THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATITICS

MAT 1100 FOUNDATION MATHEMATICS TUTORIAL SHEET 10-2021

1. Find the derivative of each function with respect to the given variable:
   (a) $f(x) = 2\sin^2 x$ (b) $f(x) = xcosx$ (c) $f(x) = \sqrt{x}sinx$ (d) $f(t) = \frac{\cos t}{\sqrt{t}}$
   (e) $f(t) = \sin t \cos^2 t$ (f) $f(x) = cos^3 x sin^2 x$ (g) $f(x) = \frac{1}{\sin x + \cos x}$
   (h) $f(x) = \frac{\sin x}{1+\cos x}$ (i) $f(x) = (\cos 3x + \sin 5x)^5$ (j) $f(t) = t^2 \cos(3t^2 - 1)$
2. Find $\frac{dy}{dx}$ given that (a) $y = \frac{\cos 3x}{\sin 5x}$ (b) $y = \sqrt{\cos\sqrt{x}}$ (c) $y = \sin 2\sqrt{x}$ (d) $y = \sin^2 x^2$
   (e) $y = \cos 3\sqrt[3]{x}$ (f) $y = \sqrt{x}(x - \cos x)^3$ (g) $y = \sqrt{x}\sin\sqrt{x + \sqrt{x}}$
   (h) $y = \cos(\sin x^2)$ (i) $y = \frac{\sec 5x}{\tan 3x}$ (j) $y = \sec^2 x - \tan^2 x$ (k) $y = x^3 \tan^3 x^3$
   (l) $y = \cot\left(\frac{1}{\sqrt{x}}\right)$ (m) $y = \sqrt{1 + \cot 5x}$ (n) $y = \sqrt{cosec\sqrt{x}}$ (o) $y = (\sec 2x)^7$
3. Find all points on the curve $y = f(x)$ where the tangent is horizontal:
   (a) $y = \cos 2x$ (b) $y = x - 2\sin x$ (c) $y = \sin x \cos x$ (d) $y = \frac{1}{3\sin^2 x + 2\cos^2 x}$.
4. Find $\frac{dy}{dx}$ given that (a) $y = \left(\frac{1}{3}\right)^x$ (b) $y = 11^{x^3}$ (c) $y = x3^{4x-2}$ (d) $y = e^{\cos 2x}$
   (e) $y = e^{\sqrt{1-x}}$ (f) $y = \frac{e^{2x}}{1+e^{2x}}$ (g) $x^2 y e^y - 10x + 3y = 0$ (h) $e^{xy} + x^2 - y^2 = 10$.
5. Find the derivative of each function given by $y = f(x)$:
   (a) $y = \ln x^2$ (b) $y = \ln 7x$ (c) $y = \ln(x^2 + 3)$ (d) $y = \ln\sqrt{4 - x}$ (e) $y = 2x\ln x$
   (f) $y = (\ln x^2)^2$ (g) $y = \frac{\ln x}{x^2}$ (h) $y = \ln\left(x\sqrt{x^2 - 1}\right)$ (i) $y = \ln\left(\frac{x}{x^2+1}\right)$ (j) $y = (\ln x)^4$
   (k) $y = e^{-x}\ln x$ (l) $y = \log_2 x$ (m) $y = \log(x^2 + 6x)$ (n) $y = (x+1)^x$ (o) $y = x^{\sin x}$
   (p) $y = (\ln x)^{\sqrt{x}}$ (q) $y = \left(1 + \frac{1}{x}\right)^x$ (r) $y = \ln(\cos x)$ (s) $y = \sin(\ln 2x)$ (t) $y = \ln(\ln x)$
6. Find $f(x)$ given that (a) $f'(x) = 4x; f(0) = 5$ (b) $f'(x) = 3\sqrt{x}; f(0) = 4$.
7. Find $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}$ given that (a) $y = 2x^2 + 7x - 12$ (b) $y = x\sqrt{x^2 - 1}$ (c) $y = \frac{x}{x^2+3}$.
8. Find the equation of the tangent line and the normal line to the curve given by the following equations at given points:
   (a) $y = 3x^2 - x + 1; x = 0$ (b) $y = \frac{1-x^3}{x^4}$; $(1, 0)$ (c) $y = \sqrt{x}\left(1 - \sqrt{x}\right); x = 4$.
   (d) $2xy + \pi\sin y = 2\pi; \left(1, \frac{\pi}{2}\right)$ (e) $\sqrt[3]{xy} = 14x + y; (2, -32)$. (f) $x^3 - xy - 3y^2 = 0; (2, -2)$.
9. Find the intervals on which the function given by $y = f(x)$ is increasing and decreasing:
   (a) $y = 3x + 2$ (b) $y = 8 - 2x^2$ (c) $y = 4x^2 + 8x + 13$ (d) $y = x^4 - 2x^2 + 1$
   (e) $y = \frac{x}{x+1}$ (f) $y = x\sqrt{x^2 + 1}$ (g) $y = x^2 e^{-2x}$ (h) $y = \frac{\ln 2x}{x}$ (i) $y = (x^2 - 4)^{2/3}$.
10. Find the relative maximum/minimum of each of the following functions:
    (a) $f(x) = x^2 - 6x$ (b) $f(x) = x^4 - 12x^3$ (c) $f(x) = x + \frac{1}{x}$ (d) $f(x) = \frac{1 - \ln x}{x}$.
11. Find the absolute maximum/minimum of each of the following functions on the given intervals: (a) $f(x) = 2(3 - x)$; $[-1, 2]$ (b) $f(x) = x^3 - 12x; [0, 4]$

(c) $f(x) = x^2 + \frac{16}{x}$; $[0, \infty)$ (d) $f(x) = \frac{2x}{x^2+4}$; $[0, \infty)$ (e) $f(x) = \frac{\ln(1+x)}{1+x}$; $(0, 5)$.

12. Sketch the graph of each equation, indicating the x −intercepts, y −intercept, and any stationary points: (a) $y = 2x^3 + 3x^2 - 72x + 5$ (b) $y = x(x-3)^2$ (c) $y = x - \sqrt{x}$ (d) $y = x^3 - 2x^2 - 7x + 10$ (e) $y = x^4 - 2x^2 + 1$ (f) $y = 8x^4 - x^8$ (g) $y = 1 - x^{1/3}$.
13. Determine the open intervals on which the graph of the function is concave upward or concave downward: (a) $f(x) = -3x^2$ (b) $y = -x^3 + 6x^2 - 9x - 1$ (c) $f(x) = \frac{x^2-1}{2x+1}$ (d) $f(x) = x(6-x)^2$ (e) $f(x) = x^5 + 5x^4 - 40x^2$. (f) $f(x) = \frac{24}{x^2+12}$.
14. Find the points of inflection on the graphs of each of the following:
(a) $f(x) = x^3 - 9x^2 + 24x - 18$ (b) $f(x) = (x-1)^3(x-5)$ (c) $f(x) = \frac{\ln x}{x}$
(d) $f(x) = (2x^2 - 3x - 1)e^{-x}$ (e) $f(x) = 5 - 6x - x^2$ (g) $f(x) = x^3 - 3x + 1$.
15. Sketch the graph of each of the following functions, indicating all intercepts, maxima, minima, asymptotes, points of inflection and concavity structure:
(a) $f(x) = \frac{x}{x-1}$ (b) $f(x) = \frac{3}{(x+2)^2}$ (c) $f(x) = \frac{x^2}{x^2+1}$ (d) $f(x) = \frac{1}{x^2+x-6}$
(e) $f(x) = \frac{(x-1)^2(x-3)}{x^2(x-4)}$ (f) $f(x) = \frac{1}{x^2-9}$ (g) $f(x) = 2x + e^{-x}$ (h) $f(x) = \frac{1}{x^2-x-2}$.
16. The profit P (in Kwacha) made by a cinema from selling x bags of popcorn can be modeled by $P = 2.36x - \frac{x^2}{25{,}000} - 3500$; $0 \le x \le 50{,}000$.
(a) Find the intervals on which P is increasing and decreasing.
(b) If you owned the cinema, what price would you charge to obtain a maximum profit from popcorn sales? Explain your reasoning.
17. A right circular cylinder of radius r and height h has a volume of $25m^3$. Show that the total surface area of the cylinder in terms of r is given by $S = 2\pi r\left(r + \frac{25}{\pi r^2}\right)$. Find the radius that will minimize the surface area.
18. A water tank is in the shape of an inverted cone. Water is being drained from the tank at a constant rate of $2m^3/min$. (since volume is decreasing, $\frac{dV}{dt}$ is negative).The height of the tank is $8m$, and the diameter of the top of the tank is $6m$. When the height of the water is $5m$, find, in units of cm/min, the following:
(a) the rate of change of the water level
(b) the rate of change of the radius of the surface of the water.
19. If the diagonal of a cube is increasing at a rate of $8cm/sec$, how fast is a side of the cube increasing?
20. A point P is moving along the circle with equation $x^2 + y^2 = 100$ at a constant rate of $3cm/sec$. How fast is the progression of P on the x-axis moving when P is $5cm$ above the x-axis?
21. A storage box with a square base must have a volume of 80 cubic centimeters. The top and bottom cost K2 per square centimeter and the sides cost K1 per square centimeter. Find the dimensions that will minimize the cost.
22. A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{25 - x^2}$. What length and width should the rectangle have so that its area is a maximum?
23. The sum of the perimeters of an equilateral triangle and a square is 10. Find the dimensions of the triangle and square that produce a minimum total area.