THE UNIVERSITY OF ZAMBIA
DEPARTMENT OF MATHEMATICS

M110 DEFFERED EXAMINATION 1995

INSTRUCTIONS: ANSWER ANY FIVE QUESTIONS
NO CALCULATORS ALLOWED
TIME ALLOWED: THREE (3) HOURS

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1 a. Find the value of m for which the equation $x^2 + (m + 3)x + 4m = 0$ has equal roots. For what value of m is the sum of roots zero. Suppose α, β are the roots of the given equation when $m = 1$, find the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

b. Use mathematical iduction to prove that for all integers $n \geq 1$

$$1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$

c. For what value of A does $x^3 - 4x^2 + Ax + 3$ yield the same remainder when divided by $x - 1$ or $x + 1$.

2 a) Show that if λ is positive but not greater than 3, the roots of the equation $(\lambda - 2)x^2 - (8 - 2\lambda)x + (6 - 3\lambda) = 0$ are real. Find the range of λ for which one root is real and positive and the other root is real and negative.

b) Find the first three terms and general term in the expansion in ascending powers of x of

$$\frac{(x + 5)}{(1 + 3x)(2 - x)}$$

3. a) Find the limits of the following

i) $\lim_{t \to \infty} \frac{6t + 5}{3t - 8}$ ii) $\lim_{x \to \pi/2} \frac{1 - \sin x}{1 + \cos 2x}$

b) Find the derivative of $y = x^{1/4}$ from the first principles. Hence find the equation of the tangent to $y = x^{1/4}$ at (0,0).

c) Find the derivative of $y = x\tan^{-1}x = \frac{1}{2}\ln(1 + x^2)$ with respect to x. Hence write down $\int \tan^{-1}x\,dx$.

4. a) Show that the function $y = \tan x - 8\sin x$ has two stationary values between $x = 0$ and $x = 2\pi$. Draw a rough graph of the function between these values of X.

b) By substituting $t = \tan x$, show that

$$\int_0^{\pi/4} \frac{dx}{1 + \sin 2x} = \frac{1}{2}$$

5. a) Sketch the graph of the function $f(x) = x(x - 3)^2$ showing its furning points and points of inflexion if it has.

b) Find the fourth roots of $z = 1 - \sqrt{3}\,i$

c) Find the length of the tangent from the point (5, -1) to the circle $(x - 1/2)^2 + y^2 = \frac{25}{4}$

6. a) Given the sets A, B, prove that $(A - B) \cup (B-A) = (A \cup B) - (A \cap B)$

b) Prove that $3x - 2$ is a factor of $3x^3 - 2x^2 + 3x - 2$. Hence find the solution set of the equation $3x^3 - 2x^2 + 3x - 2 = 0$ when x belongs to the set of

i) Integers Z
ii) rational numbers Q
iii) real numbers R
iv) complex number C

c Find the domain and range of the function $\frac{x + 1}{2x - 1}$. Determine if this function is one to one or many to one. Also determine if it is invertible. If yes find the domain and range of the inverse function.

7. a) Prove that $\frac{\sin 3\theta}{1 + 2\cos 2\theta} = \sin\theta$ and hence show that $\sin \pi/12 = (\sqrt{3} - 1)/(2\sqrt{2})$

b) If $Z = \cos\theta + i\sin\theta$ Show that $z + \frac{1}{z} = 2\cos\theta$ and $z - \frac{1}{z} = 2i\sin\theta$.

Prove that
$\sin^7\theta = \frac{1}{64}(35\sin\theta - 21\sin 3\theta + 7\sin 5\theta - \sin 7\theta)$. Hence find

$\int(35\sin\theta - 64\sin^7\theta)\, d\theta$

c) Find the equation of the lines with equation $2x^2 + 5xy - 12y^2 = 0$. Also find the angle between the line.

8. Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

and hence solve the system of equations

$$x + y + z = 7$$
$$X - y + 2z = 9$$
$$2x + y - z = 1$$

b) Prove that

$$\begin{vmatrix} 2\cos\theta & 1 & 0 \\ 1 & 2\cos\theta & 1 \\ 0 & 1 & 2\cos\theta \end{vmatrix} = \frac{\sin 4\theta}{\sin\theta}$$

c) If $a = 2i - 4j + 2k$ and $b = 3i + 4j - 5k$ are any two vectors, find a unit vector in the direction of vector $\frac{1}{2}a - b$

END OF EXAM

THE UNIVERSITY OF ZAMBIA
SESSIONAL EXAMINATIONS - OCT/NOV 1994/95
MATHEMATICS M110

TIME ALLOWED: THREE(3) HOURS

INSTRUCTIONS: Answer any FIVE(5) questions.
Calculators, tables are not permitted.

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1. a) Let E = $\mathbb{R}$, the set of real numbers such that

$A = \{x | x^2 \geq 1\}$,

$B = \{x | \ |x| \leq 2\}$ and

$C = \{-3, -2, -1, 0, 1, 2, 3\}$.

Find: (i) $A \cap B$ (ii) $A \cap C$ (iii) $B \cap C$.

b) (i) If $f(x)$ is a polynomial in x, show that when $f(x)$ is divided by $x - a$, the remainder is $f(a)$.

(ii) The polynomial $f(x) = x^3 + px + q$ is exactly divisible by $(x + 2)$ and $(x - 3)$, find the values of p and q. With these values of p and q, find the roots of $f(x) = 0$.

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2. a) The coefficient of x^3 in the expansion of $(1 + x)^n$ is four times the coefficient of x^2. Find the value of n. With this value of n, find the seventh term, leaving your answer in factorial form.

b) (i) Explain what is meant by the statement:

α) f is a function from set X to set Y.

β) f is an even function.

(ii) Let the function $f(x) = \dfrac{ax + b}{x + c}$

where a, b, c are real and $x \neq -c$. Show that if $f(x)$ is an even function then $ac = b$. Deduce that if $f(x)$ is an even function then $f(x)$ must reduce to the form $f(x) = k$, where k is a constant.

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3. a) (i) For the quadratic function $f(x) = ax^2 + bx + c$, where a, b, c are real and $a \neq 0$, complete the square and sketch the graph of $f(x)$ when $a < 0$ and $b^2 - 4ac < 0$.

(ii) If α and β are roots of the equation $x^2 + 2x - 5 = 0$, find the value of $\alpha - \beta$.

b) Express the complex number

$$z_1 = \frac{11 + 2i}{3 - 4i}$$ in the form $x + iy$, where x and y are real.

Given that $z_2 = 2 - 5i$, determine the real numbers p, q such that $p z_1 + q z_2 = -4 + i$.

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4. a) Prove that

$$\frac{\sin 3A}{\sin A} + \frac{\cos 3A}{\cos A} = 4 \cos 2A.$$

b) (i) Let m be a positive integer. State De Moivres theorem for $(\cos \theta + i \sin \theta)^n$ when

α) $n = m$ β) $n = \frac{1}{m}$.

(ii) If $z = 1 + i\sqrt{3}$, express z in the form $r[\cos\theta + i\sin\theta]$. Hence, find $|z|$ and $|z^5|$.

Give also the values of $\arg z$ and $\arg(z^5)$ lying between $-\pi$ and π. Show that $\mathrm{Re}(z^5) = 16$ and find the value of $\mathrm{Im}(z^5)$.

c) The coordinates of the points P, Q and R are $(4, -3)$, $(-1, -2)$ and $(2, t)$ respectively. Calculate the value of t given that P, Q and R are collinear.

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5. a) The circle S has the equation

$$x^2 + y^2 - 6x - 8y = 0.$$

(i) Find the coordinates of the centre and radius of the circle.

(ii) Show that the point A with coordinates (7,2), lies inside the circle.

(iii) Show that the chord of S which is bisected at A has equation $y = 2x - 12$.

b) If the coordinates of three points A,B and C are (1,2), (0,4), (1,-5) respectively, find the equation of the straight line on which a fourth point P(x,y) lies such that

$$AP^2 + BP^2 = 2CP^2 - 19.$$

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6. a) A is the matrix such that

$$A = \begin{pmatrix} 1 & p & q \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where p,q are real numbers. Calculate A^2 and A^3. Suggest a formula for A^n. Prove by mathematical induction that your formula is correct.

b) Find a unit vector which is in the opposite direction to the sum of the vectors $(3i + 2j + k)$ and $(-5i - 3j + 6k)$. Prove that this unit vector is perpendicular to the vector $(9i - 4j + 2k)$.

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7. (a) Express $y = \dfrac{2x^2 + 3x + 5}{(x+1)(x^2+3)}$

in partial fractions. Hence show that $\dfrac{dy}{dx} = -\dfrac{2}{3}$ when $x = 0$

(b) For the curve $y = x^3 - 6x^2 + 9x$

(i) Solve the equation $y = 0$.

(ii) Find the coordinates of the turning points

(iii) Determine the nature of the turning points.

(iv) Sketch the graph.

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8. (a) z is a complex number such that $z = \cos x + i\sin x$ where $-\pi < x < \pi$.

Show that

$$z - \frac{1}{z} = 2i \sin x$$

$$z^n - \frac{1}{z^n} = 2i \sin nx.$$

Prove that

$$\sin^4 x = \frac{1}{8}(\cos 4x - 4\cos 2x + 3)$$

Hence evaluate $\int_0^{\pi/4} \sin^4 x \, dx$.

(b) Sketch the graph of the quadratic function $y = x(x - 1)$. Hence find the total area bounded by the curve $y = x(x-1)$, the x - axis and the line $x = 2$.

E N D O F E X A M I N A T I O N

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

UNIVERSITY EXAMINATIONS - SEMESTER I -2000/01

M111- MATHEMATICAL METHODS I

INSTRUCTIONS: (i) Attempt any **five(5)** questions only.
(ii) Show all necessary working for full marks.
(iii) Write down your **computer number** on all answer booklets used.
(iv) Indicate the number of each question attempted on your main answer booklet.

TIME ALLOWED: Three **(3)** hours

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1. (a) For any two set A and B, simplify

$$[A' \cap (A' \cap B)']'$$

(b) Given A={0,5} , B=[-2,2] and C=(-3,2) find

(i) $A \cap B$
(ii) $A' \cap B$.
(iii) $B \cup C$

(c) (i) Express $\dfrac{2i}{(1-i)^2}$

in the form a + ib where a and b are rational numbers.

(ii) Express $2.\overline{13}$ in the form p/q where p and q are integers and $q \neq 0$.

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2. (a) Let $f(x) = 3x^2 - (p-4)x - (2p+1)$

(i) If α and β are roots of f(x) = 0, find the equation whose roots are $\alpha+2$ and $\beta+2$.
(ii) Find the value(s) of p for which f(x) = 0 has equal roots.

(b) Complete the square of the quadratic function $f(x) = -2x^2 + x + 5$.
Hence
(i) find the turning point, the x- and y-intercepts.
Sketch the graph of y = f(x).

(ii) On the same diagram sketch the graph of y = |f(x)|.

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3. (a) Prove the following trigonometric identities

(i) $\operatorname{cosec}\theta + \tan\theta \sec\theta = \operatorname{cosec}\theta \sec^2\theta$

(ii) $\dfrac{1}{\operatorname{cosec}\theta + \cot\theta} = \dfrac{1-\cos\theta}{\sin\theta}$

(b) Find the general solution of

$$2\sin^2\theta + \cos\theta - 1 = 0.$$

(c) Given that

$$f(x) = \sin(2x + \pi) \text{ where } 0 \le x \le 2\pi.$$

State the
(i) amplitude
(ii) period and
(iii) phase shift of y = f(x).
Hence, sketch the graph of y = f(x).

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4. (a) (i) Given that

$$f(x) = \frac{ax + b}{x + c}$$

where a, b, c are real and $x \neq \pm c$ show that if f is **even** then $ac = b$.

(iii) Given that $g(x) = \sqrt{x}$, simplify

$$\frac{g(x+k) - g(x)}{k}$$

in such a way that there are no radicals in the numerator.

(b) For the real valued function h defined by

$$h(x) = \frac{2x}{5 - x}$$

(i) Find the range of h.
(ii) Determine whether the function is one-one.
(iii) Find (hoh)(x)

(c) Given that

$$f(x) = x^3 + kx^2 - 2x + 1$$

gives a remainder k when divided by x - k, find all possible values of k.

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5. (a) Solve for x in each of the following:

(i) $\log_{16} x + \log_{16}(10x+3) = \frac{1}{2}$

(ii) $x^{\log_x x} = \frac{x^3}{100}$.

(b) (i) Find the **integer** value of the product yz given that

$$y = \log_a(x^3)$$
$$z = \log_x a$$

(ii) simplify $3^{\log_9 x}$

(c) On the same axes, sketch the three graphs:

$f(x) = e^x$; $g(x) = e^{-x}$; $h(x) = \frac{1}{2}(e^{-x} + e^x)$.

Hence or otherwise

(i) state the range of h

(ii) find the value of x for which h(x) = 1.

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6. (a) Compute the limits

(i) $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$ (ii) $\lim_{x\to\infty} \frac{x}{2x+3}$

(b) Using **first principles**, find $\frac{dy}{dx}$ given $y = \frac{1}{\sqrt{x}}$.

(c) Find the derivative dy/dx of each of the following

(i) $y = \frac{1}{1+x}$ (ii) $y = (3x^2+1)^2$.

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7. (a) Resolve into partial fractions

$$\frac{5x}{(x+2)(x^2+1)}$$

(b) Solve each of the following inequalities

(i) $\frac{1}{x+1} > \frac{2}{3-x}$ (ii) $|x+2| < 1 - \frac{x}{2}$

(c) Solve for x given that

$$\sqrt{3+2x} + \sqrt{2-2x} = 3.$$

END OF EXAMINATIONS

THE UNIVERSITY OF ZAMBIA

SECOND SEMESTER EXAMINATIONS

MAY 2000

M112-MATHEMATICAL METHODS II

INSTRUCTIONS: -Attempt *any* five (5) questions

-Show all essential working clearly

-Write the number of questions attempted on the main answer book

TIME ALLOWED: Three (3) hours.

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1. a) State the binomial theorem for ***any real*** number indicating clearly the condition(s) under which the expansion is valid.

   b) Find ***the coefficient*** of the term independent of x in the expansion

$$\left(\sqrt{x}-\frac{2}{x}\right)^{36}$$

   c) Using the *binomial expansion*, computer the approximate value of $(15)^{1/4}$.

2. a) ***Without*** using a calculator, evaluate

   i) $\log_{0.5}(16)$ ii) $\log_{10}(0.001)$

   b) Obtain the ***value of x*** given that

   i) $27^{4x^2}=\left(\frac{1}{9}\right)^{x-1}$ ii) $\ln\left(\frac{1}{x}\right)+\ln(2x^3)=\ln 8$

   c) In a medical treatment the function

$$f(t)=500e^{-\frac{t}{5}}$$

   represents the amount of drug in mg remaining in a patient after t hours of being administered.

   i) How many mg of drug where present ***initially***?

   ii) After ***how many hours*** will 200mg remain?

   iii) Find the ***rate of decrease*** when the amount of drug remaining in the body is 200mg.

3. a) Let $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

   i) Find a vector of magnitude 5 perpendicular to ***both*** vectors **a** and **b**

   ii) The *cosine of the* ***angle*** between **a** and **b**

   iii) The ***area*** of a triangle with adjacent sides **a** and **b**

   b) Let A be a (3x3) matrix

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 3 & 0 & -1 \\ 2 & -3 & 5 \end{pmatrix}$$

   i) Compute the ***inverse*** A^{-1}

   ii) ***Hence*** solve the system of equations

$$x + 2y - 3z = 2$$
$$3x - z = -8$$
$$2x - 3y + 5z = -9$$

4. a) Let P(10,3) be a point on the curve

$$y^2 - x + 1 = 0$$

   i) Find equations of ***normal*** and ***tangent*** the curve at point P.

   ii) Let Q(-2,1), find the ***co-ordinate*** of a point R dividing QP *externally* in the ratio 2:3

   b) Consider the curve

$$4y^2 - 9x^2 + 16y + 18x - 29 = 0$$

   Find

   i) co-ordinates of the ***Centre***

   ii)co-ordinates of the ***foci***

   iii) co-ordinates of the ***vertices***

   iv) equation of ***asymptotes*** and ***sketch the curve*** using above facts.

5. For the curve

$$y = 2x^3 - 3x^2$$

a) Find i) *stationary* (or turning) points

   ii)*maximum/minimum* points

   iii) *inflexion* points

b) Sketch the curve.

c) Find the ***area*** under the curve between the points –1 and 2.

6. a) Find $\frac{dy}{dx}$ for each of the following functions

i) $y = x\cos y$ ii) $y = \frac{1}{\sqrt{\cos(x^3)}}$ iii) $y = \ln(1 - xe^{-x})$

b) Let $y = x^2 \ln x$. Find

i) $\frac{dy}{dx}$ and

ii) $\frac{d^2y}{dx^2}$.

Hence find the value of

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y$$

c) A girl flies a kite at a height of 30m. If the kite moves horizontally away from the girl at 4m/s, how *fast* is the string being paid out when the kite is 50m from her?

7. a) i)Resolve *into partial* fractions

$$\frac{4 - 2x^2}{(2x-1)(x^2+1)}$$

ii) Hence evaluate the indefinite integral

$$\int \frac{4 - 2x^2}{(2x-1)(x^2+1)}\,dx$$

b) Using a method of *your choice* evaluate each of the following

i) $\int x^2 \ln x\,dx$ ii) $\int \frac{x^2}{\sqrt{x^3+2}}\,dx$ iii) $\int x^2 e^{-2x}\,dx$

END OF EXAMINATION.

THE UNIVERSITY OF ZAMBIA

FIRST SEMESTER EXAMINATIONS

DISTANCE EDUCATION

JANUARY 2004

MATHEMATICS M111

INSTRUCTIONS :
i) Attempt ***five (5)*** questions only.
ii) Show all ***essential working*** clearly.
iii) Indicate the ***number*** of each question attempted in the first column on your main answer book.
iv) Use of calculators and tables ***is not*** allowed .

TIME ALLOWED : Three (3) Hours.

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1. a). i. State the De Morgan's laws for the union of two sets A and B.

ii. Show that $\left[(A \cup B) \cup (A \cup B')\right]' = \phi$

b). Let $X = \{-5, -\frac{1}{5}, -\sqrt{5}, 0.\bar{7}, \frac{1}{\sqrt{5}}, \sqrt{5}, 5\}$.

List the elements of X which are :

i. Natural numbers
ii. Integers
iii. Rational numbers
iv. Irrational numbers

c). Given that $P = [-4, 3)$, $Q = (1, 6)$ and $R = \{-3, 3\}$ are sets on the set of reals, find :

i. $P \cap Q$
ii. $P' \cap R$
iii. $P \cup (Q \cap R)$

2. a). i. Solve for x and y given that :

$$x+iy=\frac{3+2i}{(1+i)^2}.$$

ii. Find the integer p and q with $q \neq 0$ given that

$$\frac{p}{q}=3.1\dot{2}$$

b). Express in the form $a+b\sqrt{c}$ where a, b and c are rational numbers:

$$\frac{3\sqrt{5}-4}{2\sqrt{5}+1}.$$

c). The polynomial $f(x)=x^3+ax+bx+12$ has a remainder 12 when divided by x+1 and a remainder -30 when divided by x+3.

i Calculate the value of a and the value of b
ii Using the values found in i) solve f(x) = 0.

3. a). Given that $f(x)=\frac{1}{x-2}$.

i. Show that $f(x)$ is one-to-one .
ii. find $f^{-1}(x)$
iii. find $(f\,o\,f)(x)$.

b). Complete the square of the quadratic function

$$f(x) = 3x^2-7x+2$$

Hence
i. find the turning point , the x and y intercepts.
ii. On the same diagram , sketch the graph of
$y=f(x)$ and $y=|f(x)|$.

c). Given that α and β are roots of the equation $x^2+2x-4=0$ find value of

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}.$$

4. a). Find the integer k, given that :

i. $$\frac{\sin x}{\cos ecx-\cot x}+\frac{\sin x}{\cos ecx+\cot x}=k$$

ii. Solve each of the following equation for $0 \leq x \leq 360^0$

$$2\sin^2 x = 4-5\cos x$$

b). If $\cos\alpha = -\frac{3}{5}$ and $\tan\beta = \frac{5}{12}$ where α is in the third quadrant and β is acute . Find $\cos(\alpha+\beta)$

5. a). Compute the following limits:

i. $\lim_{x\to 1}\left(x^2+4x-1\right)$

ii. $\lim_{x\to 16}\frac{x-16}{4-\sqrt{x}}$

iii. $\lim_{x\to+\infty}\frac{4x^2-2x+3}{x^2+3x+1}$

b). Find the $\frac{dy}{dx}$ of the following:

i. $y=(x^2+1)^7$ ii. $y=\frac{x^3}{x^2+1}$ iii. $y=x\sqrt{x^2+3}$

Hence find the equation of the tangent line at the point P(1,1).

6. a). i. Find the values of the constants A,B and C given that

$$\frac{12x^2-3x-9}{(x-1)(2x+1)} = A+\frac{B}{x-1}+\frac{C}{2x+1}.$$

ii. Solve the following inequality :

$$\frac{3}{3+x} < \frac{2}{x-1}$$

b). i. Solve for x given that:

$$\sqrt{x} = \sqrt{2x-1}+2$$

ii. $|2x-3| = |x+3|$

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
UNIVERSITY SECOND SEMESTER EXAMINATIONS - JANUARY 2004

M112 MATHEMATICAL METHODS IIA

INSTRUCTIONS: 1. You must write your **computer number** on each answer booklet used.
2. Indicate the **number** of each question attempted in the first column on the main answer booklet.
3. There are seven (7) questions in this paper. Candidates must answer any **five (5)** questions only. All questions carry equal marks.

TIME ALLOWED: Three (3) hours.

1. (a) Prove by mathematical induction that for all positive integers n,

$$3+6+9+\cdots+3n=\frac{3n(n+1)}{2}$$

(b) Obtain the term inependent of x in the expansion of $\left(2x^3-\frac{1}{x}\right)^{20}$

[You may leave your answer in terms of factorials.]

(c) In the expansion of $(1+ax)^n$, the first three terms are

$$1-\frac{5}{2}x+\frac{75}{8}x^2+\cdots .$$

Find n and a, and state the range of values of x for which the expansion is valid.

2. (a) The line $4x-3y+4=0$ is tangent to the circle with centre $(3,2)$.
(i) Find the equation of the circle.
(ii) Show that the circle touches the x - axis.

(b) Sketch the graph of the hyperbola

$$36x^2-9y^2=144,$$

stating (i) the centre
(ii) the points where the curve cuts the x or y - axis.
(iii) the foci
(iv) equation of the directrices
(v) equations of the asymptotes.

3. (a) Given the vectors $\mathbf{u} = 2\alpha\,\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} + \beta\,\mathbf{j} + 3\mathbf{k}$, find
(i) the values of α and β if the vector are parallel
(ii) a relation between α and β if the vectors are perpendicular.

(b) Find a unit vector that is perpendicular to both vectors $\mathbf{a} = 3\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}$ and $\mathbf{b} = -3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$

(c) Find the area of the triangle ABC given that its vertices are $A(2,1,-1)$, $B(1,-7,3)$ and $(C(-2,5,1)$.

4. (a) Given the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ 2 & 2 & 1 \end{pmatrix},$$

find A^{-1}, the inverse of A.

(b) Use Cramer's rule to solve the system of equations

$$\begin{aligned} x - 2y + z &= 6 \\ 2x + y - 3z &= 5 \\ 2y + z &= 1 \end{aligned}$$

(c) Find the dimensions of the largest open box which can be made from a sheet of cardboard of sides 60 cm by 28 cm by cutting equal squares from the corners and turning up the sides.

5. (a) (i) Use de Moivre's theorem to prove that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

(ii) Express $\dfrac{(\cos\phi - i\sin\phi)^3}{(\cos 2\phi + i\sin 2\phi)(\cos\phi - i\sin\phi)^5}$
in the form $\cos n\phi + i\sin n\phi$, where n is an integer

(b) Given that $y = x\sqrt{x+1}$, show that

$$\frac{dy}{dx} = \frac{3x+2}{2\sqrt{x+1}}$$

Hence or otherwise evaluate $\int_3^8 \dfrac{3x+2}{\sqrt{x+1}}dx$

6. (a) Find $\frac{dy}{dx}$:

(i) $y = (\ln x)^2$

(ii) $y = \frac{4x+3}{\sqrt{2x-1}}$

(iii) $y = \tan^{-1}(x^3)$

(b) Find the gradient of the curve with equation

$$5x^2 + 5y^2 - 6xy = 13$$

at the point (1,2).

(c) $f(x) = \frac{3x}{(x+2)(x-1)}$, $x \neq -2$, $x \neq 1$.

Use partial fractions to evaluate $\int_{-1}^{0} f(x)dx$.

7. (a) Differentiate the function $f(x) = \frac{x^2+9}{x^2-1}$ with respect to x.

Hence, find the value of x at which the function has a maximum and/or minimum and determine which of these it is.

(b) Evaluate the integrals:

(i) $\int \frac{(\ln x)^2}{x} dx$

(ii) $\int xe^{2x} dx$

(iii) $\int_0^3 \frac{x}{\sqrt{x+1}} dx$

(c) On the same diagram, sketch the graphs of the curves $y = 2x^2 + 3$ and $y = 10x - x^2$ and state their points of intersection. Hence, find the area between the two curves.

END OF EXAMINATION

THE UNIVERSITY OF UNIVERSITY

SCHOOL OF NATURAL SCIENCES

2004 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

M 111: MATHEMATICAL METHODS I

TIME: THREE (3) HOURS

INSTRUCTIONS:
(i) ANSWER ANY FIVE (5) QUESTIONS
(ii) NO CALCULATORS TO BE USED
(iii) INDICATE THE NUMBER OF EACH QUESTION ANSWERED ON YOUR MAIN ANSWER BOOK.

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1. (a) (i) Express the complex number $Z = \dfrac{(4+3i)(2+i)}{3-2i}$

in the form $a + ib$ where a and b are real numbers.

(ii) Show that $4.3\overline{57}$ is a rational number.

(iii) Draw a Venn diagram showing the following sets:

C = {complex numbers}, R = {real numbers}
Q = {rational numbers}, Z = {integers}

(b) (i) If the roots of the equation $x^2 + 5x + K = 0$ are α and $\alpha + 1$, find the value of K.

(ii) Find the integer m given that

$$\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta} = m$$

(iii) Solve for x given that

$$\frac{x+2}{x-3} \geq 3$$

(c) Simplify as much as possible where A and B are any sets

$$[(A \cap B') \cup (A' \cap B)]'$$

2. (a) Given that $f(x) = \sqrt{-x+4}$

(i) Determine the domain of f(x).

(ii) Sketch the graph of f(x). Indicate on your graph, the y-intercept.

(iii) Determine the range of values of x for which $f(x) < 4 - x$.

(b) Given the polynomial $p(x) = 6x^4 + x^3 - 8x^2 - x + 2$

(i) Factorize p(x) completely.

(ii) Find values of x for which $p(x) = 0$

(iii) Find values of x for which $p(x) < 0$

(c) Let the universal set E be all students registered at UNZA in the first semester 2004 academic year. Given the following sets:

A = Graduate students

B = Students registered in School of Law

C = Undergraduate students

D = Students who are on GRZ sponsorship.

Represent this information in a Venn diagram.

3. (a) Given that $f(x) = 3\cos(2x) + 1$

(i) Solve $f(x) = \frac{5}{2}$, $x \in [-2\pi, 2\pi]$

(ii) Determine the amplitude, period and phase of the graph of f(x).

(iii) Sketch the graph of f(x) on the interval $[0, 2\pi]$.

(b) Let $g(x) = \begin{cases} x^2 - 3x + 3 & \text{if } x \leq 2 \\ \frac{2}{x} & \text{if } x > 2 \end{cases}$

(i) Determine whether the function $g(x)$ is continuous at $x = 2$.

(ii) Determine the equations of the horizontal and vertical asymptotes of $g(x)$, if there are any.

(iii) Find the equation of the tangent line to the graph of $g(x)$ at $x = 1$.

(c) Find the equation whose roots are minus the roots of the quadratic equation

$$2x^2 - x + 3 = 0$$

4. (a) Given that $\sin\theta = \frac{-3}{5}$ and $\text{Tan}\,\alpha = \frac{3}{4}$, where θ is a third quadrant angle and α is acute angle.

(i) Find $\sin\alpha$, $\cos\theta$ and $\cos\alpha$.

(ii) Find $\sin(\theta + \alpha)$.

(iii) Find $\text{Tan}(\theta - \alpha)$.

(b) (i) Find the derivative of $f(x) = \frac{2}{x}$ from the first principles.

(ii) Solve $\frac{2x-6}{x+3} < -x - 2$.

(iii) Sketch the graph of

$$g(x) = |x - 3| - |2x - 1|$$

(c) Find the following limits:

(i) $\lim\limits_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$

(ii) $\lim\limits_{x \to -\infty} \frac{x^2 + 1}{x^2 + 2x + 1}$

5. (a) (i) Solve for x and y given that

$$\frac{x}{1+3i} + \frac{y}{1-i} = 3$$

(ii) When the polynomial $p(x) = x^3 + kx^2 - 2x + 1$ is divided by $x - k$, the remainder is k. Find the possible values of k.

(iii) Given that α and β are the roots of the equation $3x^2 + x + 2 = 0$, find the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(b) Differentiate the following:

(i) $y = x^2(2x^2 - 1)^6$.

(ii) $y = \frac{x^2 - 1}{\sqrt{x+1}}$

(iii) $y = \text{cosec}^2 3x$

(c) Prove that $\frac{\sin x \sec x}{\cot x + \tan x} = 1 - \cos^2 x$

6. (a) Given that $f(x) = \sqrt{-x^2 + 4x + 21}$

(i) Determine the domain of f(x).

(ii) Solve for $f(x) \geq 9$.

(iii) Solve for $f(x) = \frac{4}{3}(x - 2)$.

(b) Let $f(x) = \frac{x}{x-1}$ and $g(x) = x^2$.

(i) Find $(fog)(x)$.

(ii) Show whether $(fog)(x)$ is even or not.

(iii) Find $\lim_{x \to \infty} (fog)(x)$.

(c) The Zambia National Service (ZNS) has a rectangular Car Park 20 meters wide and 35 meters long. They plan to increase the area of the car park by 236 square meters by adding a strip of equal width to one side and one end. Find the width of the strip to be added.

7. (a) (i) Given that $\sqrt{3}$ is irrational number prove that $2 + \sqrt{3}$ is irrational number.

(ii) Express the complex number $\frac{1}{(1-i)^2} - \frac{1}{(1+i)^2}$

in the form $a + ib$ where a and b are real numbers.

(iii) Find the general solution for θ for the equation

$$2\sin^2\theta - 3\cos\theta = 0.$$

(b) (i) Find $\frac{dy}{dx}$ given that $y = x^2 \cot x$.

(ii) Find the values of x which satisfy the inequality

$$|2x + 5| < 3x - 5.$$

(iii) Find values of x for which $\frac{3}{x+3} < \frac{2}{x-1}$.

(c) The rate of photosynthesis of a plant denoted by p(I) is given by

$$p(I) = \frac{I}{a + bI}, \quad I \geq 0$$ where a, b are costants, and I is the light intersity.

(i) Find $p'(I)$

(ii) Find $p'(0)$

(iii) Find $\lim_{I \to \infty} p'(I)$

END OF EXAMINATION

The University of Zambia

School of Natural Sciences

Department of Mathematics and Statistics

2007 Academic year

First Semester Examinations

M111 Mathematical Methods I

Time Allowed: Three (3) Hours **3rd September 2007**

Instructions:

1. You must write your **Computer Number**, Your **TG Number** on each answer booklet used.
2. Indicate the number of each question attempted in the first column on the main booklet
3. There are **six (6)** questions in this paper. Candidates must answer any **five (5)** questions only. All questions carry equal marks.
4. No **Calculators** to be used

1. (a) (i) Show that $2.5\overline{17}$ is a rational number

(ii) Given that $z = \dfrac{\sqrt{3}+i}{1-i}$, find $z^2 - \dfrac{1}{z^2}$ giving the answer in the form $a+ib$, where a and b are real numbers

(b) Let $f(x) = x^2 - 4x - 5$

(i) Is the function f, even or odd or neither?. Justify your answer.

(ii) Find the minimum or maximum value of f

(iii) Find the values of x for which $f(x) > 0$

(iv) Sketch the graph of f

(c) (i) Find the solution to the equation $2\sin^3 x - \sin x = 0$, $0 \le x < 2\pi$

(ii) Find $\dfrac{dy}{dx}$, given that $y = \sqrt{x^2-1}\sec x$

2. (a) Let $f(x) = \cos(2x - \pi)$, $0 \le x < 2\pi$

(i) Sketch the graph of f(x)

(ii) Find the values of x for which $f(x) = \dfrac{\sqrt{3}}{2}$

(b) Simplify the following as much as possible where A and B are any sets

(i) $(A \cup B) \cap (A \cup B')$

(ii) $[A' \cup (A \cap B)']'$

(c) (i) Find the derivative from first principles of $f(x) = \dfrac{1}{\sqrt{3x-1}}$

(ii) Prove that $\tan(x - \dfrac{\pi}{4}) = \dfrac{\tan x - 1}{\tan x + 1}$

3. (a) (i) Find the set of values of x for which $\frac{1+x}{1-x} > \frac{2-x}{2+x}$

(ii) Find the derivative from the first principles of $f(x) = \frac{1}{x-1}$

(b) (i) The complex numbers z_1 *and* z_2 are given by $z_1 = 2+3i$ and $z_2 = 3-2i$ $z_2 + (p+iq)z_1 = 1$ where p and q are real numbers, find the values of p a

(ii) Find $\frac{dy}{dx}$, if $y = x^{\frac{3}{2}} - 3x^{\frac{2}{3}}$

(c) Given the polynomial $p(x) = 2x^3 + x^2 - 5x + 2$

(i) Find all rational roots of the polynomial equation $p(x) = 0$

(ii) Find the range of values of x for which $p(x) \leq 0$

4. (a) Let f be given by $f(x) = \frac{25}{3x-2}$, find the following

(i) The domain and range of f

(ii) The inverse function of f

(iii) The solution of the equation $(fog)(x) = \frac{2}{x-1}$ where $g(x) = x+1$

(b) (i) Given that $y = \sec x + \tan x$, prove that $\frac{dy}{dx} - y\tan x = 1$

(ii) Given that $x-k$ is a factor of $p(x) = kx^3 - 3x^2 - 5kx - 9$, and $k \in R$, find possible values of k.

(c) (i) Given that $\sqrt{2}$ is an irrational number, prove that $\frac{1}{\sqrt{2}+1}$ is an irrational number.

(ii) Let $u = [-5,5]$ be the universal set, $A = (-2,1]$, B = [0 , 5] and C =(-3, 2). $A \cup (B' \cap C)$ and illustrate your answer on the number line.

(iii) On the same axis sketch the graphs of $f(x) = |3x+1|$ and $h(x) = |2-x| -$

5. (a) (i) Given that the roots to the equation $2x^2 - 3x + 2 = 0$ are denoted by α and β. Without solving for α and β, find the quadratic equation whose roots are $\frac{1}{\alpha\beta}$ and $\frac{1}{\alpha+\beta}$.

(ii) Find the derivative of $y = \frac{x^2 - 1}{x^2 + 1}$

(b) Let $f(x) = \frac{1}{x^2 - x - 2}$

(i) Find the domain and range of f

(ii) Find the vertical and horizontal asymptotes (if any)

(iii) Sketch the graph of f

(c) Let $f(x) = \begin{cases} 2x+1, & if\ x \le -1 \\ x^2 - 2 & if\ x > -1 \end{cases}$

(i) Is the function f(x) continuous at x = -1 ? Justify your answer,

(ii) Find $\lim_{x \to 2} f(x)$

(iii) Sketch the graph of $f(x)$

6. (a) Given $f(x) = (x - \alpha)(x - \beta)$, $\alpha > \beta > 0$. Sketch on separate diagrams; indicate the coordinates at which each curve meets the coordinate axes.

(i) $y = f(x)$

(ii) $y = -f(x + \alpha)$

(b) Let $f(x) = 2\sin(\pi - 2x)$

(i) Find the period, amplitude and phase shift of f(x)

(ii) Sketch the graph of f(x) , $0 \le x < 2\pi$

(c) Let an operator * on a set of real numbers be defined as follows $a * b = b^2 - a^2$, for any $a, b \in R$.

(i) Is * a binary operation on R ? Justify your answer

(ii) Compute 2*(3*(-2))

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
Department of Mathematics & Statistics

2008 ACADEMIC YEAR
FIRST SEMISTER FINAL EXAMINATIONS

M 111 : MATHEMATICAL METHODS I

TIME ALLOWED: Three (3) Hours

INSTRUCTIONS: **(a)** Answer any **Five (5)** questions.
(b) Write down **your COMPUTER No.** and **TG. No.** on **ALL** your answer booklet.
(c) Use of Calculators is not **allowed** in this examination.

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1. (a) Let $X=[1,10]$ be the Universal set and $A=[1,4]$, $B=(2,8)$ and $C=[3,6)$ be the subsets of X. Find each of the following sets and display them on the real line.

(i) B' (ii) $A\cap(B-C)$ (iii) $A\cap B$

(b) (i) Solve the equation

$$\cos x\cos 30^\circ-\sin x\sin 30^\circ=\frac{1}{2},\qquad -180^\circ\le x\le 180^\circ$$

(ii) If α and β are roots of the equation $x^2-4x+2=0$, Without solving the equation, find the value of :

$$\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}$$

(iii) Let $f(x)=\begin{cases}2x+1 & if\ x\le -1\\ x^2-2 & if\ x>-1\end{cases}$

Show that $f(x)$ is continuous at $x=-1$

(c) Solve the inequality $\left|\frac{x}{x-3}\right|\le 2$ for real values of x.

2. (a) Let $f(x) = 3\cos(2x - \pi)$, $0 \le x \le 2\pi$.

(i) Find the amplitude of f(x)

(ii) Find the phase displacement of f(x)

(iii) Find the period of f(x)

(iv) Sketch the graph of f(x)

(v) Find the values of x for which $f(x) = \frac{3}{2}$

(b) Find the derivative of the function $f(x) = \frac{1}{x-1}$, from the first principles.

(c) (i) Express $1.5\dot{1}\dot{4}$ in the form $\frac{r}{q}$ where $r, q \in Z$, $q \neq 0$.

(ii) Given that $z = \frac{\sqrt{3}+1}{1-i}$, find $z^2 - \frac{1}{z^2}$, giving the answer in the form a + ib, where a and b are real numbers.

3. (a) Solve the following equations

(i) $\sqrt{3-x} - \sqrt{7+x} = 0$ (ii) $2x^3 - 3x^2 - 3x + 2 = 0$

(b) Let $f(x) = -2x^2 + 11x - 15$, find the maximum or minimum point and value of $f(x)$. Hence, sketch the graph of $f(x)$.

(c) Find $\frac{dy}{dx}$ of the following functions,

(i) $y = 12\sqrt{x} - x^{\frac{3}{2}}$

(ii) $y = x^2 \sin(3x^2 - 1)$

(iii) $y = (x - \frac{1}{x})^3$

(iv) $y = \frac{\sin x}{1 + \cos x}$

4. (a) Let $f(x)=4+2\cos(x+k)^{o}$, $0 \le x \le 360^{o}$, where k is a constant value, $0 < k < 360^{o}$. The curve with equation $y = f(x)$ passes through the point $(30^{o},5)$.

(i) find the value(s) of k

(ii) Solve the equation $f(x)=2$.

(iii) Find the greatest and least values of f(x).

(iv) Sketch the graph of f(x)

(v) Write the coordinates of the points where the curve meets the y-axis.

(b) Let the two functions f and g be defined by:

$f(x)=\dfrac{25}{3x-2}$, $1 < x \le 9$

$g(x)=x^{2}$, $1 < x \le 3$.

(i) Show that f is a one to one function.

(ii) Find the domain of f^{-1}.

(iii) Find the composite function fog.

(iv) Hence, solve the equation $(fog)(x)=\dfrac{2}{x-1}$.

(v) Find $(fog)(-1)$.

(c) Express $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ in the form $a+b\sqrt{2}$, where a and b are real numbers.

5. (a) Given the function $f(x) = -\sqrt{2-x}$,

(i) State the domain of f(x)

(ii) State the range of f(x)

(iii) Sketch the graph of $y = f(x)$

(iv) Find the solution set of the inequality $f(x) \ge 0$

(b) (i) Given that $y = \sec x + \tan x$,

Prove that $\dfrac{dy}{dx} - y\tan x = 1$.

(ii) Evaluate $\lim\limits_{x\to 3}\dfrac{x^{3}-27}{x^{2}-9}$

(c) Let $f(x)=\dfrac{2x+1}{x-1}$.

(i) Find the domain of f(x).

(ii) Find the range of f(x).

(iii) Find the vertical and horizontal asymptotes (if any).

6. (a) Let $f(x) = -|3x+1| + 2$.

(i) Sketch the graph of the function f(x).

(ii) Find the points at which the graph cuts the x-axis.

(iv) Find the range of values of x for which $|3x+1| - 2 \leq 0$.

(b) The complex numbers z_1 *and* z_2 are given by:

$z_1 = 24 + 7i$, $z_2 = 4 - 3i$

(i) Given that $z_1 + \alpha z_2$ is real, where α is real , find the value of α.

(ii) Given that $z_1 + (p+iq)z_2 = 0$, where p and q are real numbers find p and q.

(c) Let $(x-2)$ be a factor of $f(x)$ where $f(x) = x^3 - x^2 + Ax + B$.

(i) Find an equation satisfied by the constants A and B

(ii) Given further that when f(x) is divided by x-3 the remainder is 10, find a second equation satisfied by A and B.

(iii) find the values of A and B

(iv) Using your values of A and B solve the equation $f(x) = 0$.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

UNIVERSITY SEMESETER I EXAMINATIONS- 2009

M114 – MATHEMATICAL METHODS II -B

INSTRUCTIONS:
1. Answer any **four (4)** questions.
2. All questions carry equal marks.
3. Show all the necessary work to earn full marks.
4. Write down the questions attempted in one of the columns on the front page of the main booklet.
5. Use of calculators is **not** allowed.

TIME ALLOWED: Three (3) hours.

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1. (a) By using the substitution $t = \tan\frac{1}{2}x$, prove that

(i) $\operatorname{cosec} x = \frac{1+t^2}{2}$ and $\cot x = \frac{1-t^2}{2t}$

(ii) Use the result that $\operatorname{cosec} x - \cot x = \tan\frac{1}{2}x$, show that $\tan 15^0 = 2 - \sqrt{k}$, and state the value of k,

(b) The function f is defined for all real x by $f(x) = \cos x^0 - \sqrt{3}\sin x^0$

(i) Express $f(x)$ in the form $R\cos(x+\theta)^0$, where $R > 0$ and $0^0 < \theta < 90^0$

(ii) Solve the equation $|f(x)| = 1$ giving your answers in the interval $0^0 \le x \le 360^0$

(c) Find the equation of the tangent line to the curve $x^3 + y^3 = 2xy$ at the point $P(1, 1)$

(d) Find the value of k such that $4x^2 - 15xy + ky^2 = 0$ represents a pair of straight lines.

2 (a) Given the equation of the circles

$$x^2 + y^2 - 6x + 7 = 0 \text{ and } x^2 + y^2 + 2x - 8y - 1 = 0$$

(i) Show that the circles touch externally.
(ii) Find the coordinates of their point of contact.
(iii) Find the equation of their common tangent.

(b) $\underline{a} = 3i - 2j - k$, $\underline{b} = 3i - 5j + 2k$ and $\underline{c} = i + pj + qk$,

(i) Find the angle between $\underline{a}$ and $\underline{b}$
(ii) Find the constants p and q given that $\underline{c}$ is perpendicular to $\underline{a}$ and $\underline{c}$ is perpendicular to $\underline{b}$
(iii) Using values of p and q found in (ii), find a unit vector perpendicular to both $\underline{a}$ and $\underline{c}$

(c) Given the equation of the curve $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x$

(i) Find the x and y intercepts
(ii) Find the stationary points.
(iii) Test the nature of the stationary points
(iv) Find if any the point(s) of inflection.
(v) Sketch the graph of the curve labeling all the necessary features.

(d) Evaluate $\int_{\ln 2}^{\ln 3} e^{3x}\,dx$

3 (a) A square cardboard ABCD is of side 8 meters. A square of x meters is removed from each of corners and the remainder is folded to form an open tray of depth x meters and volume V metres^3

(i) Show that $V = 64x - 32x^2 + 4x^3$

(ii) Find the value of x for which $\frac{dV}{dx} = 0$

(iii) Show that the value of x gives the maximum value of V

(iv) Find this maximum value of V.

(b) Find the area A of the region in the XY plane bounded by the graphs of $2y = 16 - x^2$ and $x + 2y = 4$

(c)

(i) Find $r > 0$ and θ given that $rcis\theta = (-\sqrt{3} - i)^8$.

(ii) Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$

(d)

(i) Find $\int e^{3x}\sqrt{1 + e^{3x}}\, dx$

(ii) Show that $\frac{d}{dx}\left[\sin^{-1}(\cos x)\right] = -1$

4 (a) Discuss and sketch the graph of the equation labeling the direct ices, center and focus (foci)

$$9x^2 + y^2 - 36x + 8y + 43 = 0$$

(b) Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -2 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

and hence solve using the **inverse method** the system of equations

$$\begin{aligned} x + z &= 3 \\ -2x + 2y + z &= 2 \\ x + y + 2z &= 4 \end{aligned}$$

(c) A spherical balloon is blown up so that its radius increases at a constant rate of 0.01cm s^{-1}. Find the rate of increase in the volume when the radius is 5cm.

5 (a) Prove by mathematical induction that for all positive integers $n \geq 1$

$5^{2n} - 1$ is divisible by 24

(b) In the binomial expansion of $\left(1+\frac{x}{n}\right)^n$ in ascending powers of x , the coefficient of x^2 is $\frac{7}{16}$. Given that n is a positive integer,

(i) Find the value of n.

(ii) Evaluate the coefficient of x^3 the expansion.

(c) Solve for x the equation
$4\cosh x + 8\sinh x = 1$
for all real values of x giving the root as a natural logarithm.

(d)

(i) Evaluate $\int_0^1 \frac{x^2-1}{(x^3-3x+1)^6}\,dx$

(ii) Find $\int x e^x\,dx$

END OF EXAMINATION

2 (a) (i) Find the values of x which satisfy the following inequality:

$$x^3 - 3x^2 \leq 10x$$

(ii) Find the integer value of:

$${}^6C_0 \; + \; {}^6C_2 \; + \; {}^6C_6$$

(b) (i) Given that $x^3 - 4x^2 - 3x + 10 = (x-1)(x+2)(x-c) + px + q$, find the value of p, of q and of c.

(ii) Write in the form $A + B\sqrt{C}$, where A, B and C are integers:

$$\left(2\sqrt{5}+1\right)\left(3\sqrt{5}-2\right)$$

(c) (i) Express as a single term in its lowest terms:

$$\frac{2}{2x-1} - \frac{1}{2x+1} - \frac{2}{4x^2-1}$$

(ii) Express in terms of $\log_a x$, $\log_a y$ and $\log_a z$:

$$\log_a\left(\frac{x^3 y}{z^6}\right)$$

3 (a) (i) Given the function $f(x) = \dfrac{3x+1}{x-3} \quad x \neq 3$,

find $(f \; o \; f)^{-1}$ (2)

(ii) How many different four – digit numbers can made from the digits

3 , 4 , 5 , 6 , 7 , 8

if no digit may be repeated?

(b) (i) Let the universal set E, be the set of letters in the English alphabet such that

$$A = \{b, r, e, a, k, f, s, t\}$$
$$B = \{l, u, n, c, h\}$$
$$C = \{s, u, p, e, r\}$$

Find $(A \cap C) \cap (B' \cap C)$

3 (b) (ii) Find the value of p if the equation

$$x^2 + (p-2)x + 10 - p = 0$$

has equal roots.

(c) Solve for x such that:

(i) $\sqrt{x+5} - 2 = \sqrt{x-7}$

(ii) $4^{2x-1} = \left(\frac{1}{16}\right)^{1+x}$

4 (a) Given $f(x) = x^3 + x^2 - 3x + 1$,

(i) Express $f(x)$ as a product of a linear function and a quadratic function

Hence,

(ii) Resolve into partial fractions $\frac{3x+1}{x^3 + x^2 - 3x + 1}$.

(b) Find the positive integer n such that

(i) $\left(2+\sqrt{2}\right)^3 + \left(2-\sqrt{2}\right)^3 = n$

(ii) $\binom{n}{2} = 15$

(c) (i) Express in the form $\frac{a}{b}$ where a, b are integers and $b \neq 0$:

1. 3222222222.....

(ii) If α and β are roots of the quadratic equation $x^2 + 3x - 2 = 0$, find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

5 (a) The polynomial $P(x) = 2x^3 + ax^2 + bx + 6$ is a exactly divisible by $x - 2$ and on division by $x + 1$ leaves a remainder -12.

(i) Calculate the values of a and b.

(ii) Using the values found in (i), solve $f(x) = 0$

(b) (i) Find the real values of x and y when :

$$\frac{x}{1-i} + \frac{y}{1+3i} = 2$$

(ii) Solve the simultaneous equations

$$x - y = 21$$
$$\log x + \log y = 2$$

(c) (i) Evaluate :

$$\frac{1}{\log_2 36} + \frac{1}{\log_3 36}$$

(ii) Solve the inequality:

$$\frac{3x}{x-1} > x$$

END OF EXAMINATION

The University of Zambia
School of Natural Sciences
Department of Mathematics & Statistics

2008 ACADEMIC YEAR
FIRST SEMESTER FINAL EXAMINATIONS

M 211 - MATHEMATICAL METHODS III

INSTRUCTIONS:
1. Answer any **Five (5)** of the seven Questions only.
2. Show all essential working to obtain full marks.
3. Indicate the question number for each question attempted on the cover main answer booklet.
4. All questions carry equal marks.

TIME ALLOWED: Three (3) hours

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1. The equation of the conic section is given by

$$x^2 + xy + y^2 = 6.$$

(a) Transform the equation in standard form, and hence, identify the curve.

(b) Find its vertex (or vertices), focus (or foci) and the directrix (or directrices).

Hence,

(c) sketch the curve.

2. (a) Identify the conic section

$$r = \frac{6}{3 - 4\cos\theta},$$

and give its eccentricity and distance of the directrix from the origin. Hence, sketch the conic, indicating the position one of the foci, the x and y - intercepts and the directrix.

(b) The position of a planet in elliptical orbit around the sun is given by the polar equation

$$r = \frac{k}{1 + e\cos\theta},$$

for some value of the eccentricity e. Find
(i) the distance when the planet is closest to a sun;
(ii) the distance when the planet is furthest from the sun;
(iii) hence, show that

$$r = \frac{a(1 - e^2)}{1 + e\cos\theta},$$

where a, is the length of the semi-major axis of the elliptic path.

(c) A satellite elliptically orbits around the earth (radius ≈ 6360 km) so that the maximum distance from the satellite is 20,000 km and minimum distance is 10,000 km. Find the eccentricity of the orbit and give a polar formula for this position.

3. (a) Evaluate the limit if it exists:

(i) $\lim_{x \to 2} \dfrac{x^3 - 8}{x - 2}$

(ii) $\lim_{x \to \infty} \dfrac{\ln(1 + e^{3x})}{x}$

(iii) $\lim_{x \to \infty} (1 + x^2)^{5/x^2}$.

(b) Find the curvature and the radius of curvature of the curve

$$2xy + x + y = 4,$$

at the point (1,1).

(c) Given that the arc length is measured from the point $(0, \frac{1}{4})$, find the intrinsic equation of the curve with Cartesian equation $4y = \cosh 4x$.

4. (a) (i) Define the continuity of a function $f(x)$ at a point $x = a$.

(ii) State Rolle's theorem.

(iii) State the Mean Value theorem.

(b) (i) Find the value of c prescribed in the Rolle's theorem for the function

$$f(x) = x^3 - 12x$$

on the interval $0 \le x \le 2\sqrt{3}$.

(ii) Use the Mean value theorem to approximate $\sqrt[6]{65}$

(c) Use the linearization method to approximate the value of $\dfrac{1.05}{0.95}$.

5. Evaluate the integrals:

(a) $\int \dfrac{(x+3)}{(x^2+6x)^{1/3}}\,dx$

(b) $\int \dfrac{1}{1+\sin x-\cos x}\,dx$

(c) Find the volume generated by revolving the plane within the curve $4x^2+9y^2=36$ about the $x-$axis.

6. (a) At every point of a certain curve, $y''= x^2-1$. Find the equation of the curve if it passes through the point $(1,1)$ and is tangent to the line $x+12y=13$ at the point.

(b) Evaluate $\int_{\theta=0}^{\theta=2\pi} y\,dx$, given that $x=\theta-\sin\theta$, $y=1-\cos\theta$.

(c) Find the area of the surface of revolution generated by revolving about the $x-$axis the arc of the parabola $y^2=12x$, from $x=0$ to $x=3$.

7. Evaluate

(a) $\int \ln(1+x^2)\,dx$

(b) $\int_3^5 \frac{dx}{\sqrt{x^2+16}}$.

(c) Find the length of the arc of the curve $x = t^2$, $y = t^3$ from $t = 0$ to $t = 4$.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

DEPARTMENT OF MATHEMATICS AND STATISTICS

SEMESETER I EXAMINATIONS - 2008
DISTANCE EDUCATION
M211 – MATHEMATICAL METHODS III

INSTRUCTIONS:
1. Answer any **four (4)** questions.
2. All questions carry equal marks.
3. Show all the necessary work to earn full marks.
4. Write down the questions attempted on the front page of the main booklet.
5. Use of calculators is **not** allowed.

TIME ALLOWED: Three **(3)** hours.

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1. (a) Given the function $f(x) = \ln x - \frac{1}{8}x^2$, find a and b given that

$$\sqrt{1+[f'(x)]^2} = \frac{a}{x} + \frac{x}{b}$$

(b) Show that the polar equation of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{is} \quad r^2 = \frac{-b^2}{1 - e^2\cos^2\theta}$$

(c) Consider the polar equation of the conic given by,

$$r = \frac{6}{3 + 2\cos\theta},$$

(i) Identify the conic
(ii) Discuss the conic stating its focus, vertex and directrix
(iii) Sketch the curve stating clearly the points of intercepts.

2. (a) Given the equation

$$xy = 1$$

(i) Identify the conic given by this equation.
(ii) Use a suitable rotation of axes to find an equation for the graph in an $X'Y'$ plane.
(iii) Sketch the graph labeling vertices.

(b) Let $f(x) = \begin{cases} x+1, & x < -1 \\ (x+1)^2, & x \geq -1 \end{cases}$

(i) Determine whether f is continuous at $x = -1$
(ii) Sketch the graph of f

2 (c) Find $\lim_{x\to+\infty} \dfrac{x^2+1}{x\sqrt{3x^2+1}}$

3 (a) Evaluate $\lim_{x\to 0+}\left(\dfrac{1}{x}-\dfrac{1}{\tan^{-1}x}\right)$

(b) If $f(x)=x^3-8x-5$

(i) Show that f satisfies the hypothesis of the mean value theorem on the interval $[1\ ,4\ ]$.

(ii) Hence find the number c in the open interval $(1\ ,4\ )$ that satisfies the conclusion of the theorem.

(c) Find two pints on the graph of the equation

$x^2-xy+y^2=4$

at which the slope of the tangent line is 1.

4 (a) Let $f(x)=\ln(1+x)$

(i) Find a formula for the 4*th* Taylor polynomial of f about 0.

(ii) Hence calculate $p_4(1)$.

(b) (i) Find $\int \dfrac{1}{x^2\sqrt{x^2+1}}\,dx$

(ii) Given that $f(x)=\sqrt{a^2-x^2}$ where $-\pi\le x\le\pi$, find the volume of generated by the function when rotated about the x-axis.

5 (a) Evaluate the integral:

(i) $\int(2x+9x^3)\sqrt{1+4x^2+9x^4}\,dx$

(ii) $\int\tan x\sin^2 x\cos^5 x\,dx$

(b) Find the curvature of the function $y=\sin x$ at the point $T\left(\dfrac{\pi}{2},1\right)$

(c) Given the graphs of the curves

$y=e^x$ and $y=\sqrt{x}$ for $x\ge 0$,

(i) sketch on the same diagram, the graphs of the curves.

(ii) shade the area A, the area bounded by the two curves and the lines $x=0$ and $x=1$.

(iii) Hence, find the area A

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
Directorate of Distance Education

2008/9 ACADEMIC YEAR
SECOND SEMESTER FINAL EXAMINATIONS

M111: Mathematical Methods I

TIME ALLOWED: Three (3) Hours

INSTRUCTIONS : Answer any **Five** (5) questions from this paper
Omission of essential working may result in loss of marks
Calculators and Mathematical tables are **NOT** allowed in this paper

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1. (a) Let U = [–7 , 10] be the universal set, and let A = (–2 , 8), B = [1 , 8] and C = (–7 , 6) be subsets of U.
Find
(i) $A \cap (B - C)$
(ii) C'
(iii) $(A \cup C)'$

(b) (i) Express $\frac{2\sqrt{3}+1}{2-\sqrt{3}}$ in the form $a+b\sqrt{3}$ where a and b are rational numbers.

(ii) If α and β are roots of the equation $x^2+4x-2=0$, find the sum $\frac{1}{\beta^2}+\frac{1}{\alpha^2}$

(c) Solve the inequality $\frac{x}{x-3} \geq \frac{1}{2}$

2. (a) (i) Let $(1 + 3i)z = 5(1 + i)$. Express z in the form $a + bi$ where a and b are rational numbers.

(ii) Express $1.45\overline{45}$ in the form $\frac{a}{b}$ where a and b are integers.

(b) Prove the identity $\frac{1-\cos 2A+\sin 2A}{1+\cos 2A+\sin 2A} = \tan A$

(c) Let $f(x) = -2x^2 + 11x - 15$. Find the maximum or the minimum point of $f(x)$. Hence sketch the graph of $f(x)$.

3. (a) The table below shows an operation '*' on a set G = { 1 , 5 , 7 , 11 }.

*	1	5	7	11
1	1	5	7	11
5	5	1	11	7
7	7	11	1	5
11	11	7	5	1

(i) Is the operation a binary operation? Give reason for your answer.
(ii) Is the operation commutative?
(iii) Evaluate (5 * 1) * 11 and (5 * 7) * (7 * 11)

(b) Let $f(x) = \frac{2x}{3x-2}$ and $g(x) = \frac{1}{x^2}$
(i) State the domain of f and the range of g.
(ii) Find the composite function $(fog)(x)$.
(iii) Find the inverse of $f(x)$.

(c) The graph of $f(x) = 5 + 2\cos(x+k)^0$, $0 \le x \le 360^0$ passes through the point (30° , 5).
(i) Find the smallest value of k
(ii) Solve the equation $f(x) = 4$.

4. (a) Evaluate the following limits:

(i) $\lim_{x \to -2} \frac{x^2}{3x^2+7}$

(ii) $\lim_{x \to 3} \frac{x^2-9}{2x^2-5x-3}$

(iii) $\lim_{x \to \infty} \frac{3+2x-5x^2}{2x^2+7}$

(b) Differentiate the function $f(x) = \frac{1}{x}$ from the first principle.

(c) Let $f(x) = -3\cos(2x-\pi)$, $0 \le x \le 2\pi$
(i) Find the amplitude, the shift and the period of f(x).
(ii) Sketch the graph of $f(x)$.
(iii) Find the values of x such that $f(x) = \frac{3}{2}$.

5. (a) Given that $x = 2$ is a root of the equation $\alpha^2 x^2 + 2(2\alpha - 5)x + 8 = 0$

(i) find the possible value(s) of α

(ii) Hence find the corresponding value of the other root.

(b) The complex numbers z_1 and z_2 are given by;
$z_1 = 24 + 7i$ and $z_2 = 4 - 3i$

(i) Express $\frac{z_1}{z_2}$ in the form $a + bi$ where a and b are rational numbers.

(ii) Given that $z_1 + \alpha z_2$ is real, where α is a real number, find the value of α.

(iii) Given that $z_1 + (p + iq)z_2 = 0$ where p and q are real numbers, find p and q.

(c) Determine with reasons whether each function below is even, odd or neither:

(i) $f(x) = x^3 - \frac{3}{x}$

(ii) $g(x) = x^2 - x + \sin x$

(iii) $h(x) = 5 + 3\cos 2x$

6. (a) Find $\frac{dy}{dx}$ of the following functions:

(i) $y = x\cos(3x^2 + 1)$

(ii) $y = \ln(x^3 - 2x^2)$

(ii) $y = \frac{\sin x}{1 + \cos x}$

(b) Given that $f(x) = \sqrt{2 - x}$

(i) State the domain and the range of $f(x)$

(ii) Sketch the graph of $y = f(x)$

(c) Let $x - 2$ be a factor of $f(x) = x^3 - x^2 + ax + b$.

(i) Find a relation between a and b

(ii) Given that the remainder when $f(x)$ is divided by $x - 3$ is 10, find the values of a and b

(iii) Hence factorise $f(x)$ completely.

End of Exam

The University of Zambia
School of Natural Sciences
Department of Mathematics & Statistics

2013/14 ACADEMIC YEAR FINAL EXAMINATIONS

MAT1100 – FOUNDATION MATHEMATICS

14th July, 2014

INSTRUCTIONS: (1) Write down your **Computer number** and the **TG number** on each answer booklet used.

(2) Answer any **Five (5)** questions only.

(3) Show all essential working to avoid loss of marks.

(4) Write down the **question numbers** of all the questions attempted in the first column on the cover of the main answer booklet.

(5) Calculators and tables are **not allowed** in this paper

TIME ALLOWED: Three (3) hours.

Q1. (a) The sets A, B, C all intersect and U is the universal set. Shade the part described by the set $[B \cup (A' \cap C')]'$ in a Venn diagram.

(b) The functions f and g are defined by

$$f(x) = \frac{4}{x-2}, \quad x \in \mathbb{R}, x \neq 2$$

$$g(x) = x + 2, \quad x \in \mathbb{R}.$$

Find

(i) the domain of the function $g \circ f$

(ii) $(g \circ f)^{-1}(x)$.

(c) Use Cramer's rule to solve the system of equations:

$$\begin{aligned} x + 2y + 3z &= -2 \\ -2x \quad\quad + \; z &= 3 \\ x - 3y \quad\quad &= 5 \end{aligned}$$

(d) Find the equation of the tangent to the curve

$$y = x^2 - 9x^{-1},$$

at the point $(3, 6)$.

Q2. (a) Find the solution set of each of the following inequalities:

(i) $|3x-2|>4$

(ii) $\dfrac{2}{x-3}<\dfrac{3}{2x-1}$

(b) Find the equation of a circle which passes through the points (1,4), (7,5) and (1,8).

(c) (i) Find the period and phase shift for the function

$$f(x)=1-3\sin(2x+\pi),$$

and hence or otherwise,

(ii) sketch the curve for $-\pi\le x\le\pi$.

(d) Differentiate the function $f(x)=\dfrac{2}{\sqrt{3x-1}}$ from first principle.

Q3. (a) Evaluate the limits:

(i) $\lim\limits_{x\to 3}\dfrac{x^3-27}{x^2-9}$ (ii) $\lim\limits_{x\to+\infty}\dfrac{2x^2+1}{6+x-3x^2}$

(b) The line $y=5x-13$ meets the circle $(x-2)^2+(y+3)^2=26$ at the points A and B.

(i) Show that A has coordinates $(1,-8)$ and B has coordinates $(3,2)$.

M is the midpoint of the line AB.

(ii) Find the equation of the line which passes through M and is perpendicular to the line AB. Write your answer in the form $ax+by+c=0$, where a, b and c are integers.

(c) Express each of the following in the form $r(cos\theta+isin\theta)$:

(i) $(3-3i\sqrt{3})(1-i)$

(ii) $\dfrac{-2-i\sqrt{3}}{-2+i\sqrt{3}}$

(d) Show that $x=0$ is a solution of the equation

$$\begin{vmatrix} x-1 & 4 & -1 \\ 1 & x+2 & 1 \\ 2x-4 & 4 & x-4 \end{vmatrix}=0,$$

and find the other two roots.

Q4. (a) Given the sets $A=[-7,3]$, $B=[-3,3)$, $C=(-1,6)$ and let $\mathbb{R}$ be the universal set. Find each of the following sets and represent it on a number line:

(i) $A-B$

(ii) $A\cup(B\cap C)'$

(b) Evaluate each of the following integrals:

(i) $\int\frac{3x}{\sqrt{1-2x^2}}dx$

(ii) $\int\frac{x^3+3}{x^2-1}dx$

(iii) $\int_0^{\pi/2}x\cos x dx$

(c) (i) Given that $A=\begin{pmatrix}1&1&1\\1&3&6\\1&2&3\end{pmatrix}$, find A^{-1}.

(ii) Hence or otherwise, find X given that $Y=AX$ and $Y=\begin{pmatrix}1\\2\\1\end{pmatrix}$.

(d) Solve the equation

$$|x^2-3x|=-4x+6$$

Q5. (a) For each of the following, find $\frac{dy}{dx}$:

(i) $y=e^{4x}(3x^2-5)^6$

(ii) $x^2+\sin y-y^2=7$

(b) The roots of the equation

$$9x^2+6x+1=4kx,$$

where k is a real constant, are donated by α and β.

(i) Show that the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is

$$x^2+6x+9=4kx.$$

(ii) Find the set of values of k for which α and β are real.

(c) Solve each of the following trigonometric equations for x, if $0\le x\le 2\pi$:

(i) $\cos^2 x-\sin x-1=0$

(ii) $\cos 2x+3\sin x-2=0$

(d) Use the principle of mathematical induction to prove that for $n\in\mathbb{Z}^+$,

8^n-1 is divisible by 7.

Q6. (a) (i) Express $(1-i)^6$ in the form $p+iq$, where p and q are integers.

(ii) Expand $\frac{3}{1+3x}$ as a series in ascending powers of x, up to and including the term in x^3, indicating the range of values of x for which the expansion is valid.

(b) Solve the equation for x:

$$3^{2x+1}+5=16(3^x).$$

(c) A cylindrical can, with no lid, has a circular base of radius r cm. The total surface area of the can is $300\pi\ cm^2$.

(i) Show that the volume $V cm^3$ of the can is given by

$$V=\frac{\pi}{2}r(300-r^2).$$

(ii) Find the value of r for which V is maximum.

(d) Prove each of the following identities:

(i) $\cot\theta+\tan\theta\equiv\sec\theta\csc\theta$

(ii) $\frac{\cos A+\tan A}{\sin A\cos A}\equiv\csc A+\sec^2 A$

Q7. (a) Given that $(3x+2)$ is a factor of $f(x)=3x^3+Ax^2-4x-4$.

(i) Show that $A=5$.

(ii) When $A=5$, factorize $f(x)$ completely.

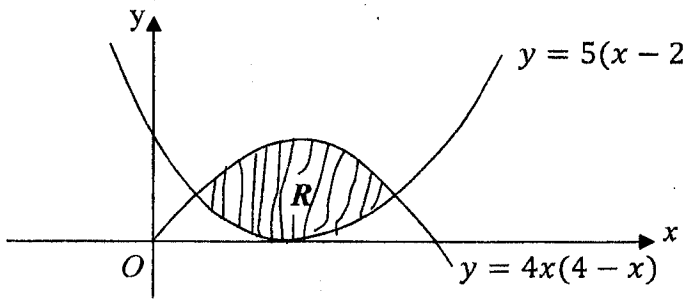
(iii) Hence, sketch the graph of $y=f(x)$, indicating the x- and y - intercepts and the turning points.

(b) Determine whether the function $f(x)=2x-5x^3$ is.even or odd or neither.

(c) Solve the equation

$$z^3+32\sqrt{3}+32i=0$$

(d) The diagram shows the shaded region ***R*** which is bounded by the curves $y=4x(4-x)$ and $y=5(x-2)^2$. Find the area of the shaded region ***R***.



END OF EXAMINATION

The University of Zambia

School of Natural Sciences

Department of Mathematics and Statistics

2012 Academic year

Second Semester Examinations

M112 Mathematical Methods II

Time Allowed: Three (3) Hours **19th August 2013**

Instructions:

1. You must write your **Computer Number**, Your **TG Number** on each answer booklet used.
2. Indicate the number of each question attempted in the first column on the right side of the main answer booklet
3. There are **seven (7)** questions in this paper. You must answer any **five (5)** questions. All questions carry equal marks.
4. No **Calculators** to be used

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1. (a) Solve the following equations;

(i) $2^{2x} - 5(2^{x+1}) + 16 = 0$

(ii) $\log_2 x + 4\log_x 2 = 5$

(b) (i) Find the term independent of x in the expansion of

$$\left(\frac{1}{3x} - \frac{3x^2}{2}\right)^9$$

(ii) Given that $y = e^x + x^2$. Prove that $\frac{d^2y}{dx^2} = y + 2 - x^2$

(c) Show that $\int_1^{\sqrt{3}} \frac{x+3}{\sqrt{4-x^2}} dx = \frac{\pi}{2} + \sqrt{3} - 1$

2. (a) (i) Express $\frac{x^2+x+1}{(x+1)(x^2+1)}$, in partial fractions

(ii) Show that $\int_0^1 \frac{x^2+x+1}{(x+1)(x^2+1)} dx = \frac{3}{4}\ln 2 + \frac{\pi}{8}$

(b) The point (6,3) divides the line segment PQ where P(4,5) and Q(x,y) in the ratio 2:5 internally. Find

(i) the coordinates of Q

(ii) the coordinates of the midpoint of PQ

(c) Use Mathematical induction to prove that $x^{2n} - y^{2n}$ is divisible by x – y for every positive integer values of n.

3. (a) (i) Given that $A=\begin{pmatrix}1 & 0 & 2\\ t & 3 & 1\\ -2 & -1 & 1\end{pmatrix}$, find the value of t if the matrix A is singular.

(ii) Given that $B=\begin{pmatrix}0 & -1 & 1\\ 2 & -1 & 2\\ 1 & 0 & 1\end{pmatrix}$. Show that $B^3 = I_3$

(iii) Use (ii) above to find B^{-1}

(b) (i) By using De Moivrés' Theorem, or otherwise , show that $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$

(ii) Find the roots of the equation $z^5 - 1 = 0$ in the form $r(\cos\theta + i\sin\theta)$ and show them on an argand diagram

(c) (i) Find in Cartesian form an equation of an equation of the circle which passes through the points (2,0), (8,0), and (10,4).

(ii) Prove that the y-axis is a tangent to the circle in (c)(i) above.

(iii) State the coordinates of the point of contact of the tangent in (ii) to the circle in (i) above.

4. (a) For the curve $y=\frac{x^2+9}{x^2-1}$, find

(i) The turning point(s) of the curve y

(ii) The vertical asymptotes

(iii) The Horizontal asymptotes

(iv) Sketch the curve .

(b) (i) The angle between the vector i + j and $2i + j + \lambda k$ is $\frac{\pi}{4}$, find the possible values of λ.

(ii) Solve the equation $3\sinh^2 x - 2\cosh x - 2 = 0$

(c) (i) Use Binomial theorem to expand $(3+10x)^4$, giving each coefficient as an integer.

(ii) Use your expansion in (i) above with appropriate value of x, to find the exact value of $(1003)^4$

5. (a) The curve C is given by $y=\frac{15-2x-x^2}{4}$. The line with equation x +2y =3 Meets the curve C at the points P and Q. Denote the region bounded by C and the line PQ by R.

(i) Determine the coordinates of P and Q

(ii) Sketch the curve C and the line PQ

(iii) Find the area of region R.

(b) Find (i) $\int \cos^3 x dx$

(ii) $\int x(3x+1)^{-\frac{1}{2}} dx$

(iii) $\int \frac{dx}{\sqrt{x}(3+\sqrt{x})^3}$

(c) Using Mathematical Induction prove that

$1^2+3^2+5^2+.....+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$, for all positive values of integer n.

6. (a) Let $A=\begin{pmatrix}1 & 0 & 1\\ 3 & 1 & 1\\ 4 & 2 & 7\end{pmatrix}$

(i) Find A^{-1}

(ii) Hence, solve the system of equations

$x+z=2$

$3x+y+z=3$

$4x+2y+7z=1$

(b) Find the equation of the circle whose centre is at (-2,-5) and passes though the Point (6,1).

(c) Given that $y=x^3-x^2-x+1$, find

(i) The stationary points

(ii) State the nature of the stationary points

(iv) Determine the intervals where the curve y is increasing and decreasing.

(v) Sketch the curve of y

7. (a) The variable point P(x,y) moves in such a way that $AP^2 = 4BP^2$ where A is the point (1,3) and B is the point (4,-3)

(i) Show that P lies on the circle C with equation

$x^2 + y^2 - 10x + 10y + 30 = 0$

(ii) The line OT is a tangent to the circle C, where O is the origin and T lies on the circle. Calculate the length OT.

(b) (i) Find the modulus and argument of the complex number $\frac{5+i}{3-2i}$

(ii) Solve the equation $Z^4 - \frac{5+i}{3-2i} = 0$, giving your answer in the form

$r(\cos\theta + i\sin\theta)$.

(a) (c) (i) USING CRAMER'S RULE.

$$X + Y + 2Z - 2 = 0$$
$$-3X + Y + 2Z - 1 = 0$$
$$6X + 2Y + Z + 4 = 0$$

(ii) Given that **a** = –i + 2j +-5k and **b** = 5i – 2j + k , find a unit vector which is perpendicular to both **a** and **b**

END of EXAMINATION

The University of Zambia
School of Natural Sciences
Department of Mathematics & Statistics

2014/15 ACADEMIC YEAR FINAL EXAMINATIONS

MAT1100 – FOUNDATION MATHEMATICS

22nd June, 2015

INSTRUCTIONS: (1) Write down your **Computer number** and the **TG number** on each answer booklet used.

(2) There are seven questions in this paper. **Answer any Five (5)** only.

(3) Show all essential working to avoid loss of marks.

(4) Write down the **question number** of each question attempted in the first column on the cover of the main answer booklet.

(5) Calculators and tables are **not allowed** in this exam.

TIME ALLOWED: Three (3) hours.

Q1. (a) The sets A, B, C all intersect and U is the universal set. Shade the part described by the set $(B \cup C) - (A \cap B)$ in a Venn diagram.

(b) (i) Express $f(x) = \frac{4-x}{(1-x)(2-x)}$ into partial fractions.

Hence,

(ii) find $\int f(x)dx$.

(c) Given that $A = \begin{pmatrix} 1 & -2 & -1 \\ 0 & 3 & 1 \\ 2 & 0 & -4 \end{pmatrix}$,

(i) find A^{-1}.

Hence,

(ii) solve the system of equations

$$\begin{aligned} x - 2y - z &= 1 \\ 3y + z &= 0 \\ 2x - 4z &= -8. \end{aligned}$$

(d) Prove that the roots of the equation

$$x^2 + (k+2)x + 2k = 0$$

are real for all real values k.

Q2. (a) (i) Solve the equation

$$|2x - 3| = |x + 3|.$$

(ii) Find the coefficient of x^{11} in the binomial expansion of $(2 - x)^{15}$.

(iii) Express the number $0.12\dot{3}$ in the form $\frac{p}{q}$, where $p, q \in \mathbb{Z}$.

(b) Differentiate the function $f(x) = \sqrt{3x + 1}$ from first principles.

(c) Given that $y = 3 + 3x^2 - x^3$, find the equation of the tangent and of the normal at the point where $x = 4$.

(d) Find the length of the straight line from the centre A of the circle

$$x^2 + y^2 + 4x - 6y - 36 = 0,$$

to point $B(6,9)$.

Q3. (a) Find the solution set of the following inequality

$$\frac{x-2}{2x-3} \geq x.$$

(b) Evaluate the limits:

(i) $\lim_{x \to 2} \frac{x^3-8}{x^2-4}$

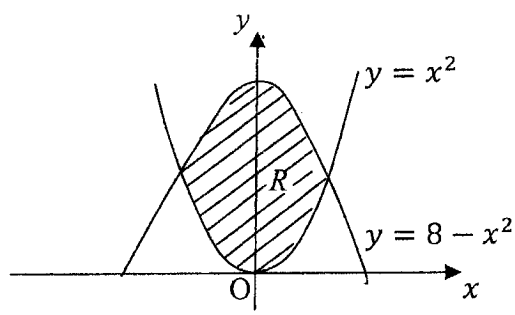
(ii) $\lim_{x \to \infty} \frac{2x^3+1}{2+x-3x^3}$

(c) (i) Solve for x in the equation

$$2^{2x+1} = 3(2^x) - 1.$$

(ii) If $xy = 64$ and $log_x y + log_y x = \frac{5}{2}$, find x and y.

(d) The diagram below shows the shaded region R which is bounded by the curves with equations $y = x^2$ and $y = 8 - x^2$. Find the area of R.



Q4. (a) Given the sets $A = (-3,2], B = [0,7], C = [-5,1)$and universal set $U = (-10,10)$, find each of the following sets and represent them on a number line:

(i) $A \cap B'$

(ii) $A - (B - C)'$

(b) Given that $f(x) = (x + 2)(x - 6)$, sketch the graph of $y = \frac{1}{f(x)}$, indicating the turning points and asymptotes, if any.

(c) Express $\frac{2-3\sqrt{5}}{1+2\sqrt{5}}$ in the form $a + b\sqrt{c}$ where, a, b and c are rational numbers.

(d) Evaluate each of the following integrals:

(i) $\int \frac{x+2}{x^2(x-1)} dx$

(ii) $\int_0^{\pi} xsinx\, dx$

Q5. (a) For each of the following, find $\frac{dy}{dx}$:

(i) $y = e^{2x} ln(2x^2 - 1)$

(ii) $coshy - 4sinh^3(x) = 7.$

(b) Evaluate the integral

$$\int x(x^2 - 4)^{\frac{1}{3}} dx.$$

(c) (i) Factorize $f(x) = x^3 - 3x^2 - 10x + 24$ completely.

(ii) Express $z = \frac{4+2i}{(1-2i)^2}$ in the form $a+ib$ where $a,b \in \mathbb{R}$.

(d) Find the period and phase shift for the function

$$f(x) = -1 + 3\cos(2x + \pi),$$

and hence or otherwise, sketch the curve for $-\pi < x \leq \pi$.

Q6. (a) Use the principle of mathematical induction to prove that for $n \in \mathbb{Z}^+$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{1}{4}n^2(n+1)^2.$$

(b) Determine whether the function $f(x) = cosx - secx$ is even or odd or neither.

(c) (i) A solid circular cylinder has fixed volume of $1000cm^3$. Show that the total surface area $A\ cm^2$ of the cylinder is related to the base radius $x\ cm$ of the cylinder by the equation

$$A = 2\pi x^2 + \frac{2000}{x}.$$

(ii) Given that x varies, show that A has a minimum when $x^3 = \frac{500}{\pi}$.

(d) (i) Show that

$$\cos(A-B) - \cos(A+B) = 2sinAsinB.$$

(ii) Hence show that

$$\cos 2x - \cos 4x = 2sin3xsinx.$$

(ii) Find all solutions in the range $0 \leq x \leq \pi$ of the equation

$$\cos 2x - \cos 4x = sinx,$$

giving all your solutions in multiples of π.

Q7. (a) The function f is defined by

$$f(x) = 3x^3 + 6x^2 - 5x - 4.$$

(i) Find the intervals in which the function is decreasing and in which it is increasing.

(ii) Find the maximum and minimum turning points.

(b) Find the exact value of x for which

$$\cos\left(\arcsin\left(\frac{\sqrt{3}}{2}\right)\right) = x + \sqrt{2}.$$

(c) A function f is defined by

$$f(x) = \frac{x-1}{x},\ x \in \mathbb{R}, x \neq 0, x \neq 1.$$

Find (i) $(fof)(x)$

(ii) $(fofof)(x)$

(iii) $f^{-1}(x)$

(d) Use Cramer's rule to solve the system of equations:

$$\begin{aligned} x + 2y + 3z &= -2 \\ -2x \quad\quad + \ z &= 3 \\ x - 3y \quad\quad &= 5. \end{aligned}$$

END OF EXAMINATION

UNIVERSITY OF ZAMBIA

DEPARTMENT OF MATHEMATICS AND STATISTICS

INSTITUTE OF DISTANCE EDUCATION

MAT1100 - FOUNDATION MATHEMATICS

END OF YEAR DEFERRED EXAMINATIONS

2014/2015 ACADEMIC YEAR

TIME ALLOWED: 3 HOURS DATE: 31 JULY, 2015

INSTRUCTIONS

1. There are **Seven(7)** questions in this paper.
   Attempt any **Five(5)** questions.
2. Write only your **computer number** on each answer booklet you have used.
3. Calculators and use of Tables are not allowed in this paper.

1. **(a)** (i) Express the complex number $\frac{2-5i}{3-i}$ in the form $a+ib$ where a and b are rational numbers.

   (ii) Given that $U = \mathbf{R}$ is the universal set, $A = [-\infty, -2)$ and $B = (-12, 15)$, find the set $(A - B) \cup (B - A)$.

   (iii) Find the term independent of x in the expansion $\left(x^2 - \frac{1}{x}\right)^{18}$.

   (b) (i) Given that α and β are the roots of the equation $2x^2 - 3x - 7 = 0$, find an equation whose roots are α^2 and β^2.

   (ii) Sketch the graph of the quadratic function $f(x) = 2x^2 - 3x + 5$.

   (iii) Let $f(x) = \frac{2x-1}{x+1}$ and $g(x) = \frac{x+1}{2}$ be functions. Find $(fog)^{-1}(x)$

2. **(a)** (i) Express $0.2\overline{53}$ in the form $\frac{a}{b}$ where a and b are integers.

   (ii) The second term in the expansion $(1-x)(1+3x)^n$ is $20x$. Find the value of n.

   (iii) Given that A and B are sets, express $[A \cap (A' \cap B)']'$ in its simplest form.

   (b) (i) Find the center and the radius of the circle $2x^2 + 2y^2 - 4x + 6y + 1 = 0$.

   (ii) Find the equation of the tangent to the circle $x^2 + y^2 - 4x + 6y + 11 = 0$ at the point $(1, -2)$.

   (iii) Given that $\sin A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$ where A is an obtuse angle and B is an acute angle, find $\tan(A + B)$

3. **(a)** (i) Solve the inequality $\frac{2x}{x+3} \leq \frac{1}{2}$.

   (ii) Express $\frac{2}{5+2\sqrt{3}}$ in the form $a + b\sqrt{3}$ where a and b are real numbers.

   (iii) Use Cramer's rule to solve the system of equations:

   $x + y + z = 7$
   $x - y + 2z = 9$
   $2x + y - z = 1$

   (b) Given that the polynomial $f(x) = 2x^3 + 3x^2 + ax + b$ leaves a remainder of -15 when it is divided by $2x - 1$ and that $2x + 3$ is one of its factors,

   (i) find the values of a and b.

   (ii) use synthetic division to find the remainder and the quotient when $f(x)$ is divided by $x + 1$.

   (iii) solve the equation $f(x) = 0$.

4. **(a)** (i) Let $\mathbf{A} = \begin{pmatrix} 2 & 3 & 4 \\ -1 & 1 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 7 & 1 \\ -1 & 1 \\ 2 & -3 \end{pmatrix}$ be two matrices. Find the matrix $\mathbf{BA}$

   (ii) Find the **domain** and the **range** of the function $g(x) = 2 - \sqrt{1 - 3x}$

   (iii) Prove the identity $\sin 3x = \sin x(4\cos^2 x - 1)$.

   (b) Find $\frac{dy}{dx}$ for each of the following:

   (i) $y = (x^2 - 1)^{\frac{5}{3}}$.

   (ii) $y = x^5 \tan x$.

   (iii) $y = \frac{3x-2}{1-4x}$.

5. **(a)** (i) Find the value of the limit $\lim_{x\to 2} \frac{2x^2-5x+2}{x^2+5x-14}$.

   (ii) Given that $\mathbf{u} = 2i - 3j + k$, $\mathbf{v} = i + 4j + 5k$ and $\mathbf{w} = i + j - k$, find $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.

   (iii) Differentiate the function $g(x) = \frac{1}{x}$ from the **First Principle**

   (b) Let $h(x) = \sqrt{3}\sin 2\left(x + \frac{\pi}{4}\right)$ be a function.

   (i) Determine the **period**, the **amplitude** and the **shift** of the function $h(x)$.

   (ii) Solve the equation $h(x) = \frac{3}{2}$ where $-\frac{\pi}{4} \leq x \leq \pi$.

   (iii) Sketch the graph of the function $h(x)$ in the interval $-\frac{\pi}{4} \leq x \leq \pi$

6. **(a)** (i) Express $\frac{5}{(1+3x)(1-2x)}$ into partial fractions.

   (ii) Solve the equation $\cos x = \sin 2x$ for $0 < x < 2\pi$.

   (iii) Sketch the graph of the function $g(x) = 2 - \sqrt{3x+1}$

   (b) (i) Prove using the principle of mathematical induction that $1+2+\ldots+n = \frac{n(n+1)}{2}$ for all $n \in \mathbf{Z}^+$

   (ii) The points $L(-3,\ 1)$ and $M(5,\ 3)$ are the end points of the diameter of a circle, with center N. Find the equation of the circle.

   (iii) Find the cube roots of the complex number $2+2i$ in the form $r(\cos\theta + i\sin\theta)$.

7. **(a)** (i) Calculate the perpendicular distance from the point $P(6,\ -6)$ to the line $3x + 4y - 12 = 0$

   (ii) Solve the equation $2^x - 9 + 8(2^{-x}) = 0$.

   (iii) Sketch the graph of the function $f(x) = x^3 - 3x^2 - 9x + 2$

   (b) Find the following integrals:

   (i) $\int x^2 e^{4x^3} dx$.

   (ii) $\int x^5 \ln x dx$.

   (iii) $\int_0^3 x\,(x+1)^{\frac{1}{2}}\,dx$

End of examination.

THE UNIVERSITY OF ZAMBIA
DEPARTMENT OF MATHEMATICS AND STATISTICS
MAT2110 FINAL EXAMINATION
3 JULY 2015
TIME: THREE HOURS

WRITE YOUR COMPUTER NUMBER ON EACH ANSWER BOOKLET
THERE ARE SEVEN QUESTIONS IN THIS EXAMINATION PAPER
ANSWER ANY FIVE QUESTIONS

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1. **(a)** The equation

$$x^2 - xy + y^2 - 4x - 4y = 20$$

defines a conic section.
(i) Identify the conic section without transforming the equation. [2marks]
(ii) Locate its centre. [11marks]

(b) The parametric equations of a certain curve are given by

$$x = t^2 + 1, \quad y = 2t$$

(i) Identify the curve. [2marks]
(ii) Determine the curvature of the curve at the point $t = 1$. [5marks]

2. **(a)** Find the expansion of the polynomial $P(x) = 12x^2 + 8x^3$ about the point $a = -3/2$. [6marks]

(b) **(i)** Given the function $z = e^{xy}\sin(x + 2y)$, prove that $z_{xy} = z_{yx}$. [6marks]

(ii) Calculate the area enclosed by the curves $y = x^2$ and $y = 3x + 4$. [8marks]

3. **(a)** Find the antiderivative of the function

$$f(x) = \frac{x^2 - 37}{x^2 + x - 12}$$

[10marks]

(b). Plane Π_1 is given by $2x - 3y + 4z = 8$ while plane Π_2 is given by $x + 2y - cz = d$. The planes are perpendicular to one another and the point $(2, 1, -1)$ lies on Π_2.
(i) Calculate the values of c and d. [5marks]
(ii) Determine the distance of the point $(2, 1, -1)$ from Π_1. [3marks]
(iii) Determine the equation of the line perpendicular to Π_2 and passing through the point $(2, 1, -1)$. [2marks]

4. **(a)** **(i)** Show that the only point of intersection between the curves $x^2 + y^2 = 10$ and $y^2 = 3x$ is $(2, \sqrt{6})$. [3marks]

(ii) Determine the angle between the curves at this point. [5marks]

(iii) Use the differental arc length $ds = \sqrt{(dx)^2 + (dy)^2}$ to calculate the length along the circle $x^2 + y^2 = 10$ from the point $(2, \sqrt{6})$ to the point $(3, 1)$. [5marks]

(b) Solve the equation

$$x\frac{dy}{dx} + 3y = 6x^3$$

[7marks]

5. **(a)** The curve $y = x^2$ is rotated about the y axis. Considering only the portion between $y = 1$ and $y = 4$, determine the following.

(i) The volume of revolution. [3marks]

(ii) The surface of revolution. [4marks]

(b) **(i)** The equation of motion of a certain damped harmonic oscillator is

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 5x = 0$$

Find the position as a function of time if the initial conditions are $x(t = 0) = 5$ and $v(t = 0) = 0$. [8marks]

(ii) Express the point of intersection of the line $x - y = 0$ and the parabola $y - 2x^2 = -1$ in polar coordinates. [5marks]

6.(a) The vertices of a triangle are at the points $A(1, 2, 4)$, $B(2, 2, 2)$ and $C(3, 0, 3)$.

(i) Determine the area of the triangle. [5marks]

(ii) Find the unit normal to the plane of the triangle. [2marks]

(iii) Find the perimeter of the triangle. [3marks]

(b) **(i)** Solve the differential equation $\left(\sqrt{x^2 - y^2} - 2y\right) dx + 2xdy = 0$ .. [6marks]

(ii) The velocity $v = dx/dt$ of a particle is given by $v = 8\sin^3 t \cos^2 t$. Find the position if $x(t = 0) = 0$. [4marks]

7. **(a)** **(i)** Find the sum of the series

$$\sum_{k=0}^{\infty} \left(\frac{5\sqrt{2}}{8}\right)^k$$

[4marks]

(ii) A particle travels along the curve $y^2 = 4x - 1$. Determine how close to the origin it passes. [6marks]

(b) (i) Determine the centre of mass of a metre rule of density $\mu(x) = 0.3x^2$ kg/m which is lying between $x = 0$ and $x = 1$ m. [4marks]

(ii) Find the Maclaurin series of $\ln[(1-x)(1-2x)]$. [6marks]

END OF EXAMINATION