The University of Zambia
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MAT1100 – Foundation Mathematics Tutorial sheet 3 (2021)

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1. (a) Find the domain of each of the following functions:

(i) $f(x) = \frac{x}{3x-1}$ (ii) $f(x) = \frac{5x+4}{x^2+3x+2}$ (iii) $f(x) = \sqrt{x} + \sqrt{4-x}$

(iv) $f(x) = \sqrt{x} - \sqrt[3]{1-x}$ (v) $f(x) = \frac{1}{\sqrt[4]{1-3x}}$ (vi) $f(x) = \frac{3x+|x|}{x}$

(b) Sketch the graph of each of the following, and evaluate the indicated function values: (i) $f(x) = \frac{3x+|x|}{x}, f(-6), f(3)$. (ii) $f(x) = \begin{cases} x, \ if\ x \le 0 \\ x+1, if\ x > 0 \end{cases}, f(-3), f(0)$.

(iii) $f(x) = \begin{cases} -1, if\ x \le -1 \\ 3x+2, if\ -1 < x < 1, \\ 7-2x, if\ x \ge 1 \end{cases}$ f(-2), f(0), f(10).

2. (a) Given that $f(x) = 2x + 3$ and $g(x) = 2 - x^2$, evaluate each of the following:

(i) $(fog)(0)$ (ii) $(gof)(0)$ (iii) (fof)(4) (iv) gog(-3) (v) f(g(-1)).

(b) For the functions f and g given, find fog, gof and their domains:

(i) $f(x) = 4x - 1, g(x) = 2 + 3x$ (ii) $f(x) = \sqrt{1+x}, g(x) = 1 + x^2$

(iii) $f(x) = \frac{2}{x+4}, g(x) = x - 1$ (iv) $f(x) = \frac{2}{x+3} - 3, g(x) = \frac{2}{x+3} - 3$.

(v) $f(x) = \frac{2x}{4-x}, g(x) = \frac{1}{x^2}$ (vi) $f(x) = \frac{3}{x^2-1}, g(x) = x + 1$.

(vii) $f(x) = x - 4, g(x) = |x|$ (viii) $f(x) = \sqrt{x}, g(x) = \sqrt{x}$.

3. (a) Determine whether f is one-one, even, odd, or neither even nor odd:

(i) $f(x) = \frac{5-2x}{3}$ (ii) $f(x) = 10$ (iii) $f(x) = (x-1)^2$ (iv) $f(x) = |x-5|$.

(b) For the pairs of functions *f* and *g* given, find , $g^{-1}, fog, (fog)^{-1}, g^{-1}of^{-1}$, and determine their domains: (i) $f(x) = \frac{2}{x-2}$, and $g(x) = \frac{1}{x+1}$,

(ii) $f(x) = \frac{x+7}{4}, g(x) = \sqrt{x}$ (iii) $f(x) = 2x^3 - 1, g(x) = \sqrt[3]{\frac{x+1}{2}}$

(iv) $f(x) = (x+3)^2, x \ge -3; g(x) = 3x + 5$ (v) $f(x) = x^5 - 2, g(x) = 7x + 1$.

4. Show that there is no real value t for which the equation $2x^2 + (2-t)x + t^2 + 3 = 0$ has real roots.

5. Find the sum and product of the roots for each of the following quadratic equations:

(a) $2x^2 + 6x - 5 = 0$ (b) $x^2 = 1 - 3x$ (c) $4x^2$-6=0 (d) $3x - \frac{2}{x} = 1$

6. If α and β are roots of the equation $3x^2 + 5x + 4 = 0$, find the values of the following expressions: (a) $\alpha^2 + \beta^2$ (b) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (c) $(\alpha - \beta)^2$ (d) $\frac{1}{\alpha} + \frac{1}{\beta}$

7. If α and β are roots of the quadratic equation $5x^2 - 3x - 1 = 0$, find a quadratic equation with integer coefficients which has the roots:

(a) $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ (b) $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. (c) α^2 and β^2.

8. Consider the quadratic equation $x^2 + 8x + k = 0$ where k is a constant.

(a) Find both roots of the equation given that one root is three times the other. And

(b) find the value of k.

9. Given the quadratic equation $x^2 + 6x + 9 = 4kx$, find the set of values of k for which it has real roots.

10. Complete the square of the given quadratic function. Hence, sketch its graph, showing clearly the x – and y – intercepts and the turning point. State

(i) the line of symmetry, and

(ii) the maximum or minimum value of the function

(a) $f(x) = 3 - 7x - 3x^2$ (b) $f(x) = 2x^2 - x - 1$ (c) $f(x) = -3x^2 + 2$

(d) $f(x) = x^2 - 2x - 1$ (e) $f(x) = -x^2 + 2x + 1$.

11. A child care center has 200m of fencing wire to enclose two adjacent rectangular safe play areas.

(a) Write an expression for the total area A of the play areas in terms of x.

(b) Find the dimensions for which the enclosed area is 1600 square meters.

12. An open box is made from a square piece of material by cutting two-cm squares from the corners and turning up the sides. The volume of the finished box is $200cm^3$. Find the size (area) of the original piece of material.

13. Find the maximum area of a rectangle inscribed in an isosceles right triangle whose hypotenuse is $20cm$ long. [HINT: If hypotenuse is $20cm$, and x is the base of the rectangle, show that the height of the rectangle is $y = 10 - \frac{x}{2}$.]

14. One number is 2 more than another number. The product of the numbers is 440. Find the numbers.

15. One number is 1 more than another number. The sum of their squares is 113. Find the numbers.