

The University of Zambia
School of Natural Sciences
Department of Mathematics & Statistics

MAT1100 Foundation Mathematics Test 1

9th November, 2020

- INSTRUCTIONS:** (1) Answer ALL the **Four (4)** questions.
(2) Show all essential working to avoid loss of marks.

TIME ALLOWED: Two (2) hours.

CALCULATORS ARE NOT ALLOWED IN THIS PAPER.

1. (a) (i) The sets A, B, C all intersect and U is the universal set. Shade the part described by the set $B \cup (A \cap C)'$ in a Venn diagram. [5]
- (ii) Simplify as far as possible the set $P - (P - Q)'$, given that the sets P and Q intersect. [5]
- (b) (i) Let $A = (-\infty, 3)$, $B = [-3, 3)$, $C = (-1, 6)$ and \mathbb{R} be the universal set. Find the set $A \cap (B - C)'$. [5]
- (ii) Simplify and express $\frac{2\sqrt{3}}{1 + \sqrt{12} + \sqrt{75} - \sqrt{108}}$ in the form $p + q\sqrt{3}$, where p and q are integers. [5]
2. (a) (i) Express in simplest form the number $2.34\overline{24}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. [5]
- (ii) Let $\frac{i-2}{z} = (5-2i)^2$. Find z , expressing your answer in the form $p+iq$, where p and q are rational numbers. [5]
- (b) The binary operation $*$ on \mathbb{R} is defined by
- $$a * b = (a - b)^2 - 2ab.$$
- (i) Show that the operation $*$ is commutative. [5]
- (ii) Find $(-1 * 3) * 2$. [5]

3. (a) (i) The functions f and g are defined by

$$f(x) = \frac{x}{x-1}, \quad x \in \mathbb{R}, x \neq 1 \text{ and } f(x) = 2x + 3.$$

Find the function $(gof)^{-1}(x)$ and determine its domain.

[5]

- (ii) Determine whether the function below is even or odd or neither even nor odd:

$$f(x) = \frac{x^3}{2x^2-3}.$$

[5]

- (b) (i) Complete the square of the quadratic function $f(x) = 2x^2 - 12x + 37$, and hence, state the turning point of the graph of $y = f(x)$. [5]

- (ii) Factorize the polynomial $f(x) = 4 + 4x - x^2 - x^3$, and hence, sketch the graph of the function $y = f(x)$, stating the x - and y - intercepts. [5]

4. (a) Find the solution set of each of the following inequalities:

(i) $\frac{x}{x-4} \leq 3$. [3]

(ii) $\left| \frac{2x+1}{x-3} \right| > 3$. [5]

- (b) (i) Find the 9th term in the expansion of $\left(2 - \frac{1}{x}\right)^{13}$. [4]

- (i) Express $(1+x)^{\frac{1}{2}}$ as a series up to and including the term in x^3 , and state the values of x for which the expansion is valid. [4]

- (ii) By substituting 0.08 for x in $(1+x)^{\frac{1}{2}}$ and its expansion find $\sqrt{3}$ correct to four significant figures. [4]

END OF TEST