

UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

MAT 1100 FOUNDATION MATHEMATICS TUTORIAL SHEET 2 -2021

- Rationalize the denominator of each of the following:
 (a) $\sqrt[7]{\frac{1}{a^2}}$ (b) $\frac{1}{\sqrt[3]{x^4}}$ (c) $\frac{3+\sqrt{8}}{2+\sqrt{18}}$ (d) $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$ (e) $\frac{h}{\sqrt{x+h}-\sqrt{x}}$ (f) $\frac{8}{\sqrt[3]{2}}$
- Rationalize the numerator of each of the following:
 (a) $\frac{\sqrt{a-3}}{a-9}$ (b) $\frac{\sqrt{x}-\sqrt{y}}{x-y}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\sqrt{x+h}-\sqrt{x}$
- Express each of the following in the form $A + B\sqrt{C}$ where A, B are rational and C is a positive integer: (a) $\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{4} + \frac{1}{\sqrt{50}}}$ (b) $\sqrt{3} + 2 + \frac{1}{\sqrt{3-3}}$
- Express the following in the form $a + ib$ where $a, b \in \mathbb{R}$
 (a) $\sqrt{-225}$ (b) $\sqrt{-36} + \sqrt{-49}$ (c) $5 + \sqrt{-4}$ (d) $7 - \sqrt{-7}$ (e) $-\sqrt{\frac{-25}{16}}$
- Simplify the following : (a) i^{11} (b) i^{105} (c) $\frac{5}{i} + \frac{2}{i^3} - \frac{20}{i^{18}}$ (d) $(4i)^3 - 4i^3$
 (e) $(1+i)^{10} - (1-i)^{10}$
- Find the real and imaginary parts of each of the following: (a) -3 (b) $-4i$ (c) $\frac{1-i}{1+i}$
 (d) $(2+i)(4+3i)$ (e) $\frac{3-i}{2+i} + \frac{3+i}{2-i}$ (f) $(1+i)^2(1-i)^2$ (g) $3i + \frac{1}{2-i}$
- Simplify each of the following, giving your answer in the form $a + bi$, where $a, b \in \mathbb{R}$:
 (a) $(2-3i) - (1+2i)$ (b) $(3-i)^2$ (c) $\frac{(1+2i)^2}{2+i}$.
- Given that $z_1 = 8 + 2i$, $z_2 = 2 + i$, $z_3 = 3 + i$, find the answer to each of the following in the form $a + bi$, where $a, b \in \mathbb{R}$:
 (a) $\frac{z_1 z_2}{z_3}$ (b) $\frac{z_1 - z_2}{z_1 + z_3}$ (c) $\frac{(z_1 - 2z_3)^2}{(3z_2)^2}$.
- Find the real values of x and y such that $(x + iy)^2 = -8 + 6i$.
- If z is a complex number, solve the equation $z^2 + 2z + 5 = 0$.
- State whether each of the following operation is a binary operation on \mathbb{Z} , the set of integers:
 (a) $a * b = a - 2b$ (b) $a * b = \sqrt{a+b}$ (c) $a * b = (a - b)^2$ (d) $a * b = a^2 - b^2$
 What happens when we replace the set \mathbb{Z} above with \mathbb{R} ?
- Let $*$ be a binary operation on the set of real numbers and a, b, c be real numbers.

Given that the operation is defined by $a * b = (a - b)^2 - 3ab$.

(a) Does the operation possess the commutative and/or associative property?

(b) For this binary operation calculate (i) $-2 * (3 * 4)$ (ii) $(-4 * 3) * 2$.

12. The binary operation $*$ is defined on the set of real numbers by

$$a * b = 3(a - b)^2.$$

Show that the binary operation is commutative.

13. Which of the following sets of ordered pairs represent functions:

(a) $\{(1,4), (3,4), (7,3)\}$ (b) $\{(1,2), (1,3), (2,3)\}$ (c) $\{(4,3), (4, 7), (3,4)\}$

14. Let $A = \{0,1,2,3\}$ and $B = \{-2, -1, 0,1,2\}$. Which of the following sets of ordered pairs represent a function from A to B ?:

(a) $\{(0,1), (1, -2), (2, 0), (3,2)\}$ (b) $\{(0, -1), (2,2), (1, -2), (3, 0), (1,1)\}$

(c) $\{(0,0), (1, 0), (2, 0), (3, 0)\}$ (d) $\{(0, 2), (3, 0), (1,1)\}$. You may represent the ordered pairs in form of arrow diagrams to see the picture.

15. Given that $f(t) = \sqrt{25 - t^2}$, find and simplify each of the following:

(a) $f(3)$ (b) $f(5)$ (c) $f(x + 5)$ (d) $f(2x)$.

16. Given that $f(x) = \frac{1}{x^2 - 2x}$, find and simplify each of the following:

(a) $f(1)$ (b) $f(-3)$ (c) $f(t+1)$ (d) $f(t^2)$.

17. Find the domain and range of each of the following functions:

(a) $f: \{(0,1), (3,4), (5,7), (8, 11), (9,12)\}$

(b) $g: \{(10, -4), (20, 1), (30, 6), (40,9), (50, 13)\}$

18. Find the domain of each of the following functions: (a) $f(x) = \sqrt[3]{x - 10}$

(b) $f(x) = \sqrt[4]{1 - x}$ (c) $f(x) = \frac{\sqrt{x+1}}{x-2}$ (d) $f(x) = \frac{x-4}{\sqrt{x}}$ (e) $f(x) = \frac{1}{x} - \frac{3}{x+2}$

(f) $f(x) = |x + 1|$ (g) $f(x) = \frac{x-3}{1-\sqrt{x-3}}$ (h) $f(x) = 5x^2 + 2x - 1$

19. Given that $f(x) = \begin{cases} 3x - 1; & x < 0 \\ 2x + 3; & x \geq 0 \end{cases}$, find (a) $f(-1)$ (b) $f(0)$ (c) $f(-2)$ (d) $f(2)$.

20. An open box is to be made from a square piece of material 18cm on a side by cutting equal squares from the corners and turning up the sides.

(a) Write down the volume V of the box as a function of its height x .

(b) What is the domain of the function?

(c) Determine the volume of a box with a height of 4cm.

