is the time (in years) and h is the height (in cm). The seedlings are 6cm tall when planted $(t = 0)$.

(a) Find the height function.

(b) How tall are the shrubs when they are sold?