

**MEC 3102 – PRODUCTION ENGINEERING I AND
ELECTRICITY & ELECTRONICS II**

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2nd Series Lecture 2[3]

✓ **Theorem 6** (Associative Laws)

(a) $X + (Y + Z) = Y + (Z + X) = Z + (X + Y)$ and

(b) $X \cdot (Y \cdot Z) = Y \cdot (Z \cdot X) = Z \cdot (X \cdot Y)$

✓ Theorems 6(a) and (b) are further illustrated by the logic diagrams in Figures 4.8 (a) and (b).,

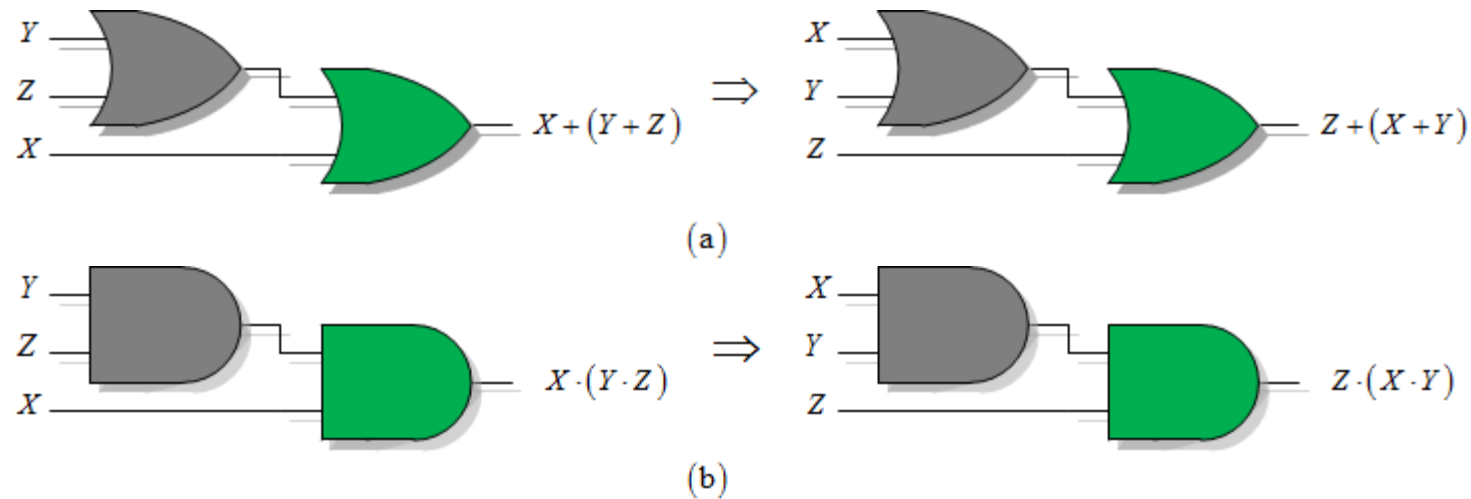


Figure 4.8: Associative laws.

✓ **Example 3** Illustration of Distributive Laws

Simplify the expressions $\bar{A} \cdot \bar{B} + \bar{A} \cdot B + A \cdot \bar{B} + A \cdot B$ and $(\bar{A} + \bar{B}) \cdot (\bar{A} + B) \cdot (A + \bar{B}) \cdot (A + B)$.

✓ **Solution.** Using the theorem 7(a) and (b) we have

$$\bar{A} \cdot \bar{B} + \bar{A} \cdot B + A \cdot \bar{B} + A \cdot B = \bar{A} \cdot (\bar{B} + B) + A \cdot (\bar{B} + B) = \bar{A} \cdot 1 + A \cdot 1 = \bar{A} + A = 1$$

$$\begin{aligned} (\bar{A} + \bar{B}) \cdot (\bar{A} + B) \cdot (A + \bar{B}) \cdot (A + B) &= (\bar{A} + \bar{B} \cdot B) \cdot (A + \bar{B} \cdot B) = (\bar{A} + 0) \cdot (A + 0) \\ &= \bar{A} \cdot A = 0 \end{aligned}$$

✓ **Theorem 8**

$$(a) \quad X \cdot Y + X \cdot \bar{Y} = X \quad \text{and} \quad (b) \quad (X + Y) \cdot (X + \bar{Y}) = X$$

✓ This is a special case of theorem 7. Theorem 8 can be usefully employed to simplify many complex Boolean expressions.

✓ **Example 4** Illustration of theorem 8.

Simplify the expressions

$$\begin{aligned} &A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + A \cdot \bar{B} \cdot \bar{C} \cdot D + A \cdot \bar{B} \cdot C \cdot \bar{D} + A \cdot \bar{B} \cdot C \cdot D \\ &+ A \cdot B \cdot \bar{C} \cdot \bar{D} + A \cdot B \cdot \bar{C} \cdot D + A \cdot B \cdot C \cdot \bar{D} + A \cdot B \cdot C \cdot D \end{aligned}$$

and

$$(A + \bar{B} + \bar{C}) \cdot (A + \bar{B} + C) \cdot (A + B + \bar{C}) \cdot (A + B + C)$$

✓ **Solution.**

- ✓ In the above expression, variables B , C and D are present in **all eight possible combinations**, and the **variable A** is **common** in all eight product terms. By **theorem 8(a)**, this expression reduces to A .
- ✓ By **theorem 8(b)** the **second expression** reduces to A as the variables B and C are present in all four possible combinations in sum terms and **variable A** is the **common factor** in all the terms.

✓ **Theorem 9**

$$(a) \quad (X + \bar{Y}) \cdot Y = X \cdot Y \quad \text{and} \quad (b) \quad X \cdot \bar{Y} + Y = X + Y$$

✓ The **Proof of theorem 9(a)** is quite trivial.

$$(X + \bar{Y}) \cdot Y = X \cdot Y + \bar{Y} \cdot Y = X \cdot Y$$

✓ Theorem 9(b) is the dual of theorem 9(a) and hence stands proved.

✓ **Theorem 10** (Absorption Law or Redundancy Law)

$$(a) \quad X + X \cdot Y = X \quad \text{and} \quad (b) \quad X \cdot (X + Y) = X$$

✓ The **Proof of absorption law** is trivial.

$$X + X \cdot Y = X \cdot (1 + Y) = X$$

✓ **Theorem 10(b)** is the dual of theorem 10(a) and is thus implied.

✓ Theorem 10 implies that, if a smaller term appears in a larger term, then the larger term is redundant.

✓ To illustrate the underlying concept of theorem 10 consider an example.

✓ **Example 5**

By the absorption theorem, we have the expression simplified as

$$A + A \cdot \bar{B} + A \cdot \bar{B} \cdot \bar{C} + A \cdot \bar{B} \cdot C + \bar{C} \cdot B \cdot A = A$$

and

$$(\bar{A} + B + \bar{C}) \cdot (\bar{A} + B) \cdot (C + B + \bar{A}) = \bar{A} + B$$

✓ **Theorem 11**

(a) $Z \cdot X + Z \cdot \bar{X} \cdot Y = Z \cdot X + Z \cdot Y$ and

(b) $(Z + X) \cdot (Z + \bar{X} + Y) = (Z + X) \cdot (Z + Y)$

✓ The **Proof of theorem 11(a)** is done using the **method of perfect induction**.
Theorem 11(b) is the dual of theorem 11(a) and is thus implied.

Proof of theorem 11(a).

X	Y	Z	ZX	ZY	$Z\bar{X}$	$Z\bar{X}Y$	$ZX + Z\bar{X}Y$	$ZX+ZY$
0	0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0	0
0	1	0	0	0	0	0	0	0
0	1	1	0	1	1	1	1	1
1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	1	1
1	1	0	0	0	0	0	0	0
1	1	1	1	1	0	0	1	1

✓ **Theorem 12** (Consensus Theorem)

$$(a) \quad X \cdot Y + \bar{X} \cdot Z + Y \cdot Z = X \cdot Y + \bar{X} \cdot Z \quad \text{and}$$

$$(b) \quad (X + Y) \cdot (\bar{X} + Y) \cdot (Y + Z) = (X + Y) \cdot (\bar{X} + Z)$$

✓ The **Proof of theorem 12(a)** is done using the **method of perfect induction**. Theorem 12(b) is the dual of theorem 12(a) and thus proof is implied.

✓ **By theorem 12**, if in a **given Boolean expression** we can **identify two terms** with one having a **variable** and the **other having its complement**, then the term that is **formed by the product** of the **remaining variables** in the two terms, in the case of a **sum-of-products expression** or by the sum of the remaining variables in the case of a **product-of-sums expression** will be **redundant**.

✓ **Example 6** Illustration of theorem 12.

Prove that

$$\begin{aligned} &A \cdot B \cdot C \cdot D + A \cdot B \cdot \bar{C} \cdot \bar{D} + A \cdot B \cdot C \cdot \bar{D} + A \cdot B \cdot \bar{C} \cdot D \\ &+ A \cdot B \cdot C \cdot D \cdot E + A \cdot B \cdot \bar{C} \cdot \bar{D} \cdot \bar{E} + A \cdot B \cdot \bar{C} \cdot D \cdot E \end{aligned}$$

can be simplified to $A \cdot B$.

✓ **Solution.**

$$\begin{aligned} &A \cdot B \cdot C \cdot D + A \cdot B \cdot \bar{C} \cdot \bar{D} + A \cdot B \cdot C \cdot \bar{D} + A \cdot B \cdot \bar{C} \cdot D \\ &+ A \cdot B \cdot C \cdot D \cdot E + A \cdot B \cdot \bar{C} \cdot \bar{D} \cdot \bar{E} + A \cdot B \cdot C \cdot \bar{D} \cdot E + A \cdot B \cdot \bar{C} \cdot D \cdot E \\ &= A \cdot B \cdot C \cdot D + A \cdot B \cdot \bar{C} \cdot \bar{D} + A \cdot B \cdot C \cdot \bar{D} + A \cdot B \cdot \bar{C} \cdot D \\ &= A \cdot B \cdot (C \cdot D + \bar{C} \cdot \bar{D} + C \cdot \bar{D} + \bar{C} \cdot D) = A \cdot B \end{aligned}$$

- $A \cdot B \cdot C \cdot D$ appears in $A \cdot B \cdot C \cdot D \cdot E$, $A \cdot B \cdot \bar{C} \cdot \bar{D}$ appears in $A \cdot B \cdot \bar{C} \cdot \bar{D} \cdot \bar{E}$ and $A \cdot B \cdot \bar{C} \cdot D$ appears in $A \cdot B \cdot \bar{C} \cdot D \cdot E$.
- As a result, all three five-variable terms are redundant.
- Also, variables C and D appear in all possible combinations, thus redundant.

✓ **Theorem 13 (DeMorgan's Theorem)**

$$(a) \quad \overline{[X_1 + X_2 + X_3 + \dots + X_n]} = \overline{X_1} \cdot \overline{X_2} \cdot \overline{X_3} \cdot \dots \cdot \overline{X_n} \quad \text{and}$$

$$(b) \quad \overline{[X_1 \cdot X_2 \cdot X_3 \cdot \dots \cdot X_n]} = \overline{X_1} + \overline{X_2} + \overline{X_3} + \dots + \overline{X_n}$$

✓ The **Proof of DeMorgan's theorem**. Firstly, let us assume that all variables are in a logic '0' state. In that case

$$\text{LHS} = \overline{[X_1 + X_2 + X_3 + \dots + X_n]} = \overline{[0 + 0 + 0 + \dots + 0]} = \overline{0} = 1$$

$$\text{RHS} = \overline{X_1} \cdot \overline{X_2} \cdot \overline{X_3} \cdot \dots \cdot \overline{X_n} = \overline{0} \cdot \overline{0} \cdot \overline{0} \cdot \dots \cdot \overline{0} = 1 \cdot 1 \cdot 1 \cdot \dots \cdot 1 = 1$$

✓ Therefore, **LHS = RHS**.

✓ Secondly, let us assume that any one of the n variables, say 1, is a logic HIGH state:

$$\text{LHS} = \overline{[X_1 + X_2 + X_3 + \dots + X_n]} = \overline{[1 + 0 + 0 + \dots + 0]} = \overline{1} = 0$$

$$\text{RHS} = \overline{X_1} \cdot \overline{X_2} \cdot \overline{X_3} \cdot \dots \cdot \overline{X_n} = \overline{1} \cdot \overline{0} \cdot \overline{0} \cdot \dots \cdot \overline{0} = 0 \cdot 1 \cdot 1 \cdot \dots \cdot 1 = 0$$

- ✓ Once again **LHS = RHS**.
- ✓ Therefore, theorem 13(a) stands proved. Since theorem 13(b) is the dual of theorem 13(a), the proof is implied.
- ✓ **DeMorgan's theorems** can thus be **interpreted** as follows: the first theorem says that a multi-input **NOR gate** can be implemented as a multi-input bubbled **AND gate**.
- ✓ The second theorem, which is the dual of the first, says that a multi-input **NAND gate** can be implemented as a multi-input bubbled **OR gate**.

✓ **Theorem 14 (Involution Law)**

$$\overline{\overline{X}} = X$$

- ✓ Involution law says that the complement of the complement of an expression leaves the expression unchanged. Also the dual of the dual of an expression is the original expression.

✓ **Theorem 15** (Transposition Theorem)

$$(a) \quad X \cdot Y + \bar{X} \cdot Z = (X + Z) \cdot (\bar{X} + Y) \quad \text{and}$$

$$(b) \quad (X + Y) \cdot (\bar{X} \cdot Z) = X \cdot Z + \bar{X} \cdot Y$$

- ✓ This theorem can be applied to any sum-of-products or product-of-sums expression having two terms, provided that a given variable in one term has its complement in the other.
- ✓ The proof of theorem 15(a) can be done using the method of perfect induction.

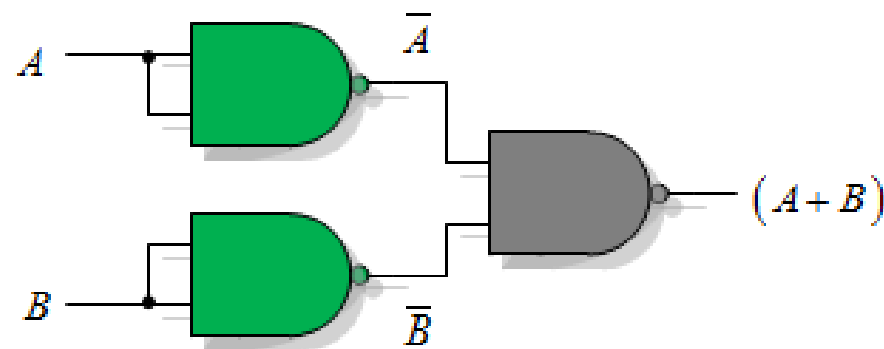


Figure 4.9: Example 7.

✓ Example 7

Starting with the Boolean expression for a two-input OR gate, apply Boolean laws and theorems to modify it in such a way as to facilitate the implementation of a two-input OR gate by using two-input NAND gates only.

Solution.

- A two-input OR gate has the Boolean equation $Y = (A + B)$, where A and B are the input logic variables and Y is the output.
- Now, $(A + B) = \overline{\overline{A + B}}$ Involution law
 $= \overline{\overline{A} \cdot \overline{B}}$ DeMorgan's theorem
 $= \overline{[(\overline{A \cdot A}) \cdot (\overline{B \cdot B})]}$ Idempotent law
- Figure 4.9 shows the NAND gate implementation of a two-input OR gate.

✓ **Example 8**

Apply suitable Boolean laws and theorems to modify the expression for a two-input EX-OR gate in such a way as to implement a two-input EX-OR gate by using the minimum number of two-input NAND gates only.

Solution.

□ A two-input EX-OR gate has the Boolean equation $Y = \bar{A} \cdot B + A \cdot \bar{B}$,

□ Now,
$$\begin{aligned} \bar{A} \cdot B + A \cdot \bar{B} &= \overline{\overline{\bar{A} \cdot B + A \cdot \bar{B}}} && \text{Involution law} \\ &= \overline{\overline{\bar{A} \cdot B} \cdot \overline{A \cdot \bar{B}}} && \text{DeMorgan's theorem} \\ &= \overline{[B \cdot (\bar{A} + \bar{B})] \cdot [A \cdot (\bar{A} + \bar{B})]} \\ &= \overline{(B \cdot A \cdot B) \cdot (A \cdot A \cdot B)} \end{aligned}$$

□ Figure 4.10 shows the NAND gate implementation of a two-input EX-OR gate.

✓ EX-OR gate

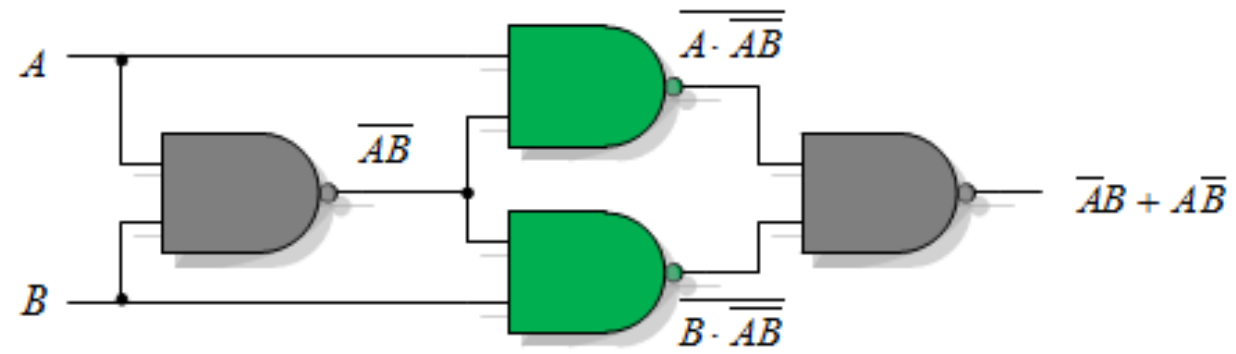


Figure 4.10: Example 8.