MEC 3102 – PRODUCTION ENGINEERING I AND ELECTRICITY & ELECTRONICS II

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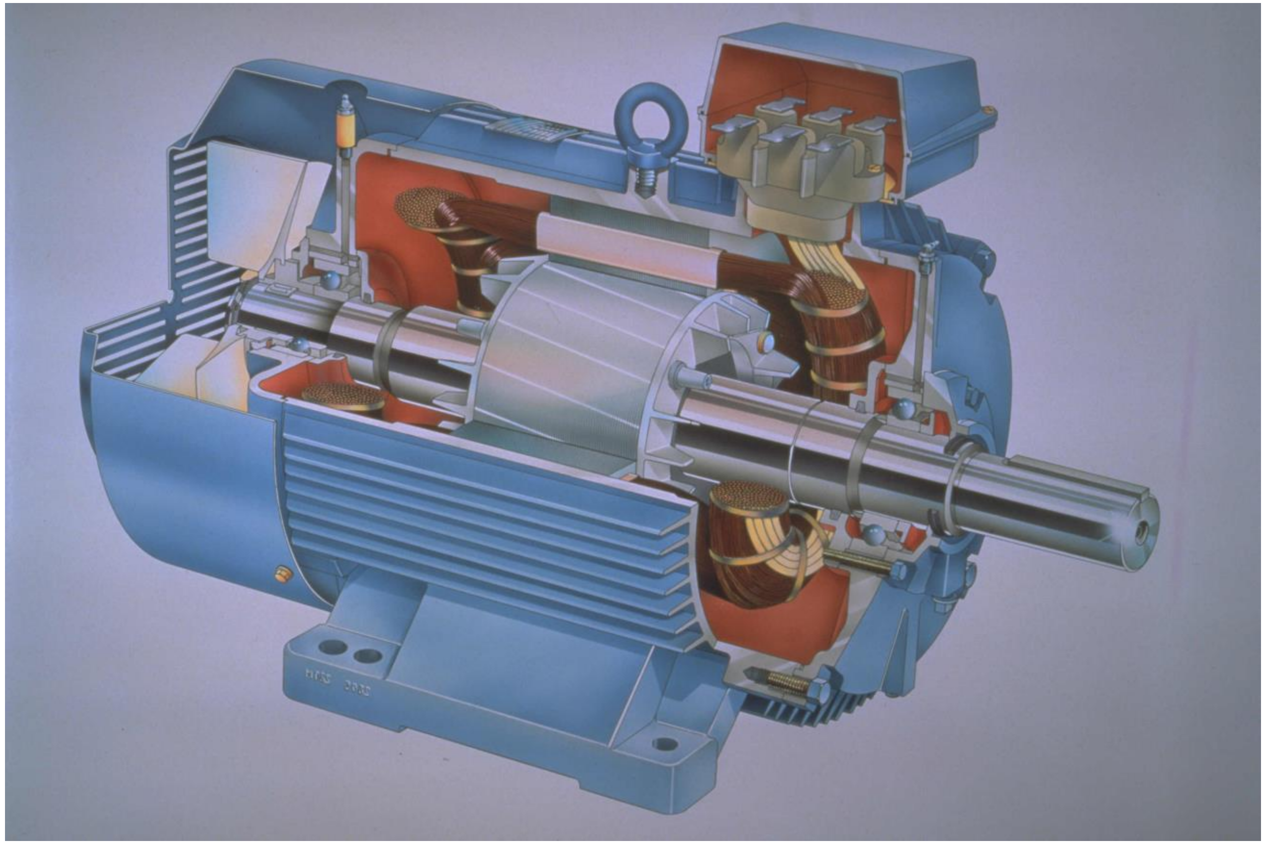
2nd Series Lecture 6[1]

4. Induction motors

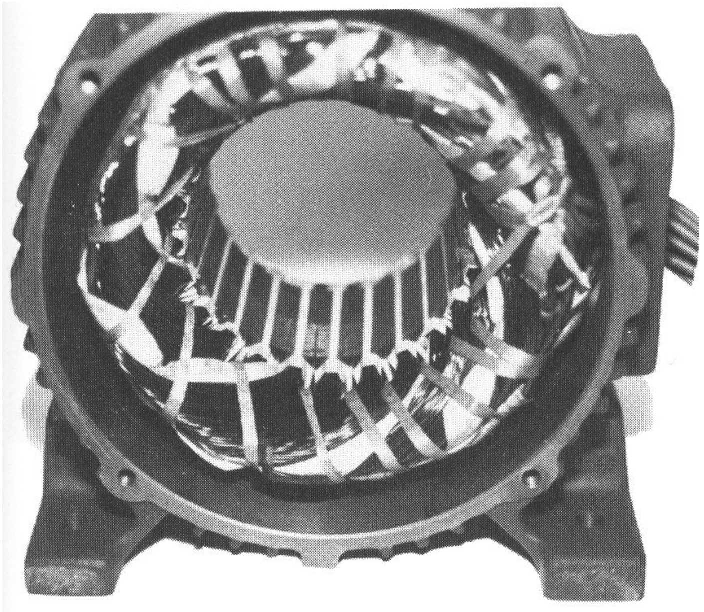


Introduction

- Three-phase induction motors (IM) are the most common and frequently encountered machines in industry. The are mostly referred to as the workhorses of the industry.
- This is due to:
  - ✓simple design, rugged in construction, low-price, easy maintenance
  - ✓wide range of power ratings: fractional horsepower up-to 10 MW
  - ✓run essentially as constant speed from zero to full load
  - ✓speed is power source frequency dependent
    - not easy to have variable speed control. This is because power source has a constant frequency source of 50 Hz or 60 Hz
    - requires a variable-frequency power-electronic drive for optimal speed control (so as to alter the source frequency)

Construction

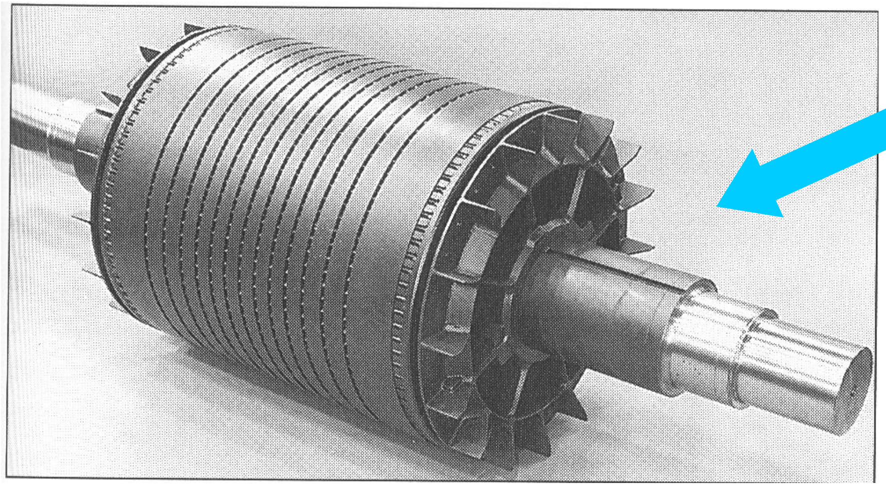
- An induction motor has two main parts
  - a stationary stator:
    - consisting of a steel frame that supports a hollow, cylindrical core.
    - core, constructed from stacked laminations (why?), having a number of evenly spaced slots, providing the space for the stator winding



4.1 Stator of IM

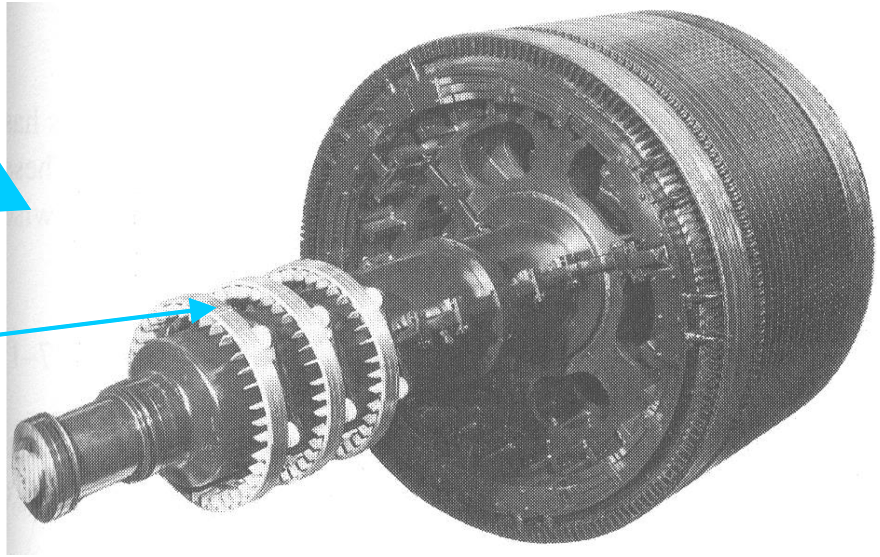
❖ Stacked lamination reduce Eddy currents, thereby allowing the stator core to maintain contant power.

- a revolving rotor
  - composed of punched laminations, stacked to create a series of rotor slots, providing space for the rotor winding
  - one of two types of rotor windings
  - conventional 3-phase windings made of insulated wire (wound-rotor) » similar to the winding on the stator
  - aluminum bus bars shorted together at the ends by two aluminum rings, forming a squirrel-cage shaped circuit (squirrel-cage)
- Two basic design types depending on the rotor design
  - squirrel-cage
  - wound-rotor



Squirrel cage rotor

Wound rotor



Notice the slip rings

Fig. 4.2 rotor tyrpes

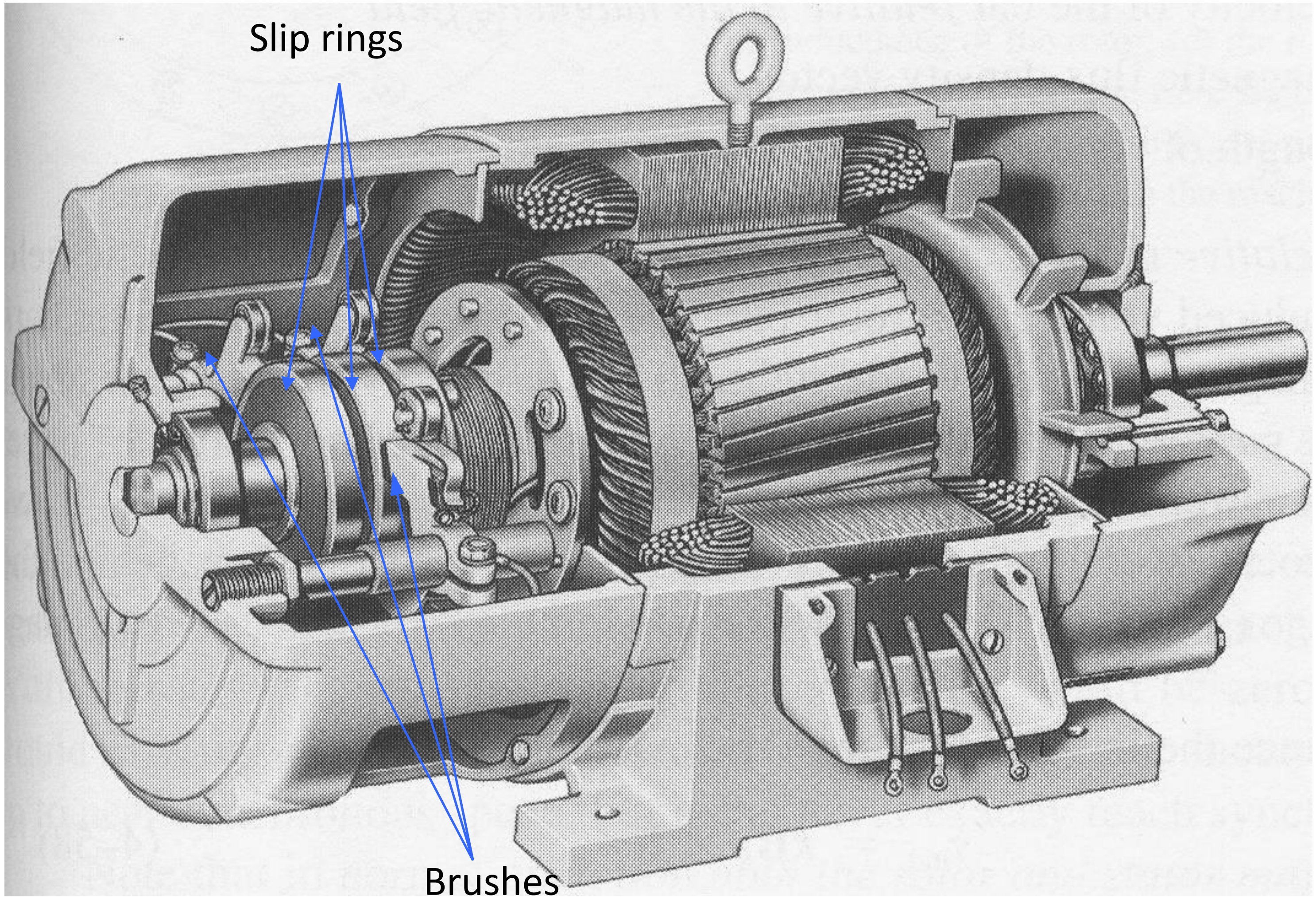
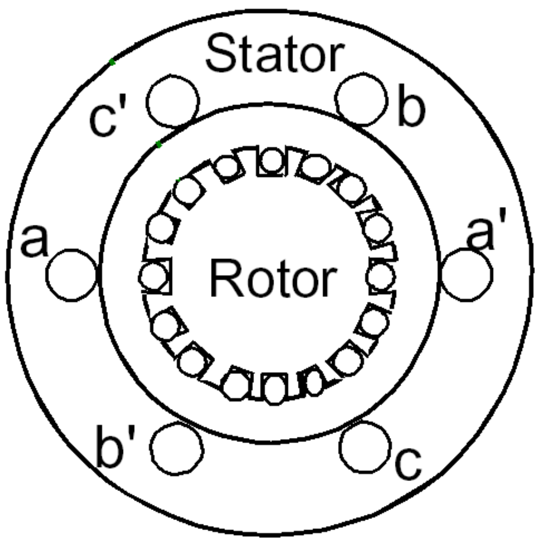


Fig. 4.3
Cutaway in a typical wound-rotor IM.
Notice the brushes and the slip rings

Rotating Magnetic Field

- Balanced three phase windings, i.e. mechanically displaced 120 degrees from each other, fed by balanced three phase source.
- A rotating magnetic field with constant magnitude is produced, rotating with a speed



$$n_s = \frac{120 f_e}{P} \quad [rpm] \qquad (4.1)$$

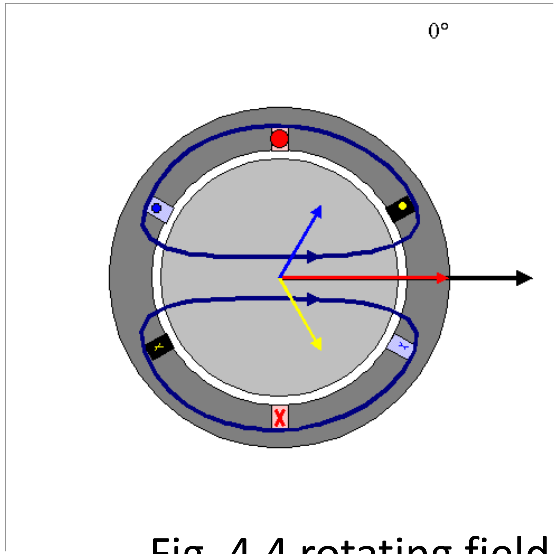


Fig. 4.4 rotating field

Where f_e is the supply frequency and P is the no. of poles and n_s is called the synchronous speed in *rpm* (revolutions per minute)

Principle of operation

- The general principle of operation of IM is similar to that of d.c. motors.

- This rotating magnetic field cuts the rotor windings and produces an induced voltage in the rotor windings.
- Due to the fact that the rotor windings are short circuited, for both squirrel cage and wound-rotor, and induced current flows within the rotor windings.
- The rotor current produces another magnetic field opposite to the main field in order to preserve the m.m.f. balance in the iron core
- A torque is produced as a result of the interaction of those two magnetic fields

$$\tau_{ind} = kB_R \times B_s$$

Where, T_{ind} is the induced torque and B_R and B_S are the magnetic flux densities of the rotor and the stator respectively

Induction motor speed

- At what speed will the IM run?
  - Can the IM run at the synchronous speed, why?
  - If rotor runs at the synchronous speed, which is the same speed of the rotating magnetic field, then the rotor will appear stationary to the rotating magnetic field and the rotating magnetic field will not cut the rotor. So, no induced current will flow in the rotor and no rotor magnetic flux will be produced so no torque is generated and the rotor speed will fall below the synchronous speed
  - When the speed falls, the rotating magnetic field will cut the rotor windings and a torque is produced.
- So, the IM will always run at a speed lower than the synchronous speed.
- The difference between the motor speed and the synchronous speed is called the *Slip.*

$$n_{slip} = n_s - n_r \qquad (4.2)$$

Where n_{slip}= slip speed

n_s = speed of the magnetic field

n_r = mechanical shaft speed of the motor

The Slip

$$s = \frac{n_s - n_r}{n_s} \qquad (4.3)$$

Where, *s* is the *slip*

Notice that : if the rotor runs at synchronous speed

$s = 0$

if the rotor is stationary

$s = 1$

Slip may be expressed as a percentage by multiplying the above eqn. by 100.

→ Because slip is a ratio → doesn't have units.

Examples

1. A 208-V, 10hp, 4-pole, 60 Hz, Y-connected induction motor has a full-load slip of 5 percent.
   i. What is the synchronous speed of this motor?
   ii. What is the rotor speed of this motor at rated load?
   iii. What is the rotor frequency of this motor at rated load?
   iv. What is the shaft torque of this motor at rated load?

Solution

1. $$n_s = \frac{120 f_e}{P} = \frac{120(60)}{4} = 1800\ rpm$$

2. $$n_m = (1-s)n_s$$
$$= (1-0.05)\times 1800 = 1710\ rpm$$

3. $$f_r = sf_e = 0.05\times 60 = 3Hz$$

4. $$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{P_{out}}{2\pi \frac{n_m}{60}}$$
$$= \frac{10\,hp \times 746\,watt / hp}{1710\times 2\pi \times (1/60)} = 41.7\ N.m$$

2. A 220 V, 3-phase, 2-pole, 50 Hz induction motor is running at a slip 5%. Calculate:
   i. The speed of the magnetic fields in rpm.
   ii. The speed of the rotor in revolutions per minute.
   iii. The slip speed of the rotor.
   iv. The rotor frequency in Hertz

Solution

i. $n_s = \frac{120f_e}{P} = \frac{120 \times 50}{2} = \mathbf{3000\ rpm}$

ii. $n_r = (1 - s)n_s = (1 - 0.05)(3000) = \mathbf{2850\ rpm}$

iii. $n_{slip} = sn_s = (0.05)(3000) = \mathbf{150\ rpm}$

iv. $f_r = \frac{n_{slip}P}{120} = \frac{(150)(2)}{120} = \mathbf{2.5\ Hz}$ or $f_r = sf_e = (0.05)(50) = \mathbf{2.5\ Hz}$

Equivalent Circuit

- Torque is produced by the interaction of a stator-bound flux wave and an induced rotor-bound current wave.
  - Since the flux and current waves are internal to the machine, it is very difficult to measure these on load and use them for machine performance calculations.
  - However, the similarities between the induction motor and the transformer permit the adaptation of the equivalent circuit model of the transformer to represent an induction motor.
  - As such an electrical equivalent circuit model can be used to predict the torque/speed characteristics and efficiency of the motor, in terms of machine currents, voltage, resistances and reactances – instead of flux and current waves.

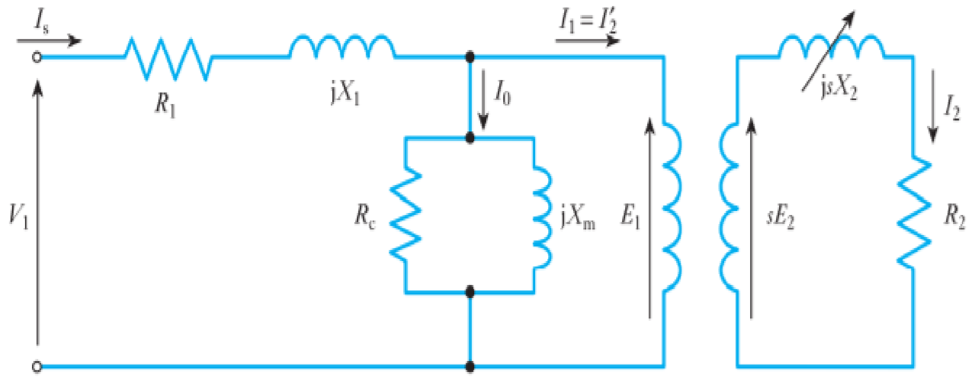


Fig. 4.5 Single-phase equivant circuit of the 3-phase IM

Where, R_1 = per-phase resistance of the stator

X_1 = per-phase reactance of the stator

X_m = per-phase magnetizing reactance of the IM

R_c = core loss resistance

The airgap of the IM is represented by the ideal transformer model with the primary being assumed to have N_1 turns and the secondary have N_2 turns.

- The magnitude of the e.m.f. wave induced in the rotor is a function of slip, sE_2 – E_2 being the rotor e.m.f. at standstill.

R_2 = series resistance

X_2 = series reactancce

- The effective reactance, sX_2, varies with the frequency of the induced rotor e.m.fs, X_2 being the value when the rotor is stationary (when s = 1).
- As the motor speeds up, the slip is reduced and the effective reactance of the rotor cage is reduced.
- The equivalent circuit is simplified by making two assumptions:

1. That the stator and rotor windings have the same number of turns per phase.
2. That the airgap flux has a constant amplitude and speed.

As there is an ampere-turn (m.m.f.) balance between stator and rotor:

$$I_1N_1 = I_2N_2$$

Using assumption (1), $N_1 = N_2$, it follows that

$$I_1 = I_2 = \frac{sE_2}{Z_2} = \frac{sE_2}{\sqrt{\{(R_2)^2 + (sX_2)^2\}}} = \frac{E_2}{\sqrt{\left\{\left(\frac{R_2}{s}\right)^2 + (X_2)^2\right\}}} \qquad (4.4)$$

- If ϕ is the phase difference between I_2 and E_2, the rotor current and voltage:

$$\tan \phi = \frac{sX_2}{R_2} \quad (4.5)$$

and

$$\cos \phi = \frac{R_2}{\sqrt{\{(R_2)^2 + (sX_2)^2\}}} \quad (4.6)$$

- Equation (4.6) suggests the following simplified circuit by referring the rotor impedance to the stator. This facilitates the calculation of the rotor current referred to the stator.

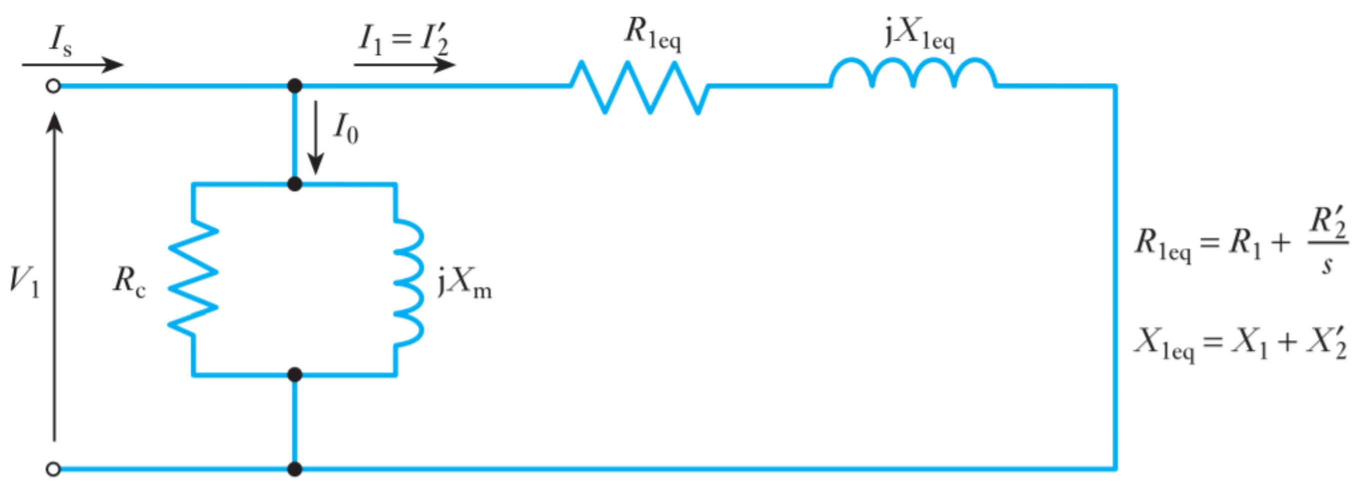
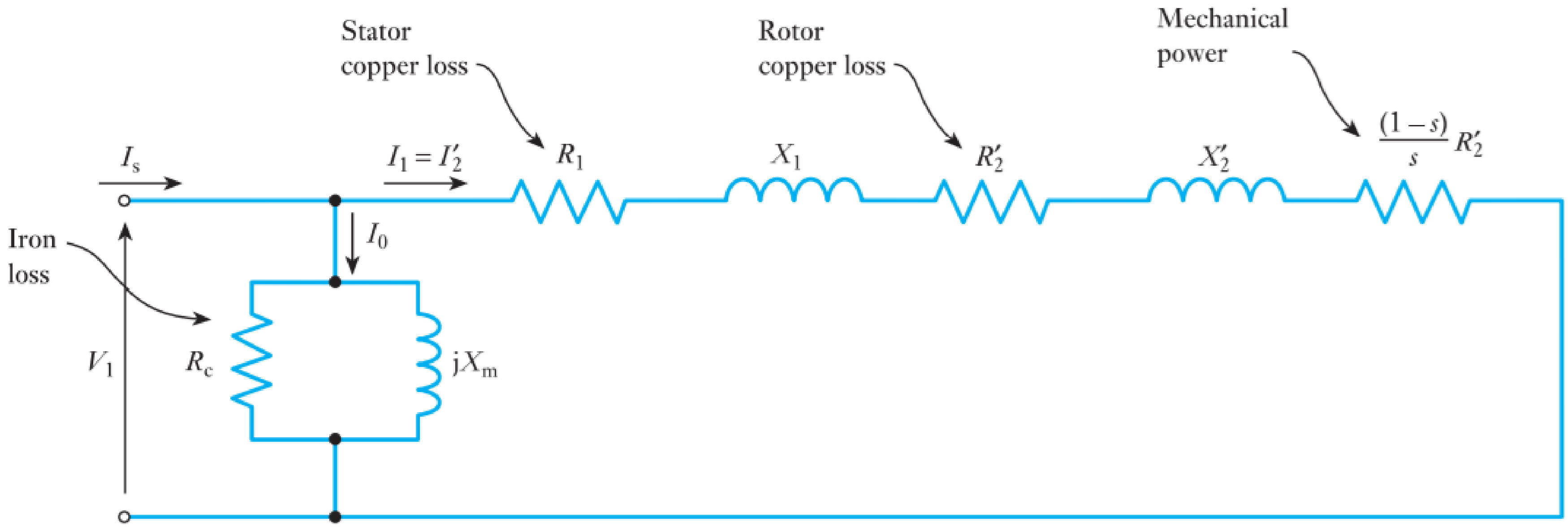


Fig. 4.6 Simplified equivalent circuit of the IM

- Since a large m.m.f. is required to get the working flux across the airgap, the magnetizing current in an induction motor can be 30 per cent of the full load current of the machine – and must be included when estimating machine performance.
  - In a transformer, the magnetizing current is usually only 5 per cent of the full load current so the magnetizing branch can often be ignored.

Mechanical power and torque

- If the power losses in the iron core of the stator are neglected, the equivalent circuit model in fig. 4.7, the real power taken from an a.c. supply is given by:



4.7 Final per-phase equivalent circuit of the induction motor

$$P_{in} = R_{eq}|I_1|^2 = \left(R_1 + \frac{R_2'}{s}\right)|I_1|^2 \quad \text{W/ph} \qquad (4.7)$$

The power dissipated in the resistance of stator and rotor windings is:

$$P_\Omega = (R_1 + R_2')|I_1|^2 \quad \text{W/ph} \qquad (4.8)$$

The difference between these two quantities must be the power dissipated in the load, i.e. the mechanical shaft power P_m:

$$P_m = P_{in} - P_\Omega$$

$$= \left(R_1 + \frac{R_2'}{s}\right)|I_1|^2 - (R_1 + R_2')|I_1|^2$$

$$= |I_1|^2\left(R_1 + \frac{R_2'}{s} - (R_1 + R_2')\right)$$

$$= |I_1|^2\left(\frac{R_2'}{s} - R_2'\right) = \left(\frac{1-s}{s}\right)R_2'|I_1|^2$$

This is a per-phase quantity so the total three-phase mechanical power developed is

$$P_{m3\phi} = 3\frac{(1-s)}{s}R_2^{'}|I_1|^2 \qquad (4.9)$$

Since,

$$\text{Power} = \text{torque}\,(\tau_m) \times \text{angular velocity}\,(\omega_m)$$

$$\therefore \tau_m = \frac{P_m}{\omega_m} \qquad (4.10)$$

For a rotor speed of n_r rpm,

$$\omega_m = \frac{2\pi n_r}{60} \quad \text{rad/s}$$

so $$\omega_m = \frac{\pi}{30}(1-s)n_s \quad \text{rad/s} \qquad (4.11)$$

Combining equations (4.8), (4.9) and (4.11), the torque is

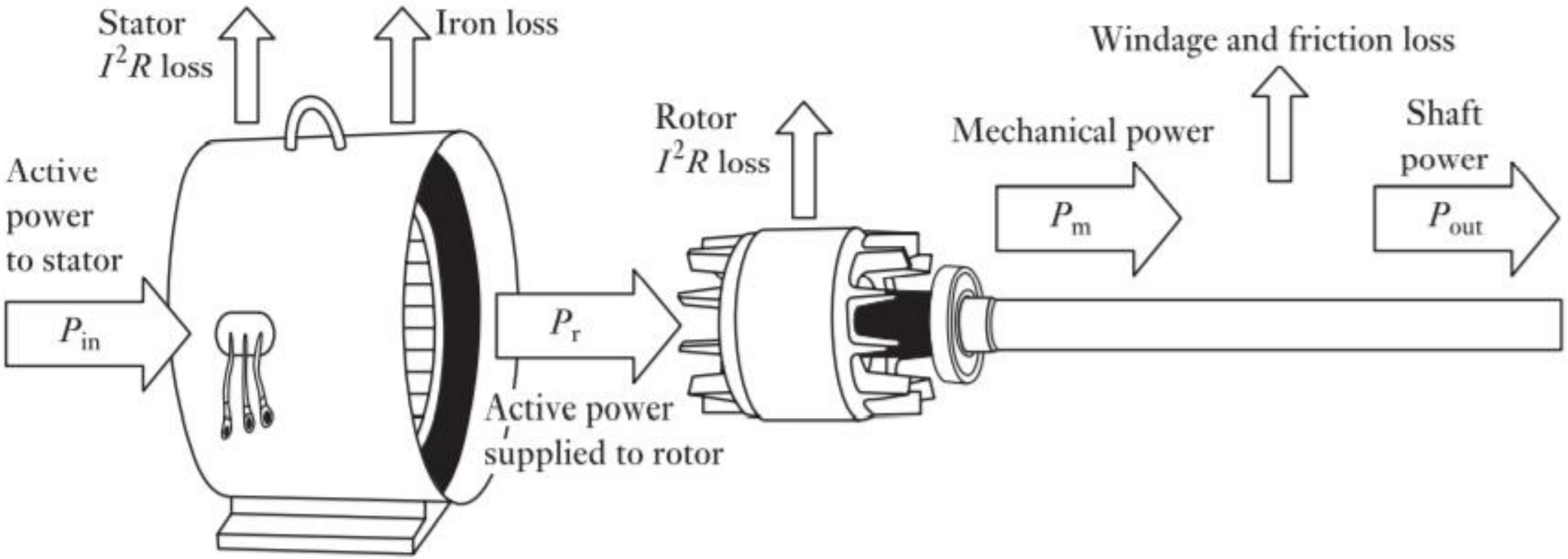
$$\tau_m = \frac{90}{\pi n_s}\frac{R_2^{'}}{s}\left|I_1\right|^2 \qquad (4.12)$$

Equation (4.9) leads to the final and most useful form of the equivalent circuit, shown in the fig. 4.7 above , which allows the mechanical power and the rotor losses to be perated.

Power flow in induction motor

The power flow through the motor and the efficiency can thus be calculated and are shown in Fig. 4.8.

- ✓ Friction and windage loss has been included
  - ➢ In the mathematical analysis above, this loss has been included in the mechanical power developed by the motor



4.8 Real power flow through an induction motor

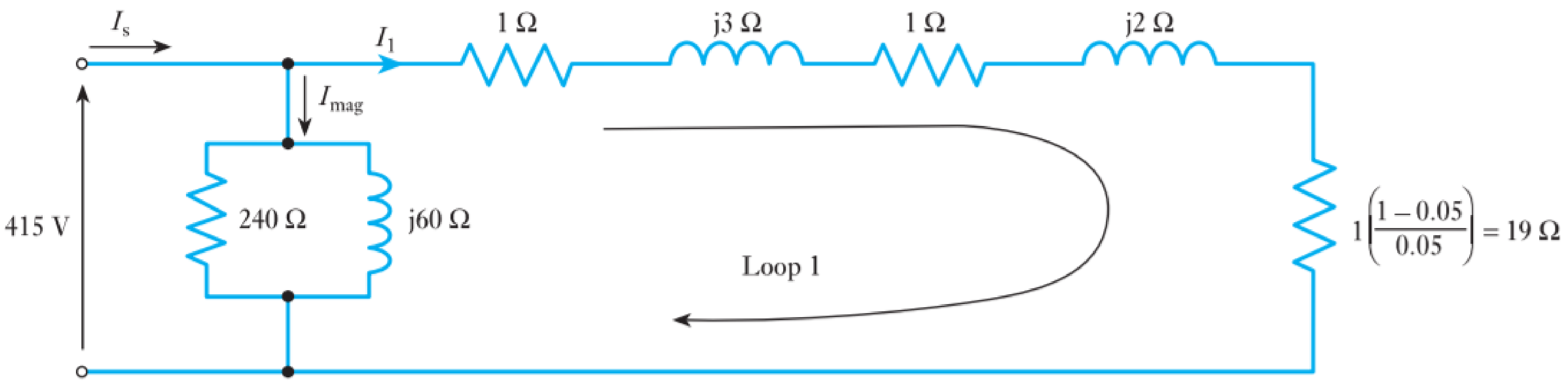
Example:

The 415 V, three-phase, 50 Hz, star-connected induction motor shown in Fig. 4.9 has the following per-phase equivalent circuit parameters:

$$R_1 = 1\Omega,\ X_1 = 3\ \Omega,\ X_m = 60\ \Omega,\ R_c = 240\ \Omega,\ R'_2 = 1\ \Omega,\ X'_2 = 2\ \Omega.$$

Using the final per-phase equivalent circuit of the machine, calculate the current drawn from the supply, and draw a diagram (as Fig. 4.8) showing the flow of real power through the machine when running with a slip of 5 per cent. The friction and windage loss in the machine is 200 W. Calculate the efficiency of the motor.

Solution:



4.9 Equivalent circuit for the Example

(a) Per-phase voltage. The stator is star-connected; the phase voltage is thus

$$\frac{415}{\sqrt{3}} = \mathbf{240\ V}$$

(b) The slip = 5 per cent. The mechanical power is

$$P_m = R_2' \frac{(1-s)}{s} = 1.0\frac{(1-0.05)}{0.05} = \mathbf{19\ \Omega}$$

(c) Currents flowing in the equivalent circuit:

Impedance of loop 1 = (1.0 + 1.0 + 19) + j(3.0 + 2.0) Ω

= (21 + j5) Ω

Thus, current flowing around loop 1 is

$$\mathbf{I}_1 = \frac{240}{(21+j5)} = \frac{240(21-j5)}{(21+j5)(21-j5)}$$

$$= \frac{240(21-j5)}{21^2+5^2}$$

$$= 10.81 - j2.57\ \text{A}$$

$$= 11.11\angle -13.4°\ \text{A}$$

The current flowing in the magnetizing branch is found by adding the current in the core loss resistance to the current in the magnetizing reactance:

$$\mathbf{I}_{mag} = \frac{240}{240} + \frac{240}{j60} = (1 - j4)\ \text{A}$$

The supply (stator) current is

$$\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_{mag} = (10.81 - j2.57) + (1 - j4)$$

$$= (11.81 - j6.57)$$

$$= \mathbf{13.51\angle 29°\ A}$$

Since the three-phase motor is star-connected, this is also the line current in the supply. The power factor is

$$\mathbf{\cos\angle 29° = 0.875}$$

(d) Input power $P_{in} = \sqrt{3}\, V_L I_L = \sqrt{3} \times 415 \times 13.51 \times \cos\angle 29° = \mathbf{8.493\ kW}$

(e) Power loss in resistances of the equivalent circuit:

$$\text{Core loss} = 3 \times \frac{240^2}{240} = \mathbf{720\ W}$$

Stator copper loss = $3\, I_1^2 R_1 = 3 \times 11.11^2 \times 1.0 =$ **370 W**

(Airgap power = 8493 − (720 + 370) W = **7403 W**)

Rotor copper loss = $3\, I_1^2 R_2' = 3 \times 11.11^2 \times 1.0 =$ **370 W**

(f) Mechanical power $P_m = 7403 - 370 = 7033$ W

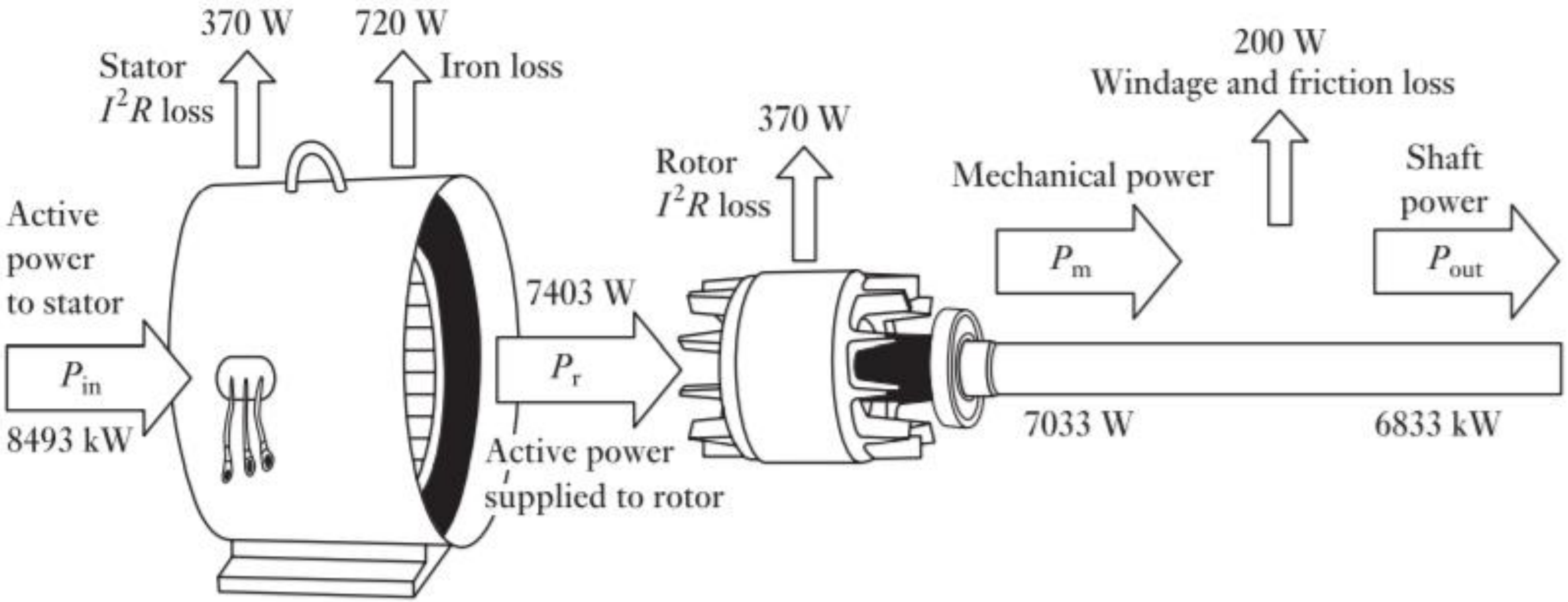
(Note: $P_{m3\phi} = 3\frac{(1-s)}{s} R_2' |I_1|^2$ equation [38.8]

$= 3 \times 19 \times 11.11^2 = 7035$ W)

Since there is a friction and windage loss of 200 W,

Useful shaft power = 7033 W − 200 W = **6833 W**

(g) Power flow diagram for the Example



(h) Efficiency of motor, $\eta = \frac{P_o}{P_{in}} = \frac{6833}{8493} \times 100$ per cent = **80.5 per cent**

- Neglecting the magnetizing component of current in the equivalent circuit of the induction motor, we can calculate the torque delivered to the mechanical load driven by the induction motor from equation (4.12).

$$\tau_m = \frac{90}{\pi n_s}\frac{R_2^{'}}{s}\left|I_1\right|^2$$

- Where I_1 is given by,

$$|I_1| = \frac{|V_1|}{\sqrt{\left\{\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2\right\}}}$$

Substituting for the current in the torque equation, we obtain the following expression;

$$\tau_m = \frac{90}{\pi n_s}\frac{R_2^{'}}{s}\frac{\left|V_1\right|^2}{(R_1 + R_2^{'})^2 + (X_1 + X_2^{'})^2} \qquad (4.13)$$

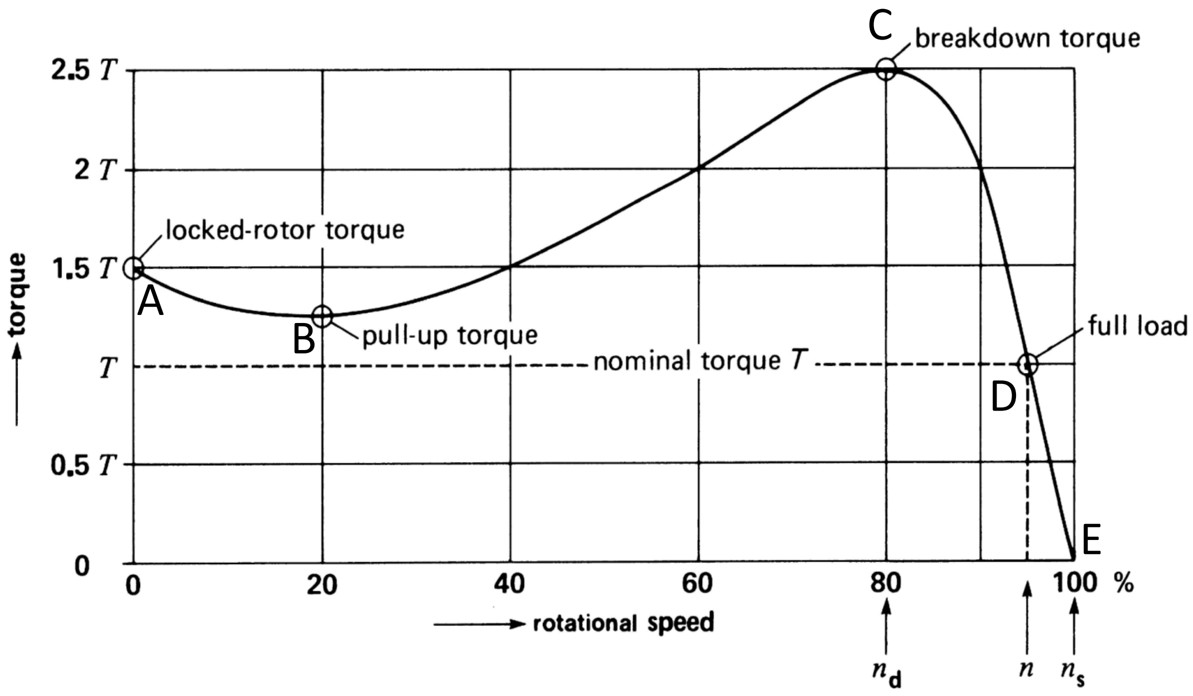
where V_1 is the r.m.s. voltage per phase

- Equation (4.13) shows the dependency of motor torque on slip (assuming that system voltage and frequency are constant) and a typical torque–speed curve

Torque-speed characteristics

The graph shows what happens in terms of output torque and motor speed when a motor is started with full voltage applied.

- The motor is initially stationary and develops locked-rotor torque (point A).
- As the motor accelerates, some motor designs produce a slight dip in torque, the lowest point being called the pull-in or pull-up torque (point B).
- As the speed increases further, the torque reaches the highest point on the curve (point C), the pull-out or breakdown torque.



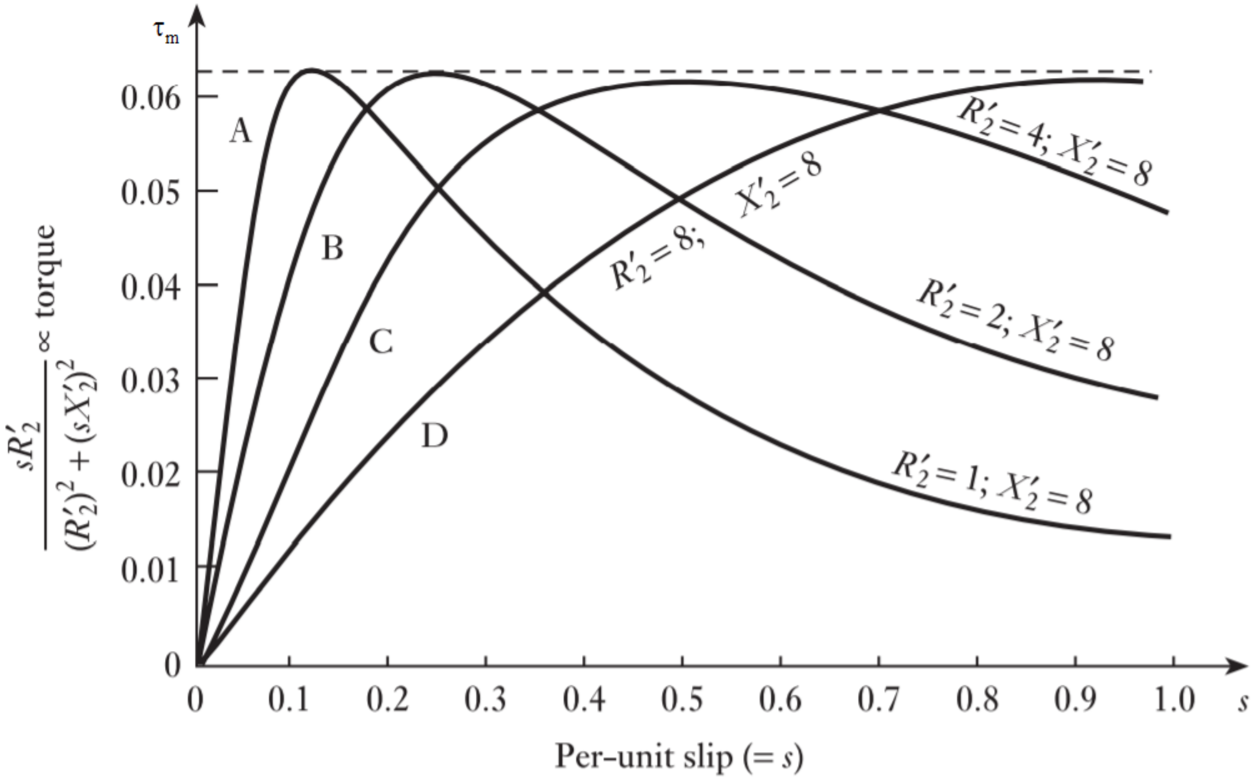
4.10 Typical torque-speed characteristics of induction motor

- Finally, when the motor is loaded to its full-load torque, the motor speed stabilizes (point D). If the motor is not driving anything, the speed increases to the no-load or synchronous speed (point E)

Assuming the impedance of the stator to be small for a given supply voltage, then, for a given a voltage and frequency, the torque equation simplifies to;

$$\tau_m = \frac{R_2^{'}}{s} \frac{K}{(\frac{R_2^{'}}{s})^2 + (X_2^{'})^2} = K \frac{sR_2^{'}}{(R_2^{'})^2 + (sX_2^{'})^2} \qquad (4.14)$$

- The value of $X_2^{'}$ is typically large compared with the rotor resistance. The dependency of τ_m on slip and rotor resistance can be illustrated by an example, in which $X_2^{'} = 8\ \Omega$.
- Calculating τ_m for different values of slip and rotor resistance, the resulting curves being close to the exact shape of typical torque–speed curves. Note that, when slip is zero, speed reaches synchronous speed.



4.11 Torgue/slip curves of an induction motor

- Important conclusions:
  i. Increasing the rotor resistance moves the maximum value of torque towards higher slip values, that is, in the direction of lower speed. Thus, a higher rotor resistance gives a higher starting torque.
  ii. A higher rotor resistance gives a lower on-load efficiency but a higher starting torque. A lower rotor resistance gives less starting torque but higher on-load efficiency.
  iii. τ_m is a constant. This can be explained by differentiating equation (4.14) for torque with respect to slip and equating to zero:

$$\frac{d}{ds}\left[\frac{sR_2'}{(R_2')^2 + (sX_2')^2}\right] = \frac{[(R_2')^2 + (sX_2')^2]R_2' - sR_2' 2s(X_2')^2}{[(R_2')^2 + (sX_2')^2]^2} = 0$$

Hence $(R_2')^2 + (sX_2')^2 = 2s^2(X_2')^2$

$$R_2' = sX_2'$$

By substituting for $R_2' = sX_2'$ back in equation (4.14), it is revealed that τ_{max} is independent of R_2'

$$\tau_{max} = \frac{K}{sX_2'}$$

Note that X_2' is the leakage reactance at standstill and is a constant for a given rotor. Hence the maximum torque is the same, whatever the value of rotor resistance.