

**MEC 3102 – PRODUCTION ENGINEERING I AND
ELECTRICITY & ELECTRONICS II**
Department of Mechanical Engineering
The University of Zambia

Name: Mr. Munkombwe Boyd

Old School of Engineering Bldg.,

Cell: 0968315273/0950435239

Email: boyd.munkombwe@unza.zm

2nd Series Lecture 2 [1]

Magnetic Hysteresis

- When a magnetic material is magnetised first in one direction and then in the other (i.e., one cycle of magnetisation), it is found that flux density B in the material lags behind the applied magnetising force H .
- Hence, the phenomenon of flux density B lagging behind the magnetising force H in a magnetic material is called magnetic hysteresis.
- 'Hysteresis' is the term derived from the Greek word hysterein meaning to lag behind.
- Consider a ring of magnetic material on which a solenoid is wound uniformly as shown in Fig. 2.7. The solenoid is connected to a DC source through a double pole double throw reversible switch (position '1')

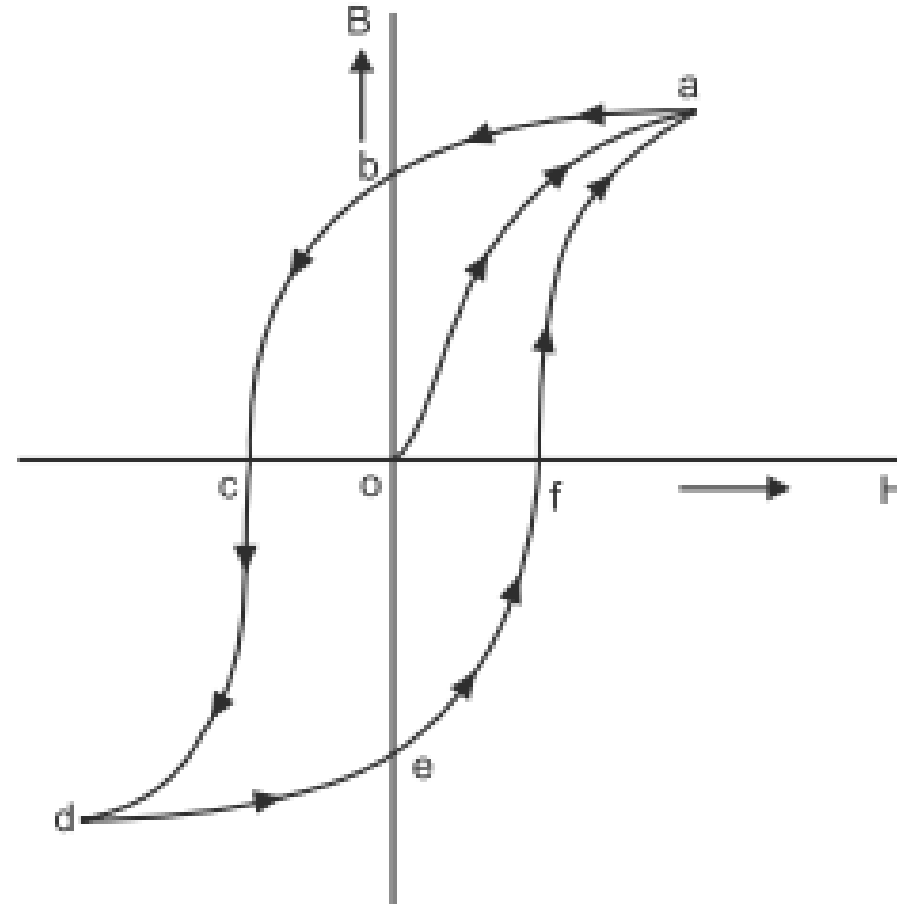
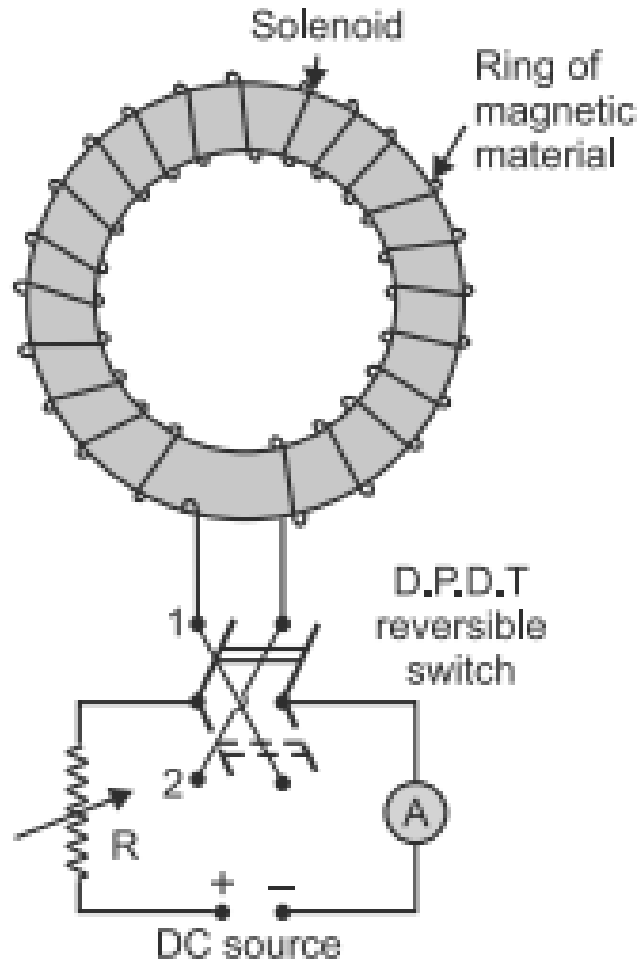


Fig. 2.7. Circuit to trace hysteresis loop an Hysteresis loop

1. When the field intensity \mathbf{H} is increased gradually by increasing current in the solenoid, the flux density \mathbf{B} also increases until saturation point a is reached and curve so obtained is oa .
2. If now the **magnetising force** is gradually reduced to zero by decreasing current in the solenoid to zero. The flux density does not become zero and the curve so obtained is ab .
 - ✓ **Residual Magnetism** (Flux density ob)
 - **Retentivity**: the power of retaining this residual magnetism of the material.
 - To demagnetise the magnetic ring, the magnetising force \mathbf{H} is reversed by reversing the direction of flow of current in the solenoid.
3. When \mathbf{H} is increased in reverse direction, the flux density starts decreasing and becomes zero and curve follows the path bc .
 - ✓ Residual magnetism of the magnetic material is wiped off by applying magnetising force oc in opposite direction.
 - **Coercive Force**
4. To complete the loop, the magnetising force H is increased further in reverse direction till saturation reaches (point ' d ') and the curve follows the path cd .

5. Again \mathbf{H} is reduced to zero and the curve follows the path de . Where oe represents the residual magnetism.
6. Then \mathbf{H} is increased in the positive direction by changing the position of reversible switch to position '1' and increasing the flow of current in the solenoid.
 - ✓ Path of efa is obtained and the loop completed. Again of is the magnetising force utilised to wipe off the residual magnetism oe .
7. Since the meaning of hysteresis is lagging behind, and in this case flux density \mathbf{B} always lags behind the magnetising force, \mathbf{H} , therefore, loop ($abcdefa$) obtained is called the **hysteresis loop**.

Hysteresis Loss

- When a magnetising force is applied, the magnetic material is magnetised and the molecular magnets are lined up in a particular direction.
- However, when the magnetising force in a magnetic material is reversed, the internal friction of the molecular magnets opposes the reversal of magnetism, resulting in *hysteresis*.
- To overcome this internal friction of the molecular magnets (or to wipe off the residual magnetism), a part of the magnetising force is used.
- The work done by the magnetising force against this internal friction of molecular magnets produces heat. This energy, which is wasted in the form of heat due to hysteresis, is called hysteresis loss.

Eddy Current Loss

- When a magnetic material is subjected to a changing (or alternating) magnetic field, an e.m.f is induced in the magnetic material itself according to Faraday's laws of electro-magnetic induction.
- Since the magnetic material is also a conducting material, these e.m.fs. circulate currents within the body of the magnetic material.
- These circulating currents are known as **eddy currents**.
 - ✓ No work is done by the currents, hence, produce $(i^2 R)$ losses in the magnetic material called **eddy current loss**.
- Like hysteresis loss, this loss also increases the temperature of the magnetic material. The hysteresis and eddy current losses in a magnetic material are called **iron losses or core losses or magnetic losses**.

3. Magnetic Circuits

- Many useful devices (such as transformers, motors, and generators) contain **coils** wound on **iron cores**.
- In many applications, we need to analyze more complex configurations (such as cores that lack symmetry and those with multiple coils) for which the direct application of Ampère's law is not feasible.

Thus, we use **magnetic circuit concepts**, which are analogous to those used to analyze **electrical circuits**.

- The **closed path** followed by magnetic flux is called a **magnetic circuit**.
- A magnetic circuit usually consists of magnetic materials having high ability to conduct magnetic lines of force through them (e.g., iron, soft steel, etc.).

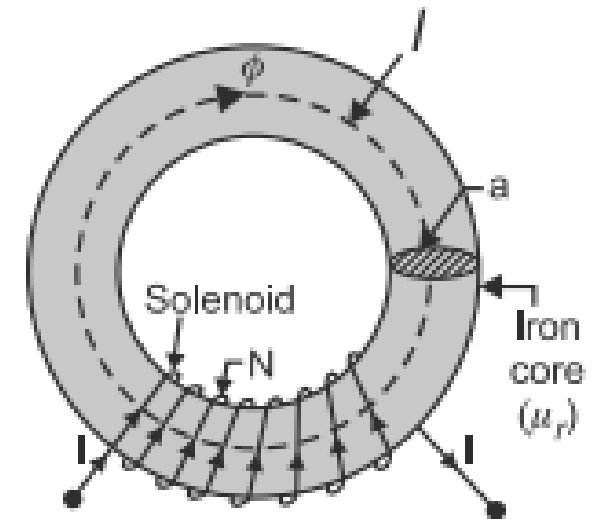


Fig. 3.1 Magnetic circuit

Magnetomotive Force (m.m.f)

- The **magnetic pressure** which sets-up or tends to **set-up magnetic flux** in a magnetic circuit is called **magnetomotive force**.
- As per work law it may be defined as: the **work done** in moving a unit magnetic pole (1 Wb) once round the magnetic circuit.

The **magnetomotive force (mmf)** of an N-turn current-carrying coil is given by:

$$F = NI \quad (3.1)$$

- Symbol: F Unit: ampere (A)

In general,

- It is analogous to emf in an electric circuit.
- ✓ Note also that in a **magnetic circuit**, there is a relationship between the **magnetomotive force [F]** and the **magnetic field intensity [H]** being given by:

$$F = HL = NI \quad (3.2)$$

Where, L is the length of the magnetic path

Relative permeability of ferromagnetic materials

- ✓ **Permeability**: this is the ability of a material to conduct magnetic lines of force through it.
- ✓ **Relative permeability**: the ratio of the permeability of material μ to the permeability of air or vacuum μ_0 .

$$\mu_r = \frac{\mu}{\mu_0} \text{ or } \mu = \mu_r \mu_0$$

- It is usually convenient to represent the relationship between the flux density and the magnetic field strength graphically as in Fig. 3.2
- Thus, using equation 1.1, we should be able to calculate the magnetomotive force for a given flux density in a magnetic circuit.

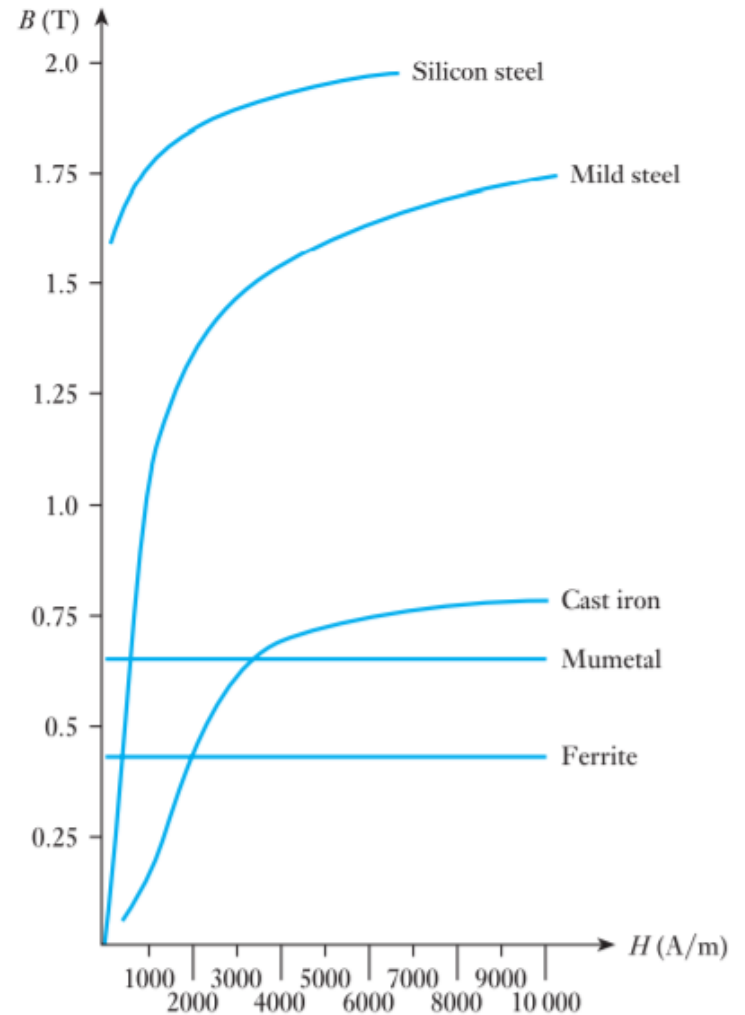


Fig. 3.2.

Reluctance

- The opposition offered to the magnetic flux by a magnetic circuit is called its **reluctance**.
- Consider a ferromagnetic ring shown in Fig. 3.1.

$$\Phi = BA \quad (3.3)$$

and

$$F = Hl \quad (3.4)$$

Dividing equation (3.3) by (3.4),

$$\frac{\Phi}{F} = \frac{BA}{Hl} = \mu_0\mu_r \times \frac{A}{l}$$

Therefore,

$$\Phi = \frac{F}{\frac{l}{\mu_0\mu_r A}}$$

Where,

$$\frac{F}{\Phi} = \frac{l}{\mu_0\mu_r A} = S \quad (3.5)$$

S is the reluctance of the magnetic circuit where

$$F = S\Phi \quad (3.6)$$

$$S = \frac{l}{\mu_0\mu_r A} \quad (3.7)$$

- The unit of reluctance is the **ampere per weber**, abbreviated to **A/Wb**
- It is analogous to resistance in an electric circuit
- The resistance of a conductor of length l , cross-sectional area A and resistivity ρ is given by

$$R = \rho l / A$$

Since electrical conductivity $\sigma = 1/\rho$, the expression for R can be rewritten as:

$$R = l / \sigma A$$

- This is very similar to equation (3.5), for the reluctance S , except permeability $\mu(\mu_0\mu_r)$ replaces σ .

Permeance, and Reluctivity

- For both electrical and magnetic circuits, increasing the length of the circuit increases the opposition to the flow of electric current or magnetic flux or vice-versa.
- **Permeance**: It is a measure of the ease with which flux can be set-up in the material. It is just reciprocal of reluctance of the material and is measured in Wb/A or henry.
- **Reluctivity**: It is specific reluctance and analogous to resistivity in electric circuit

Ohm's Law of magnetic circuit

Equation (3.9) can thus be regarded as 'Ohm's law for a magnetic circuit', since the m.m.f. (F)

$$\text{m. m. f} = \text{flux} \times \text{reluctance}$$

$$F = \Phi S$$

or

$$NI = \Phi S = HL$$

It is clear that m.m.f. (F) is analogous to e.m.f. (E) and flux Φ is analogous to current (I) in a d.c. resistive circuit

$$\text{e. m. f} = \text{current} \times \text{resistance}$$

$$V = IR$$

Comparison between Magnetic and D.C. Electric Circuits

Table 3.1

Electric Circuits		Magnetic Circuits	
Ohm's law: $V = IR$		'Ohm's law': $F = \Phi S$	
Electromotive force (e.m.f) V	volt	Magnetomotive force (m.m.f) $F (= NI)$	ampere (turn)
Current I	ampere	Magnetic flux Φ	weber
Conductivity σ	siemens	Permeability μ	henry per metre
Resistance $R = \frac{l}{\sigma A}$	ohms	Reluctance $S = \frac{l}{\mu A}$	ampere (turn) per weber
Conductance = 1/resistance		Permeance = 1/reluctance	
Resistivity		Reluctivity	
Electric field strength E	volt per metre	Magnetic field strength H	ampere (turn) per metre
Current density $J = \frac{I}{A}$	amps per m^2	Magnetic flux density $B = \frac{\Phi}{A}$	tesla

Comparison of electromagnetic and electrostatic terms

Table 3.2

Electrostatics		Electromagnetism	
Term	Symbol	Term	Symbol
Electric flux	Q	Magnetic flux	Φ
Electric flux density	\mathbf{D}	Magnetic flux density	\mathbf{B}
Electric field strength	\mathbf{E}	Magnetic field strength	\mathbf{H}
Electromotive force	E	Magnetomotive force	F
Electric potential difference	V	Magnetic potential difference	—
Permittivity of free space	ϵ_0	Permeability of free space	μ_0
Relative permittivity	ϵ_r	Relative permeability	μ_r
Absolute permittivity = $\frac{\text{electric flux density}}{\text{electric field strength}}$	ϵ	Absolute permeability = $\frac{\text{magnetic flux density}}{\text{magnetic field strength}}$	μ
i.e. $\epsilon_0\epsilon_r = \epsilon = \mathbf{D}/\mathbf{E}$		i.e. $\mu_0\mu_r = \mu = \mathbf{B}/\mathbf{H}$	

Series Magnetic Circuits

- A magnetic circuit that has a number of parts of different dimensions and materials carrying the same magnetic field is called a series magnetic circuit.
- Such as series magnetic circuit (composite circuit) is shown in Fig. 3.3.

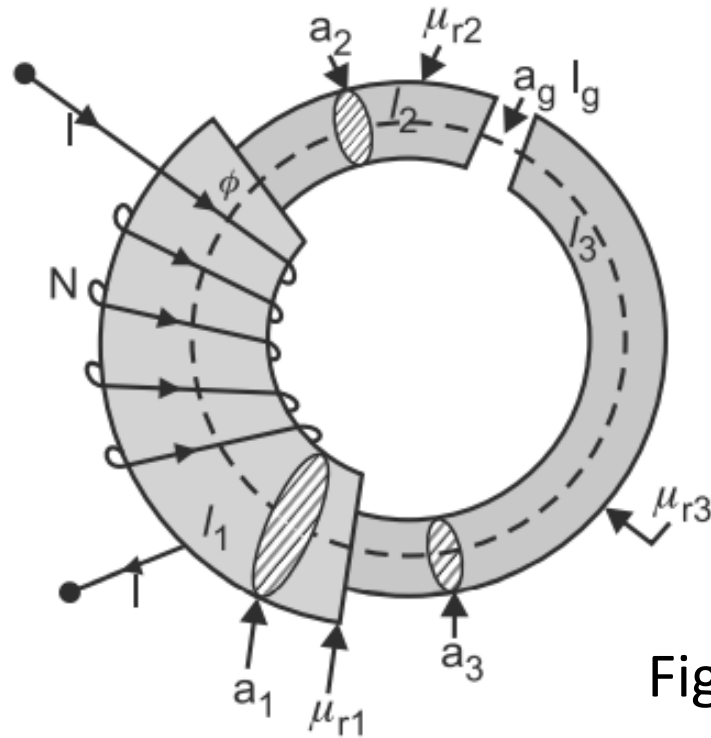


Fig. 3.3

Total reluctance of the magnetic circuit:

$$S = S_1 + S_2 + S_3 + S_g$$

$$= \frac{l_1}{\mu_0 \mu_{r1} a_1} + \frac{l_2}{\mu_0 \mu_{r2} a_2} + \frac{l_3}{\mu_0 \mu_{r3} a_3} + \frac{l_g}{\mu_0 a_g}$$

Total m.m.f = ΦS

$$= \Phi \left(\frac{l_1}{\mu_0 \mu_{r1} a_1} + \frac{l_2}{\mu_0 \mu_{r2} a_2} + \frac{l_3}{\mu_0 \mu_{r3} a_3} + \frac{l_g}{\mu_0 a_g} \right)$$

$$= \frac{B_1 l_1}{\mu_0 \mu_{r1}} + \frac{B_2 l_2}{\mu_0 \mu_{r2}} + \frac{B_3 l_3}{\mu_0 \mu_{r3}} + \frac{B_g l_g}{\mu_0}$$

$$= H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$$

Example 3.1

A magnetic circuit comprises three parts in series, each of uniform cross-sectional area (A). They are:

- (a) a length of 80 mm and $A_1 = 50 \text{ mm}^2$,
- (b) a length of 60 mm and $A_2 = 90 \text{ mm}^2$,
- (c) an airgap of length 0.5 mm and $A_3 = 150 \text{ mm}^2$.

A coil of 4000 turns is wound on part (b), and the flux density in the airgap is 0.30 T. Assuming that all the flux passes through the given circuit, and that the relative permeability $\mu_r = 1300$, estimate the coil current to produce such a flux density.

Solution:

$$\Phi = B_3 A_3 = 0.3 \times 150 \times 10^{-6} = 0.45 \times 10^{-4} \text{ Wb}$$

$$F_1 = \Phi S_1 = \Phi \times \frac{l_1}{\mu_0 \mu_r A_1} = \frac{0.45 \times 10^{-4} \times 80 \times 10^{-3}}{4\pi \times 10^{-7} \times 1300 \times 50 \times 10^{-6}} = \mathbf{44.1 \text{ At}}$$

$$F_2 = \Phi S_2 = \Phi \times \frac{l_2}{\mu_0 \mu_r A_2} = \frac{0.45 \times 10^{-4} \times 60 \times 10^{-3}}{4\pi \times 10^{-7} \times 1300 \times 90 \times 10^{-6}} = \mathbf{18.4 \text{ At}}$$

$$F_3 = \Phi S_3 = \Phi \times \frac{l_3}{\mu_0 \mu_r A_3} = \frac{0.45 \times 10^{-4} \times 0.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 150 \times 10^{-6}} = \mathbf{119.3 \text{ At}}$$

$$F = F_1 + F_2 + F_3 = 44.1 + 18.4 + 119.3 = \mathbf{181.8 \text{ At}}$$

We know that

$$F = NI$$

So that,

$$I = \frac{F}{N} = \frac{181.8}{4000} = \mathbf{45.4 \text{ mA}}$$

Parallel Magnetic Circuits

- Figure 3.4 shows a parallel magnetic circuit. A current carrying coil is wound on the central limb AB. This coil sets-up a magnetic flux Φ_1 in the central limb which is further divided into two paths i.e., (i) path ADCB which carries flux Φ_2 and (ii) path AFEB which carries flux Φ_3 .

It is clear that, $\Phi_1 = \Phi_2 + \Phi_3$

- The two magnetic paths ADCB and AFEB are in parallel as shown in figure 3.4

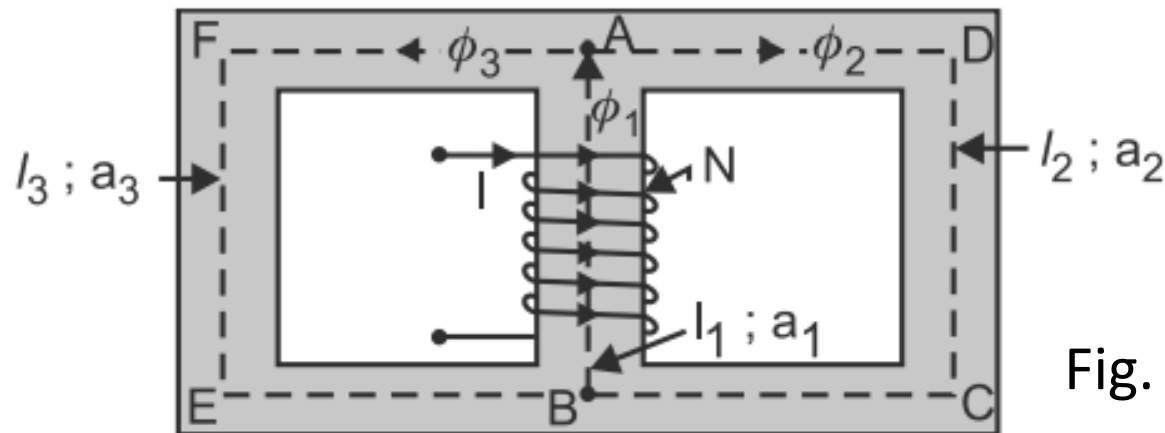


Fig. 3.4

➤ The expression for the total m.m.f(Amp-Turn)

Assuming that there are three limbs of the core as follows:

Limb-1: AB, Limb-2: ADCB, Limb-3: AFEB

Total m.m.f produced by the coil of N turns is:

$$\text{Total m.m.f} = NI \text{ (AT)}$$

In terms of reluctance,

Total mmf = Total Reluctance x Total Flux (useful)

$$F_T = \Phi_T S_T$$

m.m.f for each path:

Path 2: $F_2 = \Phi_1 S_1 + \Phi_2 S_2$

Path 3: $F_3 = \Phi_1 S_1 + \Phi_3 S_3$

❖ Thus, **Total m.m.f** = m.m.f of AB + m.m.f of ADCB (or m.m.f of AFEB)

Useful flux: This is the air gap flux that **can be utilized** for some **useful purpose**.

- Finds its **practical applications** in such as **motors, generators, moving coil and moving iron measuring instruments** etc.

Leakage Flux: It is expected that all the flux produced by the magnetizing coil completes its path only through the iron core and the air gap.

- If this was the case, all the produced flux will be available at the air gap as useful flux.
- In reality, this is not the case. Some flux actually completes its path **through the air** and/or **in general medium** surrounding the core.
- Thus, any flux that completes its path through air or the medium surrounding the magnetic circuit is referred to as **Leakage Flux**.

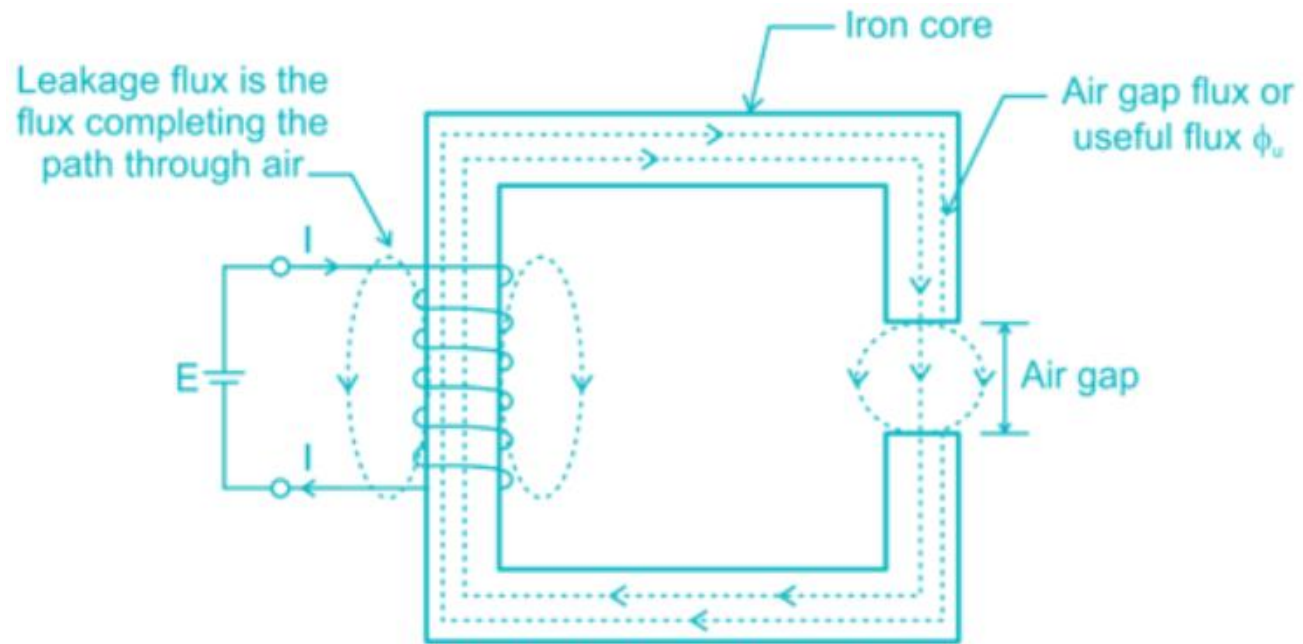


Fig. 3.5: Leakage Flux

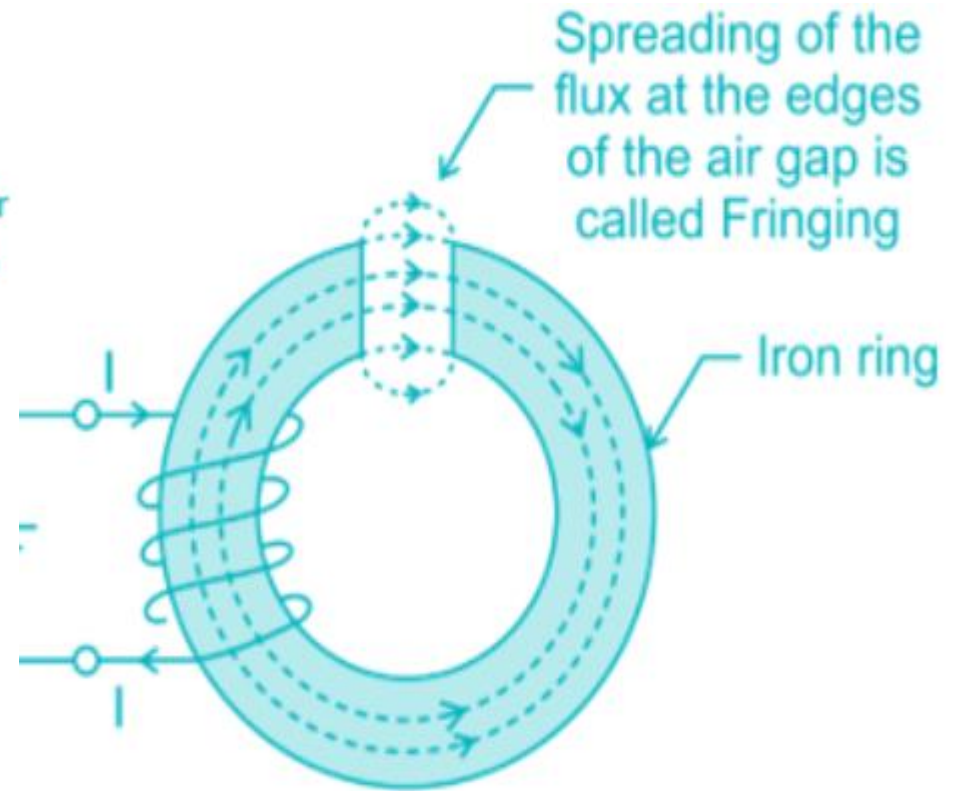


Fig. 3.6: Magnetic Fringing

Magnetic Fringing: With reference to fig. 3.5 above, the flux lines are produced inside the iron core, running parallel to each other and in the same direction.

✓ There is a **repulsive force** between them.

➤ This repulsive force causes the magnetic flux to spread out at the edges of the air gap.

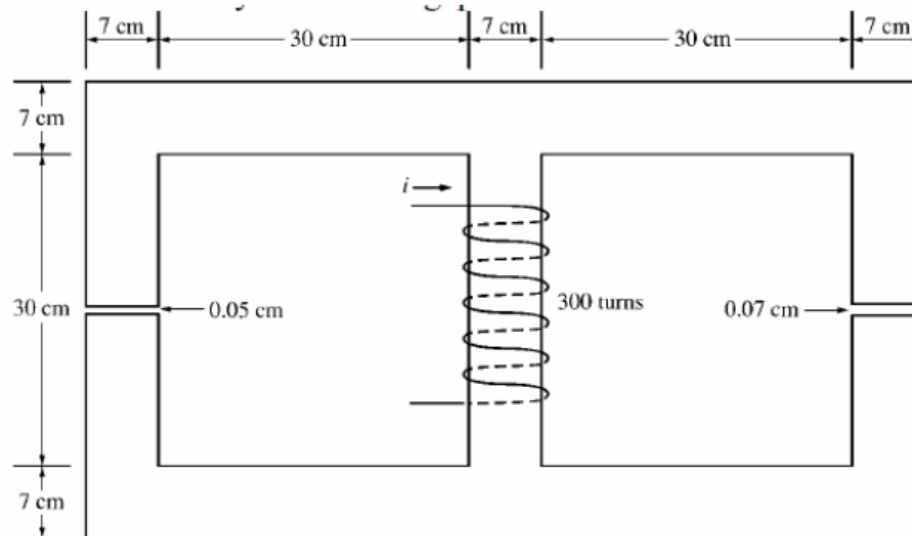
➤ Thus, this **tendency** of the magnetic flux to spread out at the edges of the air gap is what is known as **Magnetic Fringing**.

❖ **Effects of magnetic fringing:** Because of the spread out, the **effective cross-section area of the air gap is increased**, which in turn **reduces the magnetic flux density** of the air gap.

❖ **Control of Leakage flux and Magnetic fringing:** Both effects can be reduced by selecting **high-quality** magnetic materials and making the air gap as **narrower** as possible.

Example 3.2

A ferromagnetic core with a relative permeability of 2000 is shown in Figure. The dimensions are as shown in the diagram, and the depth of the core is 7 cm. The air gaps on the left and right sides of the core are 0.050 and 0.070 cm, respectively. If there are 300 turns in the coil wrapped around the center leg of the core and if the current in the coil is 1.0 A, what is the flux in each of the left, center, and right legs of the core? What is the flux density in each air gap?



Solution

Calculating the circuit elements:

$$\mathcal{R}_l = \frac{l_l}{\mu_r \mu_o A} = \frac{3 * 37 * 10^{-2}}{2000 * 4\pi * 10^{-7} * 7 * 7 * 10^{-4}} = 90.1 * 10^3 \text{ AT/wb}$$

$$\mathcal{R}_r = \frac{l_r}{\mu_r \mu_o A} = \frac{3 * 37 * 10^{-2}}{2000 * 4\pi * 10^{-7} * 7 * 7 * 10^{-4}} = 90.1 * 10^3 \text{ AT/wb}$$

$$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_o A} = \frac{37 * 10^{-2}}{2000 * 4\pi * 10^{-7} * 7 * 7 * 10^{-4}} = 30 * 10^3 \text{ AT/wb}$$

$$\mathcal{R}_{gl} = \frac{l_{gl}}{\mu_o A} = \frac{0.05 * 10^{-2}}{4\pi * 10^{-7} * 7 * 7 * 10^{-4}} = 81.2 * 10^3 \text{ AT/wb}$$

$$\mathcal{R}_{gr} = \frac{l_{gr}}{\mu_o A} = \frac{0.07 * 10^{-2}}{4\pi * 10^{-7} * 7 * 7 * 10^{-4}} = 113.7 * 10^3 \text{ AT/wb}$$

The equivalent reluctance is:

$$\mathcal{R}_{eq} = \mathcal{R}_c + (\mathcal{R}_l + \mathcal{R}_{gl}) // (\mathcal{R}_r + \mathcal{R}_{gr}) = 123 * 10^3 \text{ AT/wb}$$

The flux in the central leg is:

$$\phi_c = \frac{NI}{\mathcal{R}_{eq}} = \frac{300 * 1}{123 * 10^3} = 2.44 * 10^{-3} \text{ wb}$$

The flux in the right and left legs using flux divider rule are:

$$\begin{aligned} \phi_r &= \phi_c \frac{\mathcal{R}_l + \mathcal{R}_{gl}}{\mathcal{R}_l + \mathcal{R}_{gl} + \mathcal{R}_r + \mathcal{R}_{gr}} \\ &= 2.44 * 10^{-3} \frac{90.1 * 10^3}{(90.1 + 81.2 + 90.1 + 113.7) * 10^3} = 1.11 * 10^{-3} \text{ wb} \end{aligned}$$

$$\begin{aligned} \phi_l &= \phi_c \frac{\mathcal{R}_r + \mathcal{R}_{gr}}{\mathcal{R}_l + \mathcal{R}_{gl} + \mathcal{R}_r + \mathcal{R}_{gr}} \\ &= 2.44 * 10^{-3} \frac{90.1 * 10^3}{(90.1 + 81.2 + 90.1 + 113.7) * 10^3} = 1.32 * 10^{-3} \text{ wb} \end{aligned}$$

The flux densities are:

$$B_c = \frac{\phi_c}{A} = \frac{2.44 * 10^{-3}}{7 * 7 * 10^{-4}} = 0.49 \text{ Tesla}$$

$$B_r = \frac{\phi_r}{A} = \frac{1.11 * 10^{-3}}{7 * 7 * 10^{-4}} = 0.22 \text{ Tesla}$$

$$B_l = \frac{\phi_l}{A} = \frac{1.32 * 10^{-3}}{7 * 7 * 10^{-4}} = 0.27 \text{ Tesla}$$

References

1. Ian McKenzie Smith, John Hiley and Keith Brown,(2008), **Hughes Electrical and Electronic/ Edward Hughes**.- 10th Ed., Prentice Hall, ISBN 0-13–114397-2
2. Allan R. Hambley: (2019), **Electrical Engineering: Principles and Applications:** International Edition, 4/E, Prentice Hall,ISBN-10 0132066920
3. S. K. Sahdev, **Electical Machines**, vol. 53, no. 9. 2019.