THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATITICS

MAT 1100 FOUNDATION MATHEMATICS TUTORIAL SHEET 5 -2021

1. Solve the following inequalities for real values of x, expressing your solution set in interval notation: (a) $-10x \leq 40$ (b) $2x + 7 < 3 + 4x$ (c) $2x - 5 \leq 2x - 1$
   (d) $3(x + 2) + 7 < 2x - 5$ (e) $-11 \leq 1 - 3(x - 2) < 13$ (f) $\frac{3}{4} > x + 1 > \frac{1}{4}$
2. Solve the following inequalities for real values of x, expressing your solution set in interval notation: (a) $x^2 \leq 9$ (b) $(x + 2)^2 < 25$ (c) $x^2 + 4x + 4 \geq 9$
   (d) $x^2 + 2x > 3$ (e) $x^3 - 2x^2 - x + 2 \geq 0$ (f) $4x^3 - 12x^2 > 0$ (g) $x^2 < -1$
3. Find the domain of each of the following functions $f(x) =$:
   (a) $\sqrt{x^2 - 9}$ (b) $\sqrt{x^2 + 4}$ (c) $\sqrt[4]{-x^2 + 2x - 2}$ (d) $\sqrt{x^2 - 3x + 3}$ (e) $\sqrt{x^2 - 7x + 10}$
4. Solve the following inequalities for real values of x, expressing your solution set in interval notation: (a) $\frac{1}{x} > x$ (b) $\frac{1}{x} < 4$ (c) $\frac{x+12}{x+2} \geq 3$ (d) $\frac{x+6}{x+1} < 2$ (e) $\frac{4}{x+5} > \frac{1}{2x+3}$
   (f) $\frac{1}{x-3} - \frac{9}{4x+3} \leq 2$ (g) $|x + 3| < 5$ (h) $|5x| > 10$ (i) $\left|\frac{x-2}{x+4}\right| > 3$ (j) $\left|\frac{2x-1}{1-3x}\right| \geq 0$
   (k) $|x| - |2x - 1| - |1 - 4x| < 10$ (l) $\left|\frac{10x-1}{4x-6}\right| > -1$ (m) $\left|\frac{x+4}{2x+1}\right| + 10 < 0$
   (n) $|x^2 + 9x - 7| \leq 3x$ (o) $\left|\frac{x}{x+4}\right| < 2$ (p) $\sqrt{3x + 1} > \sqrt{x - 3}$ (q) $\sqrt{x + 1} + \sqrt{x + 4} > 4$
   (r) $|x + 6| < \sqrt{2x}$ (s) $\sqrt{15x - 1} < -15$.
5. Find the set of values of k for which the equation $(k + 2)x^2 + 2kx + k + 1 = 0$ has no real roots.
6. Show that there is no real value t for which the equation $2x^2 + (2 - t)x + t^2 + 3 = 0$ has real roots.
7. Find the set of values of k for which $3x^2 - 12x + k > 0$ for all real values of x.
8. In the vicinity of a bonfire, the temperature T in $°C$ at a distance of x metres from the centre of the fire was given by $T = \frac{600,000}{x^2+300}$. At what range of distances from the fire centre was the temperature less than $500°C$?

9. The gas mileage g (measured in km/gallon) for a particular vehicle, driven at v km/h, is given by the equation $g = 10 + 0.9v - 0.01v^2$, as long as v is between 10km/h and 75km/h. For what range of speeds is the vehicle's mileage 30km/gallon or better?

10. The revenue R and cost C for a product are given by $R = x(75 - 0.0005x)$ and $C = 25x + 100,000$ where R and C are measured in Kwacha and x represents the number of units sol

    (a) How many units must be sold to obtain a profit of at least K500, 000?

    (b) In the revenue equation, the demand equation is given by $p = 75 - 0.0005x$ where p is the price per unit. What prices per unit can the company set to obtain a profit of at least K1, 000, 000?

11. The specifications for an electronic device state that it is to be operated in a room with relative humidity h defined by $|h - 50| < 30$. What are the minimum and maximum relative humidities for the operation of this device?

12. Three students are planning to share the expense of renting a room for a week. By adding a fourth person to the group, each person could save K75 in rental fees. How much is the rent for a week?

13. A rectangular field with perimeter of 80m is to have an area of at least $380m^2$. Describe the possible lengths l and b of the field.

14. Use Pascal's triangle to expand the following: (a) $(x + y)^6$ (b) $(x + \frac{1}{x})^4$ (c) $(\frac{1}{x} - \sqrt{x})^5$ (d) $(xy^2 - 1)^5$ (e) $(\frac{3}{x} - 2\sqrt{x})^4$ (f) $(2n + \frac{1}{n^2})^6$ (g) $(\sqrt{x} - i)^6$ (h) $(\sqrt{x} + \sqrt{y})^6$

15. Evaluate the following expressions: (a) $\binom{6}{4}$ (b) $\binom{101}{99}$ (c) $\binom{7}{3} + \binom{8}{4}$ (d) $\binom{5}{2}\binom{5}{3}$ (e) $\binom{33}{0}$ (f) $\frac{5!}{3!} + \frac{7!}{5!}$ (g) $\binom{5}{0} - \binom{5}{1} + \binom{5}{2} - \binom{5}{3} + \binom{5}{4} - \binom{5}{5}$ (h) $\frac{5!4!}{6!} + \frac{8!3!}{7!}$ (i) $\frac{7!}{5!2!}$ (j) $\frac{5!}{3!0!}$

    (k) $\binom{6}{0}\left(\frac{1}{3}\right)^6 + \binom{6}{1}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right) + \binom{6}{2}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + \binom{6}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3 + - - - + \binom{6}{6}\left(\frac{2}{3}\right)^6$.

16. Given the integers r, n such that $0 \leq r \leq n$, prove that $\binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}$ and $\binom{n}{r} = \binom{n}{n-r}$. Hence express the following in the form $\binom{n}{r}$, as a single binomial coefficient: (a) $\binom{8}{3} + \binom{8}{6} + \binom{9}{4}$ (b) $\binom{10}{4} + 2\binom{10}{5} + \binom{10}{6}$ (c) $\binom{11}{7} - \binom{11}{4} + \binom{12}{7}$.

17. Verify that $n_{C_r} = \frac{n(n-1)(n-2)\ldots\ldots\ldots\ldots(n-r+1)}{r!}$.

18. Show that $\binom{n}{r-1} + 2\binom{n}{r} + \binom{n}{r+1} = \binom{n+2}{r+1}$.