# THE UNIVERSITY OF ZAMBIA

## DEPARTMENT OF MINING ENGINEERING

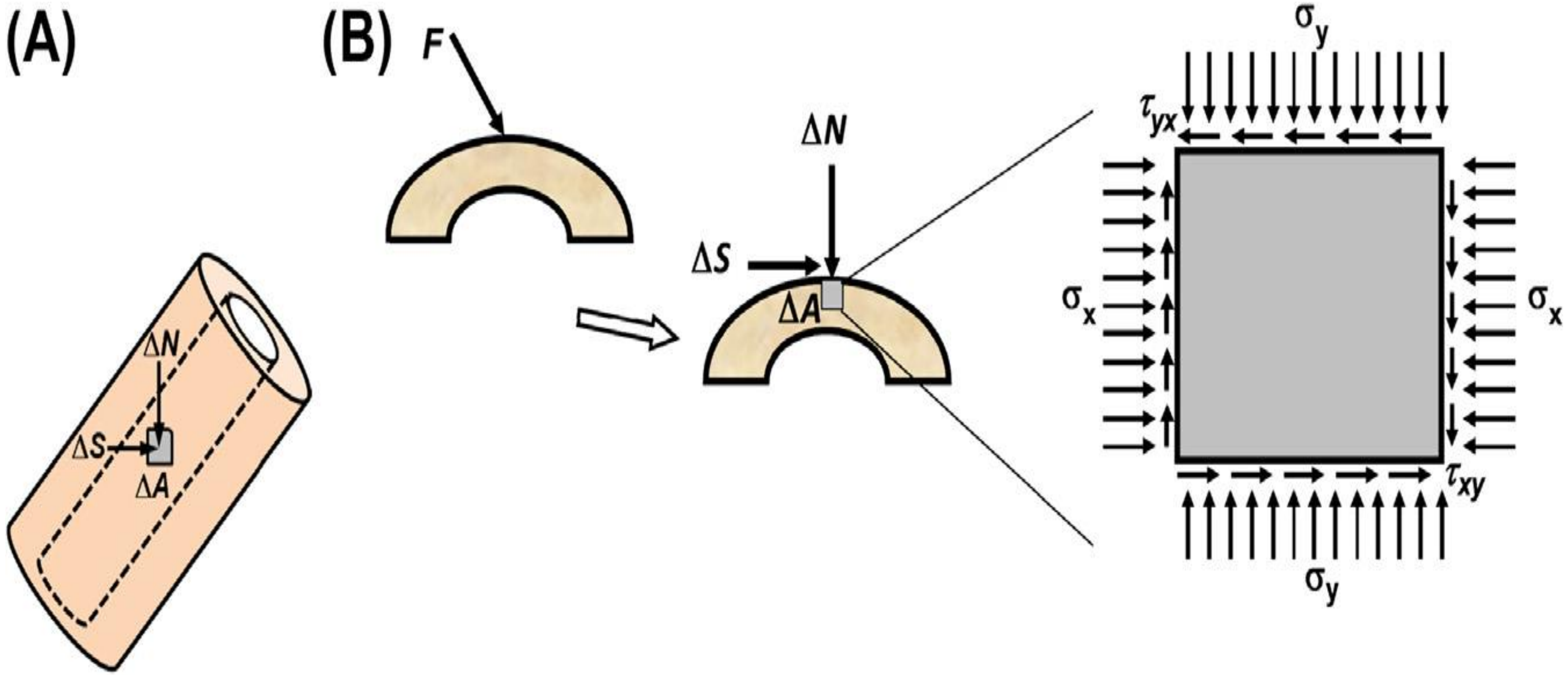

# BASIC ROCK MECHANICS

TOPIC 3-STRESS & STRAIN

# Normal and Shear stresses

- The stress is equal to the force divided by the area.
- On a real or imaginary plane through a rock, there can be normal force ($\Delta N$) and shear force ($\Delta S$ ), as shown in Fig. 1.
- The forces induce normal and shear stresses in the rock.

# Normal and Shear stresses



# Normal and Shear stresses

- The normal (shear) stress is the normal (shear) force per unit area as shown in Fig. 1.1A.
- The normal and shear forces and normal and shear stress components are shown in Fig. 1.1B.
- The normal stress is perpendicular to each of the planes, but the shear stress is parallel to each of the planes as shown in Fig. 1.1B.

# Normal and Shear stresses

- The normal and shear stresses can be mathematically defined as follows when the size of the small area is reduced to zero:

$$\text{normal stress, } \sigma_n = \lim \frac{\Delta N}{\Delta A} (\Delta A \to 0) \quad (1.1)$$

$$\text{shear stress, } \tau = \lim \frac{\Delta S}{\Delta A} (\Delta A \to 0) \quad (1.2)$$

# Stress components

- If an infinitesimal cube is cut within the rock, it will have normal and shear stresses acting on each plane of the cube.
- The compressive normal stress is positive, and the tensile normal stress is treated as negative in rock mechanics sign convention.
- Each normal stress is perpendicular to each of the planes, as shown in Fig. 1.2.

# Stress components

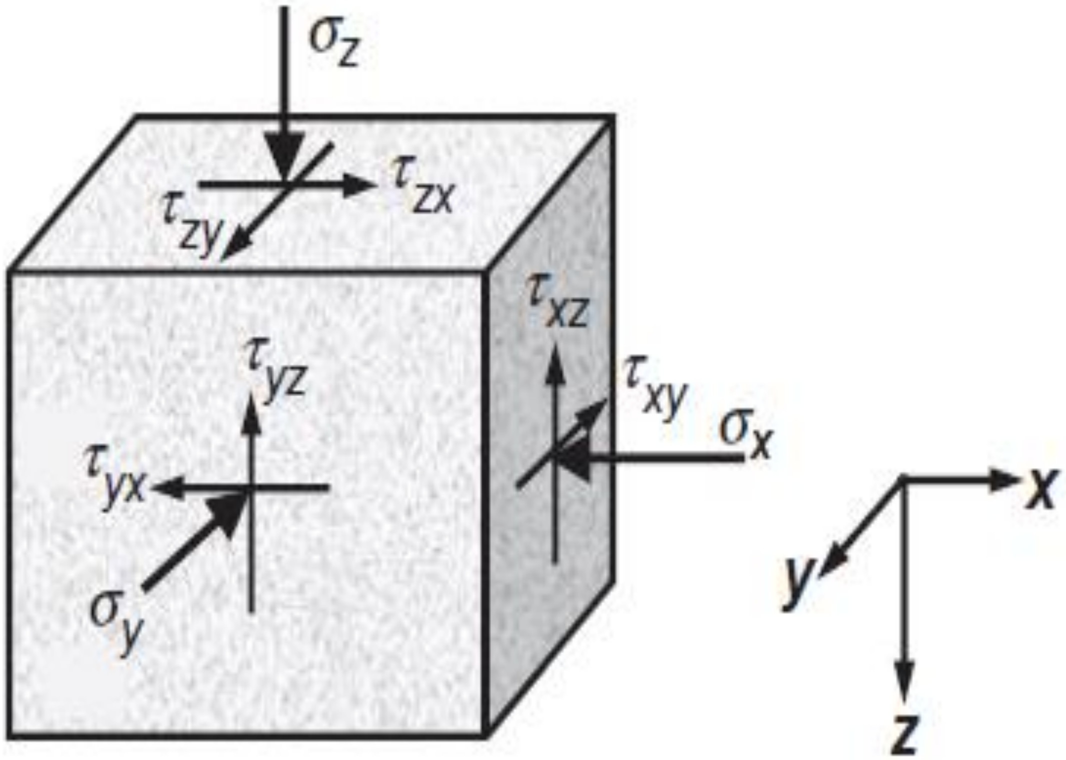



Figure 1.2 Normal and shear stress components on an infinitesimal cube in the rock.

# Stress components

- The shear stress on any face in Fig. 1.2 has two perpendicular components that are aligned with the two axes parallel to the edges of the face.
- Therefore, there are nine stress components comprising three normal components and six shear components acting on a cubic element.

# Stress components

- The stress tensor can be expressed as follows:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \qquad (1.3)$$

# Stress components

- By considering equilibrium of moments around the $x$, $y$, and $z$ axes, the shear stresses have the following relations:

$$\tau_{xy} = \tau_{yx}, \quad \tau_{yz} = \tau_{zy}, \quad \tau_{xz} = \tau_{zx} \qquad (1.4)$$

# Stress components

- Therefore, the state of stress at a point is defined completely by six independent components.
- These are three normal stress components

$$(\sigma_x, \sigma_y, \sigma_z)$$

- and three shear stress components

$$(\tau_{xy}, \tau_{yz}, \tau_{zx})$$

# Stresses in an inclined plane

- The principal stresses in two dimensions are very useful because many engineering problems of practical interest are effectively two-dimensional, which can be simplified as the state of plane strain.

# Stresses in an inclined plane

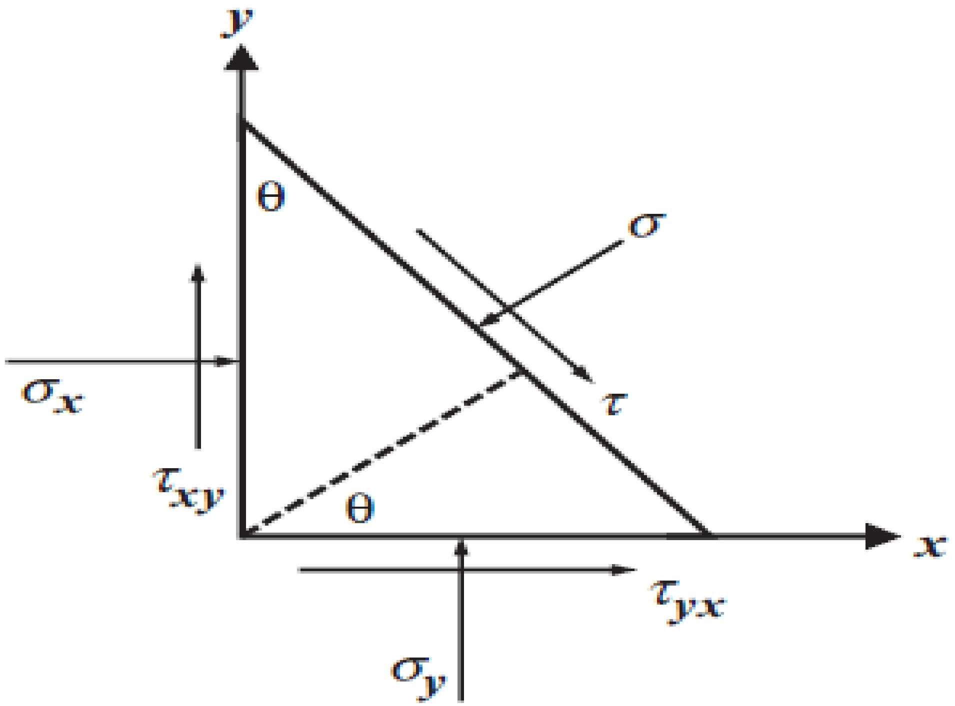



Figure 1.3 Force equilibrium on a small triangle element, assuming that all the stress components are positive.

# Stresses in an inclined plane

- Consider a two-dimensional small triangular element of the rock in which the normal stresses $\sigma_x$ and $\sigma_y$ and shear stress $\tau_{xy}$ act in the $xy$-plane.
- The normal ($\sigma$) and shear ($\tau$) stresses at a surface oriented normal to a general direction q in the $xy$-plane (Fig. 1.3) can be calculated as follows:

# Stresses in an inclined plane

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (1.5)$$

# Stresses in an inclined plane

- By proper choice of $\theta$, it is possible to obtain $\tau = 0$. From Eq. (1.5) this happens when:

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{1.6}$$

# Stresses in an inclined plane

- Eq. (1.6) has two solutions, $\theta_1$ and $\theta_2$. The two solutions correspond to two directions for which the shear stress $\tau$ vanishes. These two directions are named the principal axes of stress.
- The corresponding normal stresses, $\sigma_1$ and $\sigma_3$, are the principal stresses, and they are found by introducing Eq. (1.6) into the first equation of Eq. (1.5):

# Stresses in an inclined plane

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\tau_{xy}^2 + \frac{(\sigma_x - \sigma_y)^2}{4}}$$

$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\tau_{xy}^2 + \frac{(\sigma_x - \sigma_y)^2}{4}} \qquad (1.7)$$

# Stresses in an inclined plane

- Thus, in the direction $\theta_1$, which identifies a principal axis, the normal stress is $\sigma_1$and the shear stress is zero.
- In the direction $\theta_2$, which identifies the other principal axis, the normal stress is $\sigma_3$and the shear stress is also zero.
- The principal axes are mutually orthogonal.

# Stresses in an inclined plane

- All unsupported excavation surfaces are principal stress planes.
- This is because all unsupported excavation surfaces have no shear stresses acting on them and are therefore principal stress planes.

# Principal stresses

- It is possible to show that there is one set of axes with respect to which all shear stresses are zero, and the three normal stresses have their extreme values, as shown in Fig. 1.4.

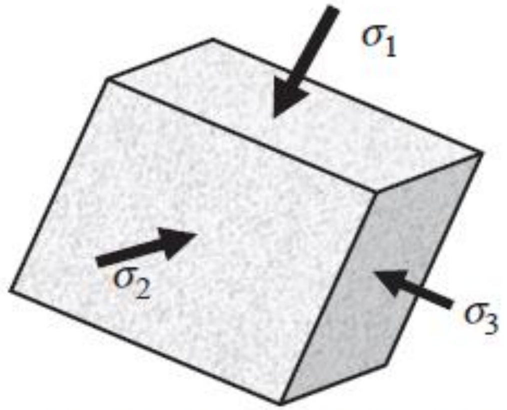



Figure 1.4 Principal stress components in the principal planes.

# Principal stresses

- These three mutually perpendicular planes are called principal planes, and the three normal stresses acting on these planes are the principal stresses.
- It is convenient to specify the stress state using these principal stresses because they provide direct information on the maximum and minimum values of the normal stress components

# Principal stresses

- The values $\sigma_1$, $\sigma_2$, and $\sigma_3$ in Fig. 1.4 are the principal stresses, and $\sigma_1 > \sigma_2 > \sigma_3$ which are three principal stress components.
- Therefore, the principal stress tensor can be expressed as follows:

# Principal stresses

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \tag{1.8}$$

# Mohr’s circle representation of stresses

- Mohr’s circle or the Mohr diagram is a useful tool to represent the stress state and rock failure.
- The Mohr circle can be used to determine graphically the stress components acting on a rotated coordinate system, i.e., acting on a differently oriented plane passing through a particular point (e.g., the point P in Fig. 1.7A).

# Mohr's circle representation of stresses

- When the principal stresses ($\sigma_1$, $\sigma_3$) are available, a two-dimensional Mohr's circle can be illustrated in Fig. 1.7A; notice that the stresses plotted in $x$-axis are the principal stresses.
- The diameter of the circle is $\sigma_1 - \sigma_3$ and the center is at $((\sigma_1 + \sigma_3)/2,0)$.

# Mohr's circle representation of stresses

- The normal stress $\sigma$ and shear stress $\tau$ at each point on the circle represent a state of stress on a plane whose normal direction is inclined at $\theta$ to $\sigma_1$ (i.e., $\theta$ is the angle between the inclined plane and the direction of $\sigma_3$), as shown in Fig. 1.7B.
- From the Mohr circle diagram, the normal and shear stresses at each point (e.g., the point P) or in each inclined plane can be easily obtained, i.e.,

# Mohr's circle representation of stresses

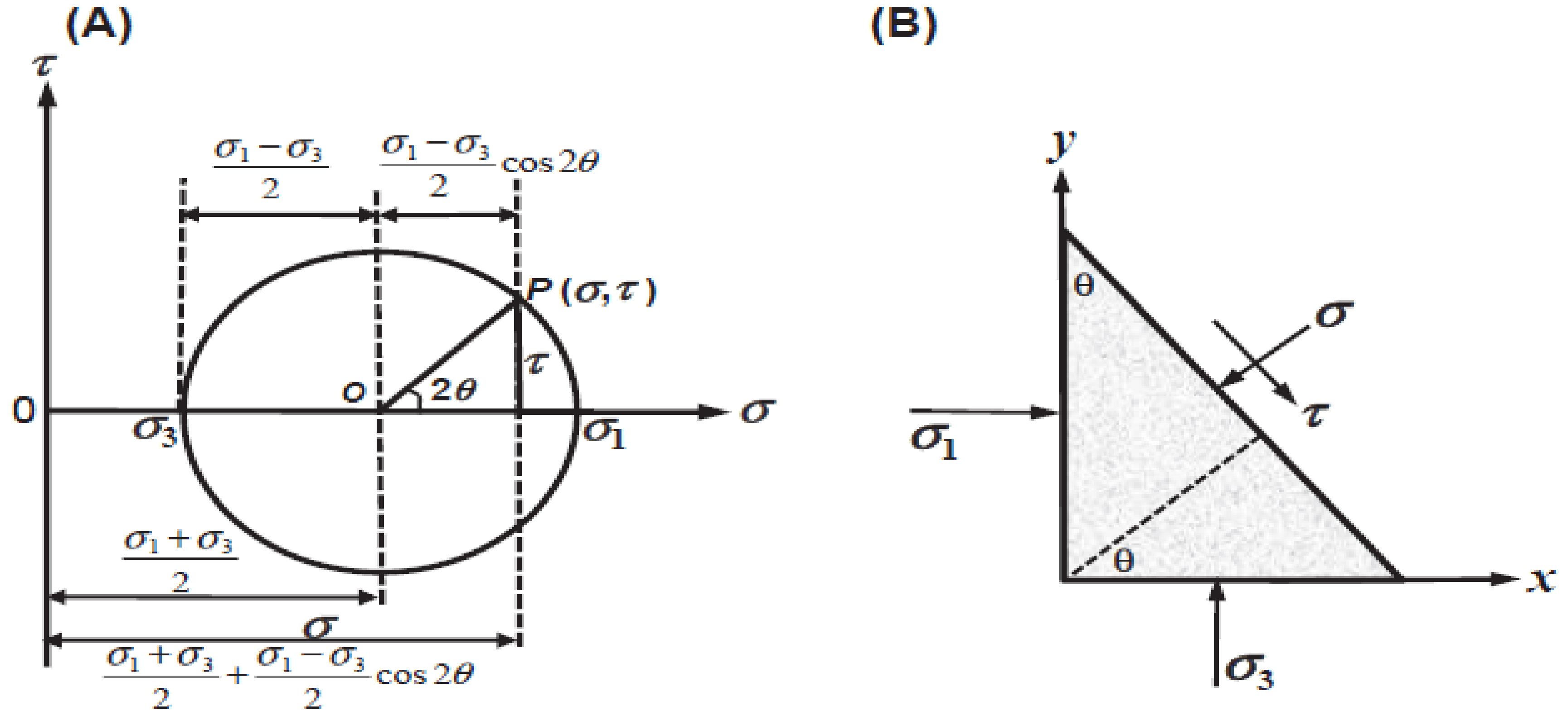



**Figure 1.7** (A) Mohr's circle diagram for a two-dimensional state of stresses (principal stresses $\sigma_1$ and $\sigma_3$). (B) Shear and normal stresses on a plane exerted by the far-field principal stresses ($\sigma_1$ and $\sigma_3$) (corresponding to the stress state at the point P).

# Mohr's circle representation of stresses

- Eqs. (1.12) and (1.5) are also very useful for analyzing stress states in fractures.

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta \qquad (1.12)$$

# Mohr's circle representation of stresses

- The maximum shear stress can be obtained from Eq. (1.12) when $2\theta = 90$ degrees, that is,

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \tag{1.13}$$

# Mohr's circles for three-dimensional stresses

- To construct Mohr's circles for a three-dimensional case of stresses at a point, the values of the principal stresses ($\sigma_1$, $\sigma_2$, $\sigma_3$) and their principal directions ($n_1$, $n_2$, $n_3$) must be first evaluated.
- When the principal stresses are available, the three-dimensional Mohr's circles can be plotted as illustrated in Fig. 1.8.

# Mohr's circles for three-dimensional stresses

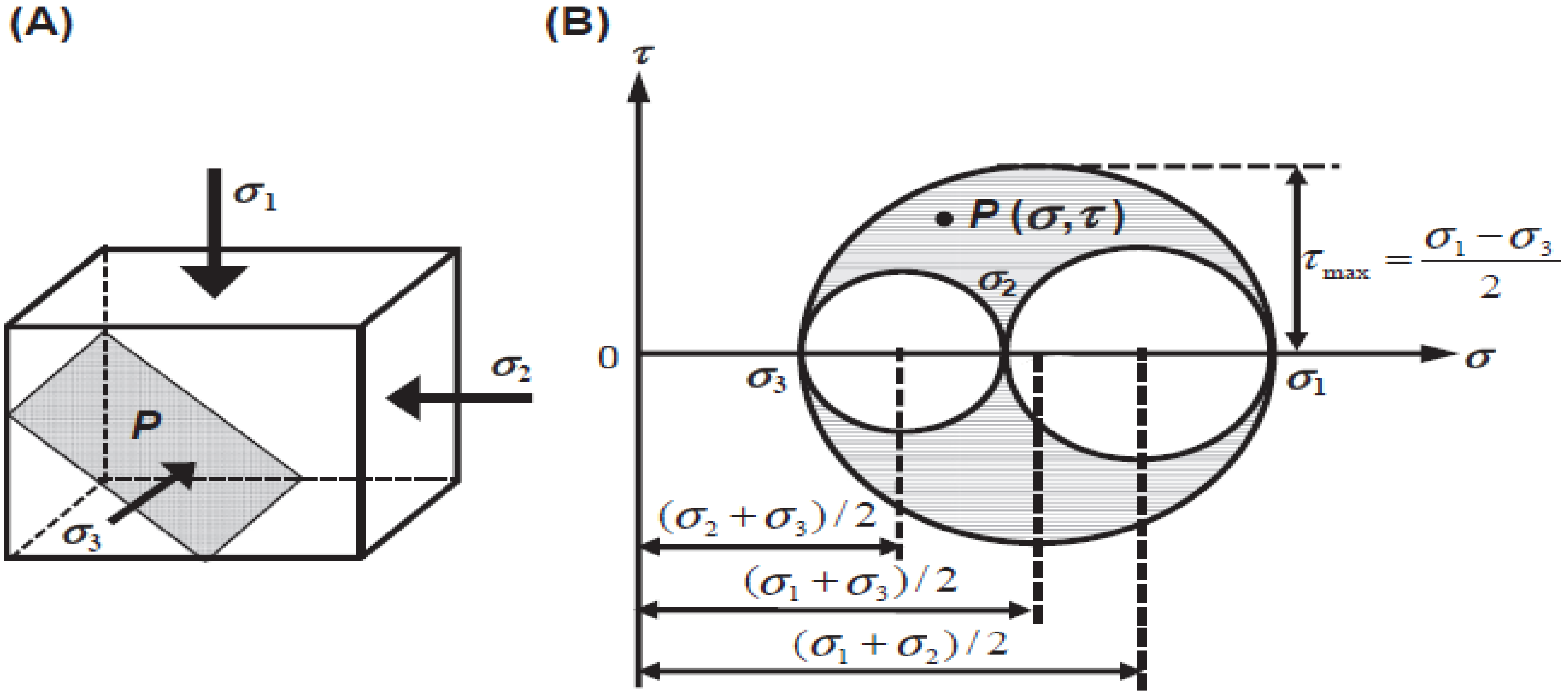



Figure 1.8 (A) Principal stresses and (B) a plane in a cube represented by Mohr's circles in a three-dimensional stress state. The *dashed vertical lines* point to the centers of the three circles.

# Mohr’s circles for three-dimensional stresses

- All admissible stress points ($\sigma$, $\tau$) lie on the three circles or within the shaded area enclosed by them, as shown in Fig. 1.8B, and each of those points (such as the point P) represents a state of stress on a plane (e.g., a weak plane or a fault plane) in the cube in Fig. 1.8A.

# Mohr's circles for three-dimensional stresses

- Three-dimensional Mohr's circles combined with shear failure envelopes can be used to analyze normal and shear stresses in fault planes for assessment of shear failures and fault reactivations.
- The maximum shear stress is the same to the one obtained from Eq. (1.13).