MAT 1100 – Foundation Mathematics

Quiz Solutions

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Note: *I created this to cushion the impact of Covid-19 on your academic performance!!!*

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1. Solve each of the following inequalities:

   (a) $3x^2 > 3 - 8x$

   Solution.

$$
\begin{aligned}
3x^2 + 8x - 3 &> 0 \\
\Longrightarrow 3x^2 + 9x - x - 3 &> 0 \\
\Longrightarrow 3x(x+3) - 1(x+3) &> 0 \\
\Longrightarrow (x+3)(3x-1) &> 0.
\end{aligned}
$$

We now find the critical values by solving the equation $(x+3)(3x-1) = 0$

solving the equation implies that either $x + 3 = 0$ or $3x - 1 = 0$, which implies that either $x = -3$ or $x = \frac{1}{3}$.

Thus, the critical values/points are $x = -3$ and $x = \frac{1}{3}$.

Finally using the critical points we form a table to test where the solution set lies.

Factors	$(-\infty, -3)$	$(-3, \frac{1}{3})$	$(\frac{1}{3}, \infty)$
$x + 3$	-	+	+
$3x - 1$	-	-	+
$(x+3)(3x-1)$	+	-	+

Thus, the solution set is $(-\infty, -3) \cup (\frac{1}{3}, \infty)$ or one can write $\{x \in \mathbb{R} : x < -3 \quad \text{or} \quad x > \frac{1}{3}\}$.

□

(b) $\frac{2x+3}{x-1} - \frac{2x}{x+1} \leq 0$

Solution.

$$\begin{aligned} \frac{2x+3}{x-1} - \frac{2x}{x+1} &\leq 0 \\ \implies \frac{(2x+3)(x+1) - 2x(x-1)}{(x-1)(x+1)} &\leq 0 \\ \implies \frac{2x^2+2x+3x+3-2x^2+2x}{(x-1)(x+1)} &\leq 0 \\ \implies \frac{7x+3}{(x-1)(x+1)} &\leq 0 \end{aligned}$$

Thus, the critical values are $x = -1$,$x = 1$ and $x = -\frac{3}{7}$. We now test for the solution set.

Factors	$x < -1$	$-1 < x < -\frac{3}{7}$	$-\frac{3}{7} < x < 1$	$x > 1$
$7x + 3$	-	-	+	+
$x - 1$	-	-	-	+
$x + 1$	-	+	+	+
$\frac{7x+3}{(x-1)(x+1)}$	-	+	-	+

Thus, the solution set is $(-\infty, -1) \cup [-\frac{3}{7}, 1)$. □

2. Solve each of the following inequalities:

(a) $\left|\frac{x+2}{x}\right| \leq 2$

Solution. By definition we have that

$$\begin{aligned} &\left|\frac{x+2}{x}\right| \leq 2 \\ \implies& -2 \leq \frac{x+2}{x} \leq 2 \\ \implies& -2 \leq \frac{x+2}{x} \quad \textbf{and} \quad \frac{x+2}{x} \leq 2 \\ \implies& -2 - \frac{x+2}{x} \leq 0 \quad \textbf{and} \quad \frac{x+2}{x} - 2 \leq 0 \\ \implies& \frac{-2x-(x+2)}{x} \leq 0 \quad \textbf{and} \quad \frac{x+2-2x}{x} \leq 0 \\ \implies& \frac{-3x-2}{x} \leq 0 \quad \textbf{and} \quad \frac{2-x}{x} \leq 0 \end{aligned}$$

We first find the solution set for $\frac{-3x-2}{x} \leq 0$ which has critical values $x = 0$ and $x = -\frac{2}{3}$ and so using these critical values we test for the solution as shown in the table below.

Factors	$x < -\frac{2}{3}$	$-\frac{2}{3} < x < 0$	$x > 1$
x	-	-	+
$-3x-2$	+	-	-
$\frac{-3x-2}{x}$	-	+	-

Thus, the solution set for $\frac{-3x-2}{x} \leq 0$ is the set $(-\infty, -\frac{2}{3}] \cup (0, \infty)$.

We now also find the solution set for $\frac{2-x}{x} \leq 0$, whose critical values are $x = 0$ and $x = 2$

Factors	$x < 0$	$0 < x < 2$	$x > 2$
$2 - x$	+	+	-
x	-	+	+
$\frac{2-x}{x}$	-	+	-

Thus, the solution set for $\frac{2-x}{x} \leq 0$ is the set $(-\infty, 0) \cup [2, \infty)$.
Hence the solution set for $\left|\frac{x+2}{x}\right| \leq 2$ is

$$\left[\left(-\infty, -\frac{2}{3}\right] \cup (0, \infty)\right] \bigcap \left[(-\infty, 0) \cup [2, \infty)\right] = \left(-\infty, -\frac{2}{3}\right] \cup [2, \infty)$$

. $\square$

(b) $\sqrt{x-3} + 1 \geq \sqrt{x+4}$

Solution. We have that the inequality is defined for $x - 3 \geq 0$ and $x + 4 \geq 0$. Thus the critical values are $x = 3$ and $x = -4$. We now find the other critical value by solving the equation
$\sqrt{x-3} + 1 = \sqrt{x+4}$. Squaring on both sides implies

$$\begin{aligned}
& (\sqrt{x-3} + 1)^2 = (\sqrt{x+4})^2 \\
\Longrightarrow \quad & x - 3 + 2\sqrt{x-3} + 1 = x + 4 \\
\Longrightarrow \quad & 2\sqrt{x-3} = 6 \\
\Longrightarrow \quad & \sqrt{x-3} = 3 \\
\Longrightarrow \quad & x - 3 = 9 \quad \text{by squaring on both sides} \\
\Longrightarrow \quad & x = 12
\end{aligned}$$

So $x = 12$ is our other critical value. Using the critical values we test for the solution as shown in the table below.

$x < -4$, say $x = -7$	$-4 < x < 3$, $x = 2$	$3 < x < 10$, $x = 4$	$x > 12$, $x = 19$
$\sqrt{x-3} + 1 \geq \sqrt{x+4}$	$\sqrt{x-3} + 1 \geq \sqrt{x+4}$	$\sqrt{x-3} + 1 \geq \sqrt{x+4}$	$\sqrt{x-3} + 1 \geq \sqrt{x+4}$
$\sqrt{-7} + 1 \geq \sqrt{-3}$	$\sqrt{-1} + 1 \geq \sqrt{6}$	$\sqrt{1} + 1 \geq \sqrt{8}$	$\sqrt{16} + 1 \geq \sqrt{24}$
Undefined	Undefined	Not True	True

Thus, the solution set is $[12, \infty)$ $\square$

3. Find the first three terms in the expansion of each of the following:

   (a) $(x - 2y)^7$

   Solution. We know that

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + ... + \binom{n}{r} a^{n-r}b^r + ...$$

   Therefore, the first three terms of

$$\begin{aligned}(x-2y)^7 &= x^7 + \binom{7}{1} x^{7-1}(-2y) + \binom{7}{2} x^{7-2}(-2y)^2 \\ &= x^7 - 14x^6y + 84x^5y^2\end{aligned}$$

$\square$

   (b) $(1 - 2y)(2 + 3y)^8$

   Solution. Now the first three terms of $(2 + 3y)^8$ are

$$\begin{aligned}&2^8 + \binom{8}{1} 2^7(3y) + \binom{8}{2} 2^6(3y)^2 \\ &= 256 + 3072y + 16128y^2\end{aligned}$$

   Thus, the first three terms of $(1 - 2y)(2 + 3y)^8$ will come from

$$(1-2y)(256 + 3072y + 16128y^2).$$

   And we have that

$$\begin{aligned}&(1-2y)(256 + 3072y + 16128y^2) \\ &= 256 + 3072y + 16128y^2 - 512y - 6144y^2 - 32256y^2 \\ &= 256 + 2560y + 9984y^2 - 32256y^3\end{aligned}$$

   Thus, the three terms are $256 + 2560y + 9984y^2$ $\square$

4. (a) Find the fifth term in the expansion of $\left(x - \frac{1}{x^2}\right)^8$

   Solution. Given $(a + b)^n$

$$r^{th}\text{term} = \binom{n}{r-1} a^{n-(r-1)}b^{r-1}$$

   Thus for the given binomial, the

$$\begin{aligned}5^{th}\text{term} &= \binom{8}{4} x^{8-4} \left(-\frac{-1}{x^2}\right)^4 \\ &= \frac{70}{x^4}\end{aligned}$$

$\square$

(b) Expand the function $\sqrt{4+2x}$ as a series of ascending powers of x up to and including the term in x^3. Hence, state the range of values of x for which the expansion is valid.

Solution. Now $\sqrt{4+2x} = (4+2x)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left(1+\frac{x}{2}\right)^{\frac{1}{2}}$.
We know that up to x^3

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3.$$

Therefore,

$$\begin{aligned} 2\left(1+\frac{x}{2}\right)^{\frac{1}{2}} &= 2\left[1 + \frac{1}{2}\cdot\frac{x}{2} + \frac{\frac{1}{2}\cdot\frac{-1}{2}\cdot\frac{-3}{2}}{3!}\left(\frac{x}{2}\right)^3\right] \\ &= 2 + \frac{x}{2} - \frac{x^2}{16} + \frac{x^3}{64} \end{aligned}$$

Now the range is $\left|\frac{x}{2}\right| < 1$ which is the set $\{x \in \mathbb{R} : -2 < x < 2\}$. □

You can do all things through Him that gives you strength...