

UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

MAT 1100 FOUNDATION MATHEMATICS TUTORIAL SHEET 2 -2022

1. Simplify the following into their simplest surd form: (a) $\sqrt[4]{48}$ (b) $\sqrt{75x^3}$
(c) $\sqrt[4]{(5x)^4}$ (d) $\sqrt{\frac{7x^2}{9}}$ (e) $\sqrt[3]{24}$ (f) $\sqrt[5]{32a^{11}}$ (g) $\sqrt[3]{-40x^6}$ (h) $\sqrt[5]{\frac{64}{y^5}}$ (i) $\sqrt{(4x^3)^2}$.
2. Without using a calculator, evaluate the following expressions involving radicals: (a) $-\sqrt[3]{-27}$ (b) $\sqrt[4]{-16}$ (c) $\frac{\sqrt[4]{81}}{6}$ (d) $\sqrt[3]{\frac{-27}{125}}$ (e) $\sqrt[3]{0}$ (f) $\sqrt[6]{-1}$.
3. Simplify the following expressions involving radicals: (a) $2\sqrt{x} - 3\sqrt{x}$
(b) $3\sqrt{x+1} + 10\sqrt{x+1}$ (c) $5\sqrt{50} + 3\sqrt{8}$ (d) $2\sqrt{27} - \sqrt{75}$
(e) $2\sqrt{108x} + \sqrt{189x}$ (f) $2\sqrt{4y} - 2\sqrt{9y}$ (g) $\sqrt{5+3}$ (h) $\sqrt{3} \cdot \sqrt[4]{3}$
4. Rationalize the denominator of each of the following:
(a) $\sqrt[7]{\frac{1}{a^2}}$ (b) $\frac{1}{\sqrt[3]{x^4}}$ (c) $\frac{3+\sqrt{8}}{2+\sqrt{18}}$ (d) $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$ (e) $\frac{h}{\sqrt{x+h}-\sqrt{x}}$ (f) $\frac{8}{\sqrt[3]{2}}$
(g) $\frac{2x}{5-\sqrt{3}}$ (h) $\frac{5x}{-5+2\sqrt{14}}$ (i) $\frac{5}{\sqrt[3]{(5x)^2}}$ (j) $\frac{5}{-5+2\sqrt{10}}$ (k) $\frac{3}{\sqrt{5}-\sqrt{6}}$ (l) $\frac{2}{x+\sqrt{7}}$ (m) $\frac{2}{\sqrt{x+7}}$.
5. Rationalize the numerator of each of the following:
(a) $\frac{\sqrt{a-3}}{a-9}$ (b) $\frac{\sqrt{x-y}}{x-y}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\sqrt{x+h} - \sqrt{x}$
6. Express each of the following in the form $A + B\sqrt{C}$ where A, B are rational and C is a positive integer: (a) $\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{4}} + \frac{1}{\sqrt{50}}$ (b) $\sqrt{3} + 2 + \frac{1}{\sqrt{3-3}}$
7. Express the following in the form $a + ib$ where $a, b \in \mathbb{R}$
(a) $\sqrt{-225}$ (b) $\sqrt{-36} + \sqrt{-49}$ (c) $5 + \sqrt{-4}$ (d) $7 - \sqrt{-7}$ (e) $-\sqrt{\frac{-25}{16}}$.
8. Simplify the following : (a) i^{11} (b) i^{105} (c) $\frac{5}{i} + \frac{2}{i^3} - \frac{20}{i^{18}}$ (d) $(4i)^3 - 4i^3$
(e) $(1+i)^{10} - (1-i)^{10}$
9. Find the real and imaginary parts of each of the following: (a) -3 (b) $-4i$ (c) $\frac{1-i}{1+i}$
(d) $(2+i)(4+3i)$ (e) $\frac{3-i}{2+i} + \frac{3+i}{2-i}$ (f) $(1+i)^2(1-i)^2$ (g) $3i + \frac{1}{2-i}$
10. Simplify each of the following, giving your answer in the form $a + bi$, where $a, b \in \mathbb{R}$:
(a) $(2-3i) - (1+2i)$ (b) $(3-i)^2$ (c) $\frac{(1+2i)^2}{2+i}$ (d) $\frac{1+2i}{(2+i)^2}$ (e) $\frac{1-i}{(1+i)(2-i)}$.
11. Given that $z_1 = 8 + 2i$, $z_2 = 2 + i$, $z_3 = 3 + i$, find the answer to each of the following in the form $a + bi$, where $a, b \in \mathbb{R}$:

- (a) $\frac{z_1 z_2}{z_3}$ (b) $\frac{z_1 - z_2}{z_1 + z_3}$ (c) $\frac{(z_1 - 2z_3)^2}{(3z_2)^2}$.
12. Find the real values of x and y such that $(x + iy)^2 = -8 + 6i$.
13. Find real numbers x and y such that $3x + 2iy - ix + 5y = 7 + 5i$.
10. If z is a complex number, solve the equation $z^2 + 2z + 5 = 0$.
11. State whether each of the following operation is a binary operation on \mathbb{Z} , the set of integers:
- (a) $a * b = a - 2b$ (b) $a * b = \sqrt{a + b}$ (c) $a * b = (a - b)^2$ (d) $a * b = a^2 - b^2$
- What happens when we replace the set \mathbb{Z} above with \mathbb{R} ?
12. Let $*$ be a binary operation on the set of real numbers and a, b, c be real numbers. Given that the operation is defined by $a * b = (a - b)^2 - 3ab$.
- (a) Does the operation possess the commutative and/or associative property?
- (b) For this binary operation calculate (i) $-2 * (3 * 4)$ (ii) $(-4 * 3) * 2$.
13. The binary operation $*$ is defined on the set of real numbers by $a * b = 3(a - b)^2$.
- (a) Show that the binary operation is commutative.
- (b) Determine whether or not $*$ is associative.
- (c) Find (i) $(-1 * 4) * 0$ (ii) $-1 * (4 * 0)$ (iii) $(-1 * -1) * (0 * 1)^2$.
14. Which of the following sets of ordered pairs represent functions:
- (a) $\{(1,4), (3,4), (7,3)\}$ (b) $\{(1,2), (1,3), (2,3)\}$ (c) $\{(4,3), (4, 7), (3,4)\}$
15. Let $A = \{0,1,2,3\}$ and $B = \{-2, -1, 0,1,2\}$. Which of the following sets of ordered pairs represent a function from A to B ?:
- (a) $\{(0,1), (1, -2), (2, 0), (3,2)\}$ (b) $\{(0, -1), (2,2), (1, -2), (3, 0), (1,1)\}$
- (c) $\{(0,0), (1, 0), (2, 0), (3, 0)\}$ (d) $\{(0, 2), (3, 0), (1,1)\}$. You may represent the ordered pairs in form of arrow diagrams to see the picture.
16. Given that $f(t) = \sqrt{25 - t^2}$, find and simplify each of the following:
- (a) $f(3)$ (b) $f(5)$ (c) $f(x + 5)$ (d) $f(2x)$.
17. Given that $f(x) = \frac{1}{x^2 - 2x}$, find and simplify each of the following:
- (a) $f(1)$ (b) $f(-3)$ (c) $f(t+1)$ (d) $f(t^2)$.
18. Find the domain and range of each of the following functions:
- $f: \{(0,1), (3,4), (5,7), (8, 11), (9,12)\}$, $g: \{(10, -4), (20, 1), (30, 6), (40,9), (50, 13)\}$.

