THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATITICS

MAT 1100 FOUNDATION MATHEMATICS TUTORIAL SHEET 8-2021

1. Find the radian measure of the angle with the given degree measure:

   (a) $40°$ (b) $330°$ (c) $-80°$ (d) $-150°$ (e) $765°$ (f) $1°$ (g) $300°$ (h) $340°$.

2. Find the degree measure of the angle with the given radian measure:

   (a) $\frac{3\pi}{4}$ (b) $-\frac{7\pi}{2}$ (c) $\frac{5\pi}{6}$ (d) $\frac{2\pi}{9}$ (e) 2rad (f) $-\frac{\pi}{12}$ (g) $-\frac{4\pi}{3}$ (h) $\frac{\pi}{5}$.

3. Find an angle in the interval $0° \leq \theta \leq 360°$ that is coterminal with the given angle:

   (a) $370°$ (b) $2223°$ (c) $-100°$ (d) $-800°$ (e) $1270°$ (f) $430°$ (g) $-280°$.

4. Find an angle in the interval $0 \leq \theta \leq 2\pi$ that is coterminal with the given angle:

   (a) $\frac{11\pi}{5}$ (b) $-\frac{\pi}{4}$ (c) $-\frac{7\pi}{3}$ (d) 89π (e) $\frac{17\pi}{4}$ (f) 10rad (g) $\frac{51\pi}{2}$ (h) $\frac{11\pi}{3}$ (i) $-\frac{2\pi}{3}$.

5. Find the reference angle for the given angle:

   (a) $225°$ (b) $-35°$ (c) $181°$ (d) $290°$ (e) $570°$ (f) $-95°$ (g) $40°$ (h) $335°$ (i) $165°$

   (j) $\frac{7\pi}{6}$ (k) $-\frac{2\pi}{3}$ (l) $\frac{17\pi}{3}$ (m) $\frac{23\pi}{11}$ (n) $-\frac{\pi}{4}$ (o) $-\frac{5\pi}{3}$ (p) 3rad (q) $\frac{5\pi}{2}$ (r) $\frac{17\pi}{7}$.

6. Find the exact values of the given trigonometric ratios: (a) $\tan(-\frac{\pi}{6})$ (b) $\sin\left(\frac{7\pi}{6}\right)$

   (c) $\cos\left(\frac{7\pi}{3}\right)$ (d) $\sec\left(-\frac{13\pi}{6}\right)$ (e) $\text{cosec}\left(\frac{\pi}{2}\right)$ (f) $\cot\left(\frac{7\pi}{4}\right)$ (g) $\sin\left(\frac{11\pi}{6}\right)$ (h) $\tan\left(\frac{5\pi}{2}\right)$.

7. Find the trigonometric ratios of the other five trigonometric functions given that:

   (a) $\cos\theta = \frac{8}{17}$ and $\sin\theta < 0$ (b) $\tan\theta = -\frac{6}{5}$ and $\sin\theta > 0$ (c) $\sec\theta = 2$, $\frac{3\pi}{2} < \theta < 2\pi$.

   (d) $\tan\theta = -\frac{3}{4}$ and $\cos\theta > 0$ (e) $\text{cosec}\theta = 2$ and θ in quadrant I. (f) $\cot\theta = \frac{1}{4}$, $\sin\theta < 0$.

8. Verify the following identities: (a) $\sin\theta\cot\theta = \cos\theta$ (b) $\frac{cos\theta sec\theta}{tan\theta} = cot\theta$

   (c) $\frac{cosx}{secxsinx} = cosecx - sinx$ (d) $(sinx + cosx)^2 = 1 + 2sinxcosx$

   (e) $\frac{cosx}{secx} + \frac{sinx}{cscx} = 1$ (f) $\frac{(sinx+cosx)^2}{sin^2x-cos^2x} = \frac{sin^2x-cos^2x}{(sinx-cosx)^2}$ (g) $\frac{1}{1-sin^2y} = 1 + tan^2y$

   (h) $sin^4x - cos^4x = sin^2x - cos^2x$ (i) $\frac{sinx}{1+cosx} = \frac{1-cosx}{sinx}$ (j) $\frac{sinx-1}{sinx+1} = \frac{-cos^2x}{(sinx+1)^2}$

   (k) $\frac{1+tanx}{1-tanx} = \frac{cosx+sinx}{cosx-sinx}$ (l) $tan^2x - cot^2x = sec^2x - csc^2x$.

9. Use the addition or subtraction formula to find the exact value of the given expression:

(a) $\sin\frac{11\pi}{12}$ (b) $\sin 15°$ (c) $\cos 165°$ (d) $\tan 105°$ (e) $\sin 18°\cos 27° + \cos 18°\sin 27°$

(f) $\cos\frac{\pi}{12}$ (g) $\frac{tan73°-tan13°}{1+tan73°tan13°}$ (h) $cos\frac{13\pi}{15}cos\left(-\frac{\pi}{5}\right) - sin\frac{13\pi}{15}sin\left(-\frac{\pi}{5}\right)$.

10. Prove the following identities: (a) $tan\left(\frac{\pi}{2}-x\right) = cotx$ (b) $sin\left(x-\frac{\pi}{2}\right) = -cosx$

(c) $cos\left(x-\frac{\pi}{2}\right) = sinx$ (d) $tan\left(x-\frac{\pi}{2}\right) = \frac{tanx-1}{tanx+1}$ (e) $tanx - tany = \frac{\sin(x-y)}{cosxcosy}$

(f) $cot2x = \frac{1-tan^2x}{2tanx}$ (g) $\frac{sin4x}{sinx} = 4cosxcos2x$ (h) $4(sin^6x + cos^6x) = 4 - 3sin^2 2x$

(i) $sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-cosx}{2}}$ (j) $cos2x = \frac{1-tan^2x}{1+tan^2x}$ (k) $cotx - tanx = 2cot2x$

11. Show that (a) $sin45° + sin15° = sin75°$ (b) $cos87° + cos33° = sin63°$.

12. Express $sin3x$ in terms of $sinx$ only.

13. Find all the solutions of the following equations: (a) $2cosx - 1 = 0$

(b) $2sinx - \sqrt{3} = 0$ (c) $4cos^2x - 1 = 0$ (d) $secx\left(2cosx - \sqrt{2}\right)$=0 (e) $csc3x = sin3x$

(f) $tan^5x - 9tanx = 0$ (g) $cos\frac{x}{2} - 1 = 0$ (h) $4sinxcosx + 2sinx - 2cosx - 1 = 0$.

14. Find all the solutions of the equation in the interval $[0, 2\pi)$:

(a) $2cos3x = 1$ (b) $3csc^2x = 4$ (c) $tanx - 3cotx = 0$ (d) $cos2x = sin^2x$

(e) $2sin^2x - cosx = 1$ (f) $cosx + 1 = sinx$ (g) $tan\frac{x}{2} - sinx = 0$

(h) $sinx + sin3x = 0$ (i) $sin5x - sin3x = cos4x$ (j) $sin3xcosx - cos3xsinx = 0$.

15. Sketch the graphs of the following function in one period: (a) $f(x) = 2 + sinx$

(b) $f(x) = -1 + cosx$ (c) $f(x) = |sinx|$ (d) $f(x) = 4 - 2sinx$ (e) $f(x) = -sinx$.

16. Find the amplitude, period and phase shift of the given function, and sketch the graph in the interval $0 \le x \le 2\pi$: (a) $y = 10sin\frac{1}{2}x$ (b) $y = -cos\frac{1}{3}x$ (c) $y = sin(-2x)$

(d) $y = cos\left(x-\frac{\pi}{2}\right)$ (e) $y = 3cos\left(x+\frac{\pi}{4}\right)$ (f) $y = 1 + cos\left(3x+\frac{\pi}{2}\right)$ (g) $y = \sin(2x+\pi)$.

17. Find the exact value of each expression, if it is defined: (a) $sin^{-1}\frac{1}{2}$ (b) $cos^{-1}\frac{\sqrt{3}}{2}$

(c) $cos^{-1}2$ (d) $sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ (e) $tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ (f) $tan^{-1}\left(tan\frac{2\pi}{3}\right)$ (g) $cos\left(sin^{-1}\frac{\sqrt{3}}{2}\right)$

(h) $tan\left(sin^{-1}\frac{\sqrt{2}}{2}\right)$ (i) $tan^{-1}\left(2sin\frac{\pi}{3}\right)$ (j) $cos^{-1}\left(\sqrt{3}sin\frac{\pi}{6}\right)$ (k) $sin\left(2cos^{-1}\frac{3}{5}\right)$

(l) $sin\left(sin^{-1}\frac{1}{2} + cos^{-1}\frac{1}{2}\right)$ (m) $cos\left(sin^{-1}\frac{3}{5} - cos^{-1}\frac{3}{5}\right)$ (n) $cos(tan^{-1}2)$.