

The University of Zambia
School of Natural Sciences
Department of Mathematics & Statistics

MAT1100 Foundation Mathematics Test 1

27th March, 2021

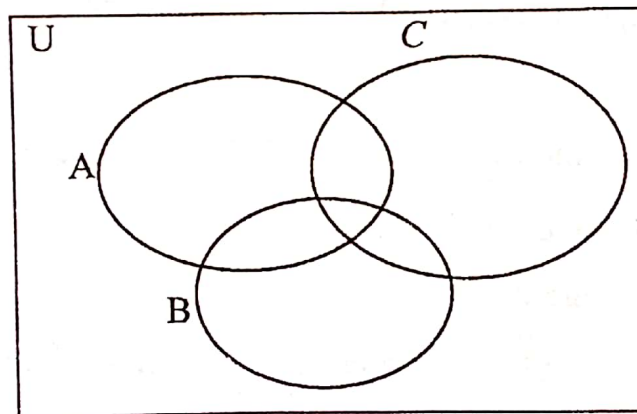
INSTRUCTIONS: 1. Write your Name, Computer Number and TG on the answer scripts.

2. There are four questions. Answer ALL.

3. Show all necessary working to earn full marks.

TIME ALLOWED: 2 hours

- 1.(a) (i) The subsets A, B, C of the universal set U are given as shown in the Venn diagram below. Shade the part described by the set $(A \cap B) \cup [C - (A \cup B)]$ in a Venn diagram.



[5]

- (ii) Simplify as far as possible the set $X - (X - Y)'$, given that the sets X and Y are disjoint. [5]

- (b) (i) Let $A = (-\infty, 3)$, $B = [-3, 3)$, $C = (-1, 6)$ and \mathbb{R} be the universal set. Find the set $[A \cap (B - C)]'$, and display it on the number line. [5]

- (ii) Simplify and express $\frac{1}{1 + \sqrt{432} - 3\sqrt{48} + \sqrt{27}}$ in the form $p + q\sqrt{3}$, where p and q are rational numbers. [5]

2. (a) (i) Express the number $12.34\overline{11}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. [5]

- (ii) Given that $\sqrt{2}$ is irrational, determine whether or not $\sqrt{2} - \frac{22}{7}$ is irrational. [5]

(b)(i) Find the real and imaginary parts of $z = \frac{5}{i} - \frac{20}{i^{18}} + (4i)^3 - \sqrt{-25}$. [5]

(ii) Express $z = \frac{(1+2i)^2}{2+i}$ in the form $z = a + ib$, where $a, b \in \mathbb{R}$. [5]

3. (a) (i) Rationalise the denominator of $\frac{7}{\sqrt[3]{2}}$. [5]

(ii) If $z = x + iy$ is a complex number, find values of $x, y \in \mathbb{R}$ such that

$$\overline{z - 2} + 2z = 1 - i. \quad [5]$$

(b) The binary operation $*$ on \mathbb{R} is defined by

$$a * b = a^2 + b^2 - 2(a - b).$$

(i) Determine whether $*$ is commutative or not. [5]

(ii) Find $(-1 * 1) * 2$ and $-1 * (1 * 2)$. Is $*$ associative? [5]

4. (a) (i) Let $A = \{0, 1, 2, 3\}$ and $B = \{a, b, c, d\}$.

Determine, giving reasons, whether the following relation represents a function:

$R = \{(1, b), (0, b), (2, c), (3, a)\}$. Further, find the range of the relation R .

[5]

(ii) Find the domain of the function given by $f(x) = \frac{x-3}{1-\sqrt{x-3}}$ and display the domain on the number line. [5]

(b) (i) The functions f and g are defined by

$$f(x) = \frac{2x}{4-x}, \quad x \in \mathbb{R}, x \neq 4 \quad \text{and} \quad g(x) = \frac{1}{x^2}, \quad x \in \mathbb{R}, x \neq 0$$

Find the function $(f \circ g)(x)$ and determine its domain. [5]

(ii) Given that $f(x) = x + 7$ and $g(x) = \sqrt{x}$, find $(f \circ g)^{-1}$ and determine its domain. [5]

END OF TEST