UNIT 3.

3.1 PARTICLE SIZE ANALYSIS.

3.1.1 Introduction.

In this Unit we are going to learn about particle size, shape and distribution, particle-size distribution functions. Interaction of solid particles and fluids in gravitational and centrifugal fields.

Methods of sub-sieve size analysis, sedimentation methods, elutriation methods, cyclosizer, coulter counter, pore-size and volume determination, permeametry, laser light scattering methods are outline ending with On-stream particle-size analysis

3.1.2 Objectives.

By the end of this unit you should be able to;

1. Define the terms used in particle size characterisation for better liberation
2. Define various equivalent diameters
3. Various methods used in sub sieve size analysis

3.1.3 Reflection.

> What is the importance of particle size analyses in mineral processing, process technology industries chemical .and pharmaceutical applications? Well, the particle size is what determines the degree of liberation after comminution. This is particularly important for reactions involving surface phenomena and more especially in briquetting pills or capsules in the pharmaceutical industries

3.2 PARTICLE SIZE ANALYSIS.

The evaluation of particle characteristics is a critical component of mineral processing as it is used to estimate efficiencies of comminution equipment or processes of separation. More important, however, is the fact that characterisation of particles allows for effective control of unit processes. In size reduction for example, the aim is to liberate the mineral values from the gangue. However, over or under liberation results in problems of separation so that it is important to achieve the correct degree of particle size reduction.

In reality, however, particle size is not easy to measure because typical products of comminution have a wide range of characteristics and consideration has to be given to

a) the size of individual particles
b) the average size of all the particles
c) the shape of the particles
d) the size distribution of particulate systems
e) the density and surface characteristics of the particles.

These and other aspects of particle size analysis are considered in the following notes.

3.3. TERMINOLOGIES USED IN PATICLE SIZE ANALYSIS.

Particle: Discrete portion of matter, small in relation to surroundings. The absolute size is not important nor is the physical state i.e. not necessarily solid.

Particulate Material: This consists of a number of particles which retain their individuality i.e. they do not adhere together.

Powder: Consists of discrete particles of material with a maximum size of about 100μm.

Particulate System: Consists of a particulate material dispersed in a continuous phase of another material.

Sizing: Refers to the division of a particulate system into groups of nearly the same size.

Classification: Operation in which differences in density and size of particles are exploited to obtain grading by settling through a fluid. The fluid is normally water or air.

Sedimentation: The process of settling of particles in fluid by (usually) gravity.

Elutriation: Washing away of finer particles from a particulate system by rising currents of fluid.

3.4 PARTICLE SIZE.

3.4.1 Equivalent Diameters.

Only regular geometric shapes can have their size conveniently described because it is difficult to define any linear size without reference to the particle shape. As the shape becomes more complex the problems of defining the particle size increase e.g.

Sphere: only 1 dimension will uniquely define size.
For a cylinder: 2 dimensions will uniquely define size.
For a cuboid: 3 dimensions will uniquely define size.

Broken material, on the other hand have irregular shapes and their sizes are therefore difficult to quantify. It is convenient to express the size of such particles in terms of the diameter of a sphere which is equivalent to the particle with regard to some stated property. The advantage is that the size of the particle can be stated by a single number.

3.4.1.1 **Projected area diameter (d_a):** Diameter of the sphere having the same projected area as the particle when viewed in a direction perpendicular to the plane of greatest stability.

$$d_a = \left(\frac{4A_p}{\pi}\right)^{\frac{1}{2}}$$, where A_p is the projected area of the particle

3.4.1.2 **Volume diameter (d_v):** Diameter of a sphere of the same volume as the particle.

$$d_v = (6V/\pi)^{\frac{1}{3}} = \left(\frac{6V}{\pi}\right)^{1/3}$$ where V is the volume of the particle

3.4.1.3 **Surface diameter (d_s):** Diameter of a sphere having the same surface as the particle.

$$d_s = (S/\pi)^{1/2} = \left(\frac{S}{\pi}\right)^{1/2}$$ where S is the surface area of the particle

3.4.1.4 **Free falling diameter (d_f):** Diameter of a sphere having the same density and free-fall velocity as the particle in the same fluid medium.

3.4.1.5 **Stokes' diameter (d_{st}):** The free-falling diameter in the laminar flow regime

$$d_{st} = \sqrt{\frac{18V_t\eta}{(\rho_s - \rho_l)g}}$$

where $\mathbf{V_t}$ is the terminal velocity of a particle falling in fluid with a viscocity of η. $\boldsymbol{\rho_s}$ and $\boldsymbol{\rho_l}$ are the densities of the solid and the liquid and g is the gravitational pull.

3.4.1.6 **Sieve diameter (d_A):** Diameter corresponding to a square aperture of side A through which the particle just passes.

It can be seen that various methods measure different diameters. It is therefore vital whenever mentioning particle size to define the nominal diameter used. Microscopic measurements give d_a. Sedimentation and elutriation methods measure d_f or d_{st}.

3.5 PRESENTATION OF SIZING DATA .

3.5.1 Tabular Form.

The most precise method of data presentation is the tabular form. Results can be tabulated in the form in which they are obtained e.g. screening results as a weight frequency distribution.

Screen Size, um	Weight %
+ 212	0.4
-212 + 150	1.0
-150 + 106	6.8
-106 + 75	11.6
-75	80.2
	100

The frequency figures can be converted to a cumulative distribution, i.e.

Particle size, um	% by wt. > particle Size	% by wt < particle size
212	0.4	99.6
150	1.4	98.6
106	8.2	91.8
75	19.8	80.2

3.5.2 Graphical Presentation.

These methods have several advantages over tables.

a) Give a quick visual picture
b) Several size distribution data can be given in one graph. This facilitates comparisons
c) Yield numerical values describing distribution e.g. median, mode etc
d) Enable the fitting and determination of an equation describing the distribution and hence the parameters of the distribution. Conventionally the particle size is plotted as the abscissa and the measured quantity as the ordinate.

3.5.2.1 Histograms: This is the simplest method of the graphical techniques. Fig. 1 shows a histogram of weight % retained against screen size d_A. Histograms give a useful visual picture.

3.5.1.2 Size Frequency Curves: Provided the size intervals are small the histograms can be presented as a continuous curve drawn through the mid points of the ranges. This is shown in Fig. 2. Area under the curve approximates the total weight of the particles. The mode and the distribution of size (i.e. dispersion, skewedness) are readily distinguished in the continuously plotted area.

3.5.1.3 Cumulative Plots: These are useful for determining the amount of material greater or smaller than a specified size. The median and also the % between any two sizes are easily read off. The difference between the two curves gives the size frequency plot. The main disadvantage, however, is that the sigmoidal shape (see Fig 3) is difficult to express mathematically. Furthermore, the ends of the vertical scale have insufficient spread.

It is sometimes useful to plot size frequency and cumulative curves on linear scales to have an intuitive picture of the spread of the material over the entire size range. This, however, compresses the size scale at the fine end. Hence a logarithmic size scale is more often used to spread the data along the abscissa.

3.6 PARTICLE SIZE DISTRIBUTION FUNCTIONS.

The above methods of presenting particle size distribution data are normally adequate for quantitative description of sizing data. However, for analytical reasons it is desirable that the size distribution data conform to some mathematical distribution law to enable, for example, extrapolation to a different size range.

Several attempts have been made to arrive at such mathematical functions. Although it would appear that some of them can be mathematically derived, they are all based on empirical relationships which over the years have been modified to improve their description of comminution products. The equations are al of the general form

$$W_f = f_n\left(\frac{d}{d^*}\right)$$

where W_f = cumulative fraction or % finer than d

d^* = a reference size.

d^* is referred to as the size modulus. The equation also include an exponent to indicate the dispersion of the data and this is appropriately called the distribution modulus. Additional parameters have been suggested to account for skewness of the size distribution but this introduces additional complexity which is hardly justifiable.

The most commonly used functions in mineral processing are the Gaudin –Schuhmann and Rosin-Rammler functions.

3.6.1 Gaudin-Schuhmann (G.S.) function.

This was introduced by Professor Gaudin (1926) and modified by Professor Schuhmann (1940). The function is defined by

$$W_f = 100\left(\frac{d}{k}\right)^m, \quad \frac{W_f}{100} = \left(\frac{d}{k}\right)^m$$

k is the size modulus and defines the maximum size for which the function is valid. For $d = k$, $w_f = 100\%$

m indicates the dispersion in the range in which the function is valid. For a particular sample, m and k will be constants.
Taking logs of the G.S. equation

$$\text{Log } (w_f/100) = m \log d - m \log k$$

If the function therefore fits the data a straight line will result if log W_f is plotted against log d. This is useful because it allows for extrapolation of the line to cover particle size ranges outside those used to establish the graph (e.g. into the sub-sieve range) – However, actual data often deviates from a straight line particularly at the fine particle size range as shown in Fig. 4 and at very coarse sizes. Nevertheless the distribution is widely used in mineral processing because of its simplicity.

The <u>weight frequency</u> curve can easily be obtained as the derivative of the G.S. function, but only for values of $m > 1$, for $m < 1$, $d \rightarrow 0$ as dW_f/dp increases.

3.6.2 Rosin - Rammler (R-R) Function.

It was introduced by Professor Rosin in 1933 and can be defined in terms of cumulative weight % retained (or finer), W_r

$$w_r = 100e^{-\left(\frac{d}{d_o}\right)^n}$$

d_o = absolute size constant or characteristic particle size.

n = dispersion coefficient or distribution modulus

$W_r = 100 - W_f$

If $d = d_o$

$W_r = 100/_e = 36.8\%$

Hence d_o is the size at which 36.8% of the particles is retained.
Taking logs twice,

$$\text{Log In } \left(\frac{W_r}{100}\right) = -\left(\frac{d}{d_0}\right)^n \; or \; ln\left(\frac{100}{W_r}\right) = \left(\frac{d}{d_0}\right)^n$$

$$\text{Log ln } (100/W_r) = n \text{ log} d - n\text{log } d_o$$

Hence if the data fits the R-R equation a plot of log ln $\frac{(100)}{W_r}$ against logd will give a straight line of slope n. If n is small the particles are spread over a wide size range but as all the particles approach the same size, n tends to infinity.

3.6.3 Comparison of the G-S and R-R functions.

Both the G-S and R-R functions are only suitable for cumulative weight data. The R-R function, however, expands both ends of the vertical scale unlike the G-S function. However, its widespread use appears to be limited by the scarcity of suitable graph paper.

In practice it is not easy to state with certainty which function suits the data best. At fine particle sizes the G-S and R-R functions became similar. From the G-S equation

$$\frac{W_f}{100} = \left(\frac{d}{k}\right)^m, or \; \frac{W_r}{100} = 1 - \left(\frac{d}{k}\right)^m$$

$$Hence \; ln\frac{W_r}{100} = \ln\left(1 - \frac{d}{k}\right)^m$$

$$= -\left(\frac{d}{k}\right)^m - \tfrac{1}{2}\left(\frac{d}{k}\right)^{2m} - \tfrac{1}{3}\left(\frac{d}{k}\right)^{3m} - - -$$

Neglecting the higher order terms of the expansion will give an equation similar to the R-R equation. In practice it may also be advantageous to use d_{80} rather than d_{100} in place of k in the G-S function. This is because of the deviation of the equation at coarse sizes.

References and Recommended Readings.

1. **Gy. P. M.,** "Sampling of Particulate Materials: Theory and Practice." Elsevier Scientific Publishing Co., Amsterdam, 1979.
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