

**THE UNIVERSITY OF ZAMBIA**  
**SCHOOL OF NATURAL SCIENCES**  
**DEPARTMENT OF MATHEMATICS AND STATISTICS**

**MAT 1100 FOUNDATION MATHEMATICS TUTORIAL SHEET 4 -2021**

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1. If  $\alpha, \beta, \gamma$  are roots of the cubic equation  $ax^3 + bx^2 + cx + d = 0$ , show that  $\alpha + \beta + \gamma = \frac{-b}{a}$  and  $\alpha\beta\gamma = \frac{-d}{a}$ .
2. Two roots of the equation  $x^3 - 3x^2 + kx + 75 = 0$  are opposites. Find the values of all the roots and the constant  $k$ .
3. One of the zeros of the equation  $x^3 - 63x + 162 = 0$  is double another zero. Find all the three zeros.
4. Use long division to divide in each case, given the dividend and divisor, respectively:
  - (i)  $x^3 + 8x^2 - 5x - 1; x - 2$     (ii)  $2x^3 + 6x^2 - x + 5; 2x^2 - 1$
  - (iii)  $x^4 - 4x^2 + 3; 3 + 2x - x^2$     (iv)  $x^5 + x^4 - 8x^3 + x + 2; x^2 + x - 7$
  - (v)  $-x^4; -2x^2 + x - 3$     (vi)  $9x^2 - x + 5; 3x^2 - 7x$
5. Use synthetic division to divide the polynomials and write the function in the form  $p(x) = (x - k)q(x) + r$ , where  $q(x)$  is the quotient and  $r$  is the remainder, given the dividend and divisor as follows:
  - (i)  $x^3 - 10x^2 + 31x - 30; x + 3$     (ii)  $x^3 + 15x^2 + 68x + 96; x - 2$
  - (iii)  $6x^3 + x^2 - 21x - 10; 2x - 1$     (iv)  $5x^4 + 30x^3 - 40x^2 + 36x + 14; x + 7$
6. Write the function in the form  $p(x) = (x - k)q(x) + r$ , where  $q(x)$  is the quotient and  $r$  is the remainder:
  - (i)  $p(x) = x^3 + x^2 - 12x + 20, k = 2$
  - (ii)  $p(x) = x^3 - 2x^2 - 15x + 7, k = -4$     (iii)  $p(x) = x^3 + 2x^2 - 3x - 12, k = \sqrt{3}$
  - (iv)  $p(x) = 3x^3 - 19x^2 + 27x - 7, k = 3 - \sqrt{2}$ .
7. Factorize the polynomial completely:
  - (i)  $p(x) = x^3 - 12x - 16$     (ii)  $p(x) = 3x^3 + 10x^2 - 27x - 10$
  - (iii)  $p(x) = x^3 + 2x^2 - 3x - 6$     (iv)  $p(x) = x^3 + 2x^2 - 2x - 4$ .
8. Given that  $(x - 1)$  and  $(x + 1)$  are factors of  $px^3 + qx^2 - 3x - 7$ , find the value of  $p$  and  $q$ .
9. The expression  $2x^3 - ax^2 + bx + 3$  gives a remainder  $-15$  when divided by

- $(x + 1)$  and a remainder  $-46$  when divided by  $(x - 3)$ . Find the value of  $a$  and of  $b$ .
10. Find the zeros of each of the following polynomial functions. Hence sketch its graph indicating the  $x$  - and  $y$  - intercepts:
- (i)  $p(x) = x^3 - 2x - 7x + 12$       (ii)  $p(x) = -x^3 + x^2 + 5x - 2$   
 (iii)  $p(x) = 15 + 5x - 3x^2 - x^3$       (iv)  $p(x) = x^3 + 5x^2 + 6x + 2$
11. Show that  $p(x) = 2x^4 - x^3 + x + 2$  has no rational zeros.
12. (a) Show that  $(x - 2)$  is a factor of  $p(x) = x^3 + x^2 - 5x - 2$ .  
 (b) Hence, or otherwise, find the exact solutions of the equation  $p(x) = 0$ .
13. [Conjugate Zeros]. If a polynomial  $P$  has real coefficients, and if  $z = a + ib$  is a zero of  $P$ , then its conjugate  $\bar{z} = a - ib$  is also a zero of  $P$ .
- (a) Given that  $x = 2 - 3i$  is a zero of  $P(x) = x^3 - 7x^2 + 25x - 39$ , find the other remaining zeros.  
 (b) Find a polynomial of lowest degree with real coefficients and the given zeros:  
 (i)  $x = -1$  and  $x = 1 - i$  (ii)  $x = 2, x = -4$  and  $x = -3i$ .
14. Sketch the graph of the rational function, indicating all the intercepts and any asymptotes: (a)  $f(x) = \frac{1}{x+2}$  (b)  $f(x) = \frac{1-4x}{1-4}$  (c)  $f(x) = \frac{2}{x^2+2x-3}$  (d)  $f(x) = \frac{x}{x^2-9}$
15. Sketch the graphs of the following functions, and determine the domain and range in each case: (a)  $f(x) = |x + 1|$  (b)  $f(x) = -|x - 2| + 3$  (c)  $f(x) = -\sqrt{x - 2}$   
 (d)  $f(x) = -|x^2 + 2x - 3|$  (e)  $f(x) = \sqrt{2x - 3} - 3$  (f)  $f(x) = -\sqrt{-x} - 1$ .
16. Solve for real values of  $x$ , expressing your answers as exact solutions. Check your solutions and recognize any extraneous solutions:  
 (a)  $\sqrt{x + 6} + 2x = 9$  (b)  $\sqrt{x + 7} + 5 = x$  (c)  $\sqrt{7x + 14} - 2 = x$  (d)  $\sqrt{x} - \frac{6}{\sqrt{x}} = 1$   
 (e)  $\sqrt{2x + 3} - \sqrt{x - 2} = 2$  (f)  $\sqrt{4 - x} - \sqrt{6 + x} = \sqrt{14 + 2x}$  (g)  $-\sqrt{3x - 7} = 14$   
 (h)  $\frac{1}{x} - \frac{1}{x+1} = \frac{1}{x+4}$  (i)  $\frac{x+1}{2x+3} = \frac{5x-1}{7x+3}$  (j)  $x^4 - 2x^2 - 15 = 0$  (k)  $2x^{\frac{2}{3}} - x^{\frac{1}{3}} - 15 = 0$   
 (l)  $|3x + 4| = 8$  (m)  $|x + 6| = |3x - 24|$  (n)  $|5x + 1| = 2x$  (o)  $\left| \frac{x+1}{x-1} \right| = 3$   
 (p)  $|x - 1| + |x| = 3$  (q)  $|4x| + |2x - 7| = 0$  (r)  $|\sqrt{2x - 5} - 2| = 3$ .
17. Solve each given system of equations:  
 (a)  $x + y = 16; y = x + 2$       (b)  $\frac{2}{x} + \frac{3}{y} = \frac{19}{15}; \frac{-2}{x} + \frac{1}{y} = \frac{-7}{15}$   
 (c)  $\frac{x+3}{4} + \frac{y-1}{3} = 1; x - y = 3$  (d)  $y = x^3 - 2x^2; 10x - y = -7$ .
18. Sketch the graph of  $f(x) = x^5 - x$  indicating  $x$ - and  $y$ -intercepts.

