# MIN 3029

## ROCK MECHANICS

# Rock and Rock mass.

## Intact rock (Chile).

- Continous
- homogeneous - Same properties in different directions.
- Isotropic - Same properties in all directions.
- elastic.

## Rock Mass (Diane)

- Discontinous
- Inhomogeneous
- Anisotropic
- Non - elastic

## Physical Properties of Rocks.

1 Bulk density

Mass density - mass per unit volume of an Object

Weight density - weight per unit volume of an Object.

$$W = m \times g$$
$$= kg \times m/s^2 \qquad 1N = kg.ms^{-2}.$$
$$= kg\,m/s^2$$
$$= N$$

**Bulk density**: Suppose that a sample of rock is under examination let further assume that was cut from an outcrop. Let the volume sample be V and let the mass of the sample M. The density $\rho$ of the rock sample can be calculated as follows;

$$\rho = \frac{M}{V}$$

In order to calculate or establish what is known as the Bulk density more samples have to be collected from the outcrop. The density of each sample should be calculated. After the average density of the rock sample will be a close estimation to actual Bulk density of the rock mass.

$$\text{Bulk density} = \sum_{i=1}^{n} \frac{Si}{n}$$

Where n = Number of samples obtained from the outcrop.

**Specific Gravity**: The specific gravity G of a substance is defined as

$$G = \frac{\rho}{\rho_w}$$

where $\rho$ = density of substance
$\rho$ = density of water

- The value of specific gravity has no units since G is a ratio of two densities.
- The two values for the density of water

That are commonly used are

$\rho_w = 1.0\ g/cm^3$

$= 1000\ kg/m^3$

In Our course we will use the value of $1000\ kg/m^3$ for the density of water for the purpose of calculating specific gravity.

Example!

A block of rock with edge length 85.5cm, 79.0 cm and 43.8 cm has a mass of 953kg. Find the specific gravity of the rock.

$$V = 0.855\ m \times 0.79\ m \times 0.438$$
$$= 0.2958471\ m^3$$

$$\rho = \frac{M}{V}$$
$$= \frac{953\ kg}{0.296\ m^3}$$
$$= 3219.59\ kg/m^3$$

$$G = \frac{\rho}{\rho_W}$$
$$= \frac{3219.59\ kg/m^3}{1000\ kg/m^3}$$
$$= 3.22$$

Unit weight : The unit weight ($\gamma$) of an Object is defined as follows

$$\gamma = \frac{W}{V} \quad \text{--- (1)}$$

where $W$ = weight of an Object.

$V$ = Volume of an Object.

Units for unit weight is $N/m^3$ The unit Weight Can also be expressed in terms of mass as follows

$$W = mg \quad \dots \quad (2)$$

as such, if we replace $w$ by equation 2 in equation 1 then.

$$\gamma = \frac{mg}{V}$$

$$\gamma = \left(\frac{m}{V}\right)g$$

$$\gamma = \rho g \quad \dots \quad (3)$$

Take note $g = 9.81\ m/s^2$

Specific gravity Can also be expressed in terms of unit weight as follows.

$$G = \frac{\rho}{\rho_w}$$

$$\frac{\rho g}{\rho_w g}$$

$$\frac{\gamma}{\gamma_w}$$

The unit weight of water, $\gamma_w = \rho_w \times g$

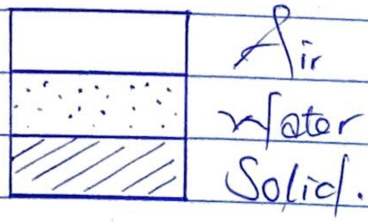
$$= 1000\ kg/m^3 \times 9.81\ m/s^2$$

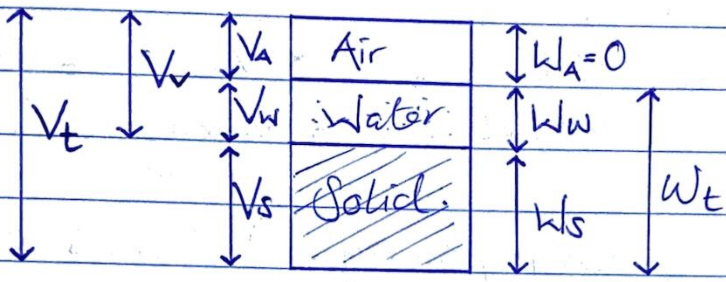
$$= 9810\ \frac{kg.m/s^2}{m^3}$$

$$= 9810\ N/m^3$$

$$= 9.81\ kN/m^3.$$

**Porosity**: Concept of Porosity can be best described using what is known as a three phase system.




$V_A$ = Volume of air
$V_W$ = Volume of water
$V_S$ = Volume of Solid
$V_V$ = Volume of Voids
$V_t$ = total volume of rock Sample.

$W_A$ = Weight of air
$W_W$ = Weight of Water
$W_S$ = Weight of Solid
$W_t$ = total weight of rock sample.

$$\text{Porosity } (n) = \frac{V_v}{V_t} \quad \ldots \quad (1)$$

$$V_t = V_v + V_s \quad \ldots \quad (2)$$

$$\text{Porosity } (n) = \frac{V_v}{V_v + V_s} \quad \ldots \quad (3)$$

Expressing Porosity (n) in terms of density.

$$\text{density of density} = \rho_s = \frac{M_s}{V_s} \quad \ldots \quad (4)$$

$$n = \frac{V_v}{V_t}$$

$$V_s = V_t - V_v$$
$$= V_t - nV_t$$
$$= V_t(1-n) \quad \ldots \quad (5)$$

Substituting equation 5 into 4

$$\rho_s = \frac{M_s}{(1-n)V_t} \quad \ldots \quad (6)$$

for dry rocks ($M_s = M$.) implying that equation 6 becomes.

$$\rho_s = \frac{m}{(1-n)Vt} = \frac{m \times \frac{1}{Vt}}{(1-n)V_t \times \frac{1}{Vt}} = \frac{\rho}{(1-n)}.$$

Expressing Porosity (n) interms of unit weight

$$\gamma_s = \frac{\gamma}{1-n} \quad \ldots \quad (8)$$

Example!

A $0.885 m^3$ block of Sandstone has a mass of 1752 kg. When the block is crushed just sufficiently to close all pores, which are empty, the volume of rock becomes $0.584 m^3$ find

(a) The Porosity of Sandstone
(b) The density of the grains.

$$\text{Porosity}(n) = \frac{V_v}{V_t}$$

$$V_t = V_{voids} + V_{solids}$$

$$V_{voids} = V_{total} - V_{solids}$$

$V_{total} = 0.885 m^3$

$V_{solids} = 0.584 m^3$

$$= 0.885 m^3 - 0.584 m^3$$
$$= 0.301 m^3.$$

$$Porosity\ (n) = \frac{0.301}{0.885} = 0.340.$$

(b) $\rho_s = \frac{M_s}{V_s} = \frac{1752\ kg}{0.584\ m^3}$ OR $\rho_s = \frac{\rho}{1-n}$

$= 3000\ kg/m^3$

## Dry and Saturated Unit weights.

The Pores of insitu rock may be filled with gas or Liquid. The densities of gases that are found in rock may vary much less than the densities of the solid/grains.

A similar statement cannot be made for liquids. The densities of liquids commonly found in the Pore of rocks, although less than the densities of grains are not very much less.

If the Porosity of the rock is large enough, and significant fraction of Pores contain liquid, the mass of liquid cannot be treated as negligable.

An important relation is that between the unit weight $\gamma_{sat}$ of saturated rock sample, the unit weight $\gamma_{dry}$ of same sample when it is dry and the unit weight $\gamma_L$ of liquid occupying the Pores of saturated Sample.

$$\gamma_{sat} = \gamma_{dry} + \gamma_L \times$$

Note that

$$W_{sat} = W_{dry} + W_L \quad \ldots \quad (9)$$

also note that the volume $V$ of the rock sample is the same whether it is dry or saturated.

Dividing equation (9) with the volume of rock Sample V, we have

$$\frac{W_{sat}}{V} = \frac{W_{dry}}{V} + \frac{W_L}{V}$$

$$\gamma_{sat} = \gamma_{dry} + \frac{W_L}{V}$$

Further note that the weight of Liquid $V_L$ is

$$\gamma_L = \frac{W_L}{V_L}$$

Where $V_L$ is the volume of Liquid with weight $W_L$. if we assume that the rock is fully Saturated.

$$V_L = V_v$$

Hence, by the defination of Porosity

$$n = \frac{V_v}{V} = \frac{V_L}{V}$$

$$V_L = nV \quad \ldots \quad (12).$$

Substitute equation (12) into equation (10) we get.

$$\gamma_{sat} = \gamma_{dry} + \frac{W_L}{\frac{V_L}{n}} = \gamma_{dry} + \frac{nW_L}{V_L} = \gamma_{dry} + n\gamma_L$$

$$\therefore \gamma_{sat} = \gamma_{dry} + n\gamma_L \quad \ldots \quad (13).$$

A Similar notation holds between densities.

$\gamma = \rho g$ (remember)

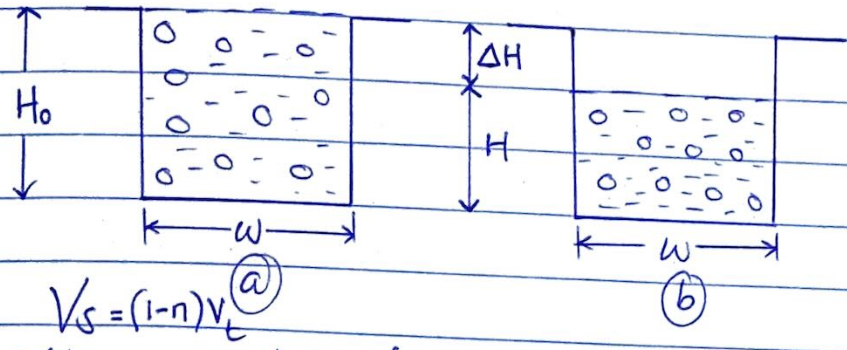
A such, equation becomes.

$$\rho_{sat}\cancel{g} = \rho_{dry}\cancel{g} + n(\rho_L\cancel{g})$$

$$\rho_{sat} = \rho_{dry} + n\rho_L \quad \ldots \quad (14).$$

- fully Saturated $V_w = V_v$
- Partially Saturated $V_w < V_v$
- dry rock $V_w = 0$.

# Subsidence




$V_s = (1-n)V_t$

Volume of Solids before Compaction

$V_s = (1-n_o)\,V_{to}$

where $n_o$ is the Porosity before self Compaction (a)
and $V_{to}$ is the total volume of material before Compaction.

then

$V_s = (1-n_o)\,H_o W L$ (where L is the length of the excavation)

Volume of Solids after Compaction / after Subsidence
and $V_t$ is the total volume after Subsidence.

then

$V_s = (1-n)\,H W L$

Volume of Solids before Subsidence is equal to the Volume of Solids after Subsidence.

i.e $(1-n_o)\,H_o W \cancel{L} = (1-n)\,H W \cancel{L}$

$H_o = \Delta H + H$

$(1-n_o)\,H_o = (1-n)\,H \qquad H = H_o - \Delta H$

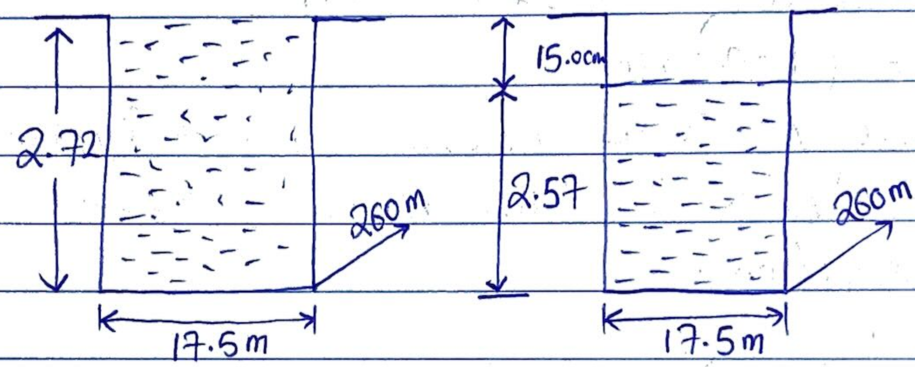
$(1-n_o)\,H_o = (1-n)(H_o - \Delta H)$

$$\Delta H = H_o\left[\frac{n_o - n}{1-n}\right]$$

Example

A layer of Clay with a Porosity of 47.0% (0.47) and saturated with Water is deposited into a rectangular trench 260 m Long and 17.5 m wide to the depth of 2.72m later. it is found that the Clay has settled by 15.0cm. Find the volume of water squeezed of clay.

$n_0 = 0.47$




$$\Delta H = H_0 \left[\frac{n_0 - n}{1 - n}\right]$$

$H_0 = 2.72$

$n_0 = 0.47$

$\Delta = 15cm = 0.15m.$

$n = 0.44$

$V_w = V_t - V_s$

# Weathering and Slaking.

## What is weathering?

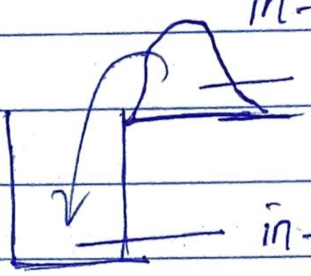
- Weathering is slow break down of rocks when exposed to atmospheric conditions.
- This weathering process can either be mechanical (also called physical) or chemical.
- Mechanical weathering as the physical breaking down of rocks when subjected to atmospheric conditions.
- Under mechanical or physical weathering, the chemical composition of the rocks remain the same.
- Chemical weathering is the breaking down of rocks as a result of chemical processes / reactions taking place in the rock as a result of atmosphere exposur
- Under chemical weathering, the chemical composition of rock is altered / changed.
- In general, chemical weathering is an exothemic reaction in nature and also results in volumetric increase.
- The following ae the common chemical weathering reaction that occur in rocks.
  - (i) Sollution : This is a type of reaction whereby a mineral disolves during weathering.
  - (ii) Hydrolysis : This type of reaction is between acidic weathering solution and many of the silicate minerals.
  - (iii) Hydration : This type of reaction corresponds to the penetration of water into the lattice structure of mineral.
  - (iv) Oxidation : This type of reation corresponds to the reaction of free oxygen with metallic elements.

Slaking is basically an index that describes the degree of alterability

determining the slake durability (Lab work)

## Swelling Potential.

Swell factor = $\frac{\text{loose Volume}}{\text{in-situ Volume / bank Volume.}}$




Swell volume = initial volume + Change in volume.

= $V_0 + \Delta V$

→ the extent of the change in volume describes the Swelling Potential.

## Hardness and abraisiveness.

The engineering application of hardness and abraisive Properties of rocks is mostly in the Prediction of rock drillability, cuttability, borability and tunnel boring machine advance rate.

## Degree of fissuring.

# Mechanical properties of Rocks.

- Unconfined Compressive Strength
- Stress
- Stress analysis. ← insitu stress.
- Strain and deformation
- Strain analysis
- Stress – Strain relationship.

## Unconfined Compressive Strength (UCS)

Suppose that, from a block of a particular rock type, a test Specimen in the Shape of right circular cylinder ( length about twice diameter is cut. the values of length and diameter are recorded. The cylinder is placed on a test bench, standing on one of it's circular ends.

A force F is applied perpendicular to the top force, area A and is directed towards it. to ensure that the action of this applied force is distributed uniformly over the face, the force actually is applied to a plate on the cylinder. The bench exerts an equal and opposite force f upwards on the bottom force ~~of~~ the cylinder. There are no forces that act on the curved sides of cylinder, Laterally, the cylinder is unconfined.

The Procedure for unconfined compressive Strength.

Example

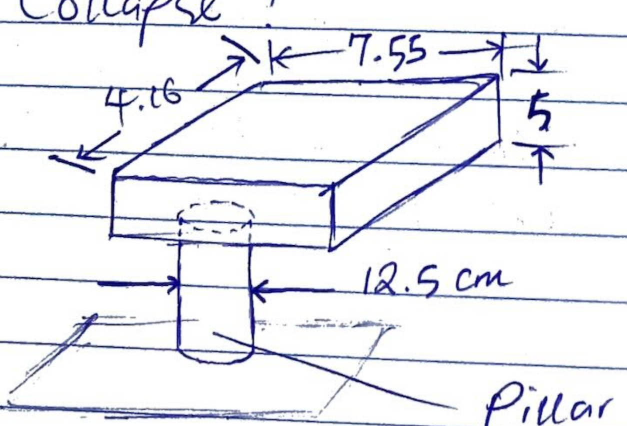
A load of rock 1730 N is applied perpendicularly to a square surface of edge length 1.43 cm. The sample fails at the applied load of 1730 N. Calculate the UCS of the sample.

$$UCS = \frac{\text{load (N)}}{\text{Surface Area } (m^2)}$$

Units : $N/m^2 \longrightarrow Pa$.

Class Exercise.

A rectangular block of rock with base length 7.55 m and base width of 4.16 m rests on a pillar of diameter 12.5 cm. The unconfined compressive strength of pillar rock is 62.3 MPa. The unit weight of block is 29.3 KN/m³. For what height h of the block is the pillar on the point collapse?



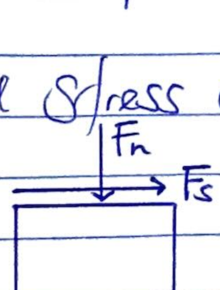
Stress.

Justification to the study of Stress in Rock mechanics.

The difference between Scaler, Vector and tensor

- A scalar is a quantity with magnitude only.
- A vector quantity has both magnitude and direction
- tensor is a quantity with magnitude, direction and the plane under consideration.

Normal Stress component and Shear Stress component.



- The normal and shear stress components are the normal and shear force per unit Area.
- We use $F_n$ and $F_s$ notation to represent the normal and shear force respectively.
- $\sigma$ and $\tau$ to represent the normal stress and shear respectively

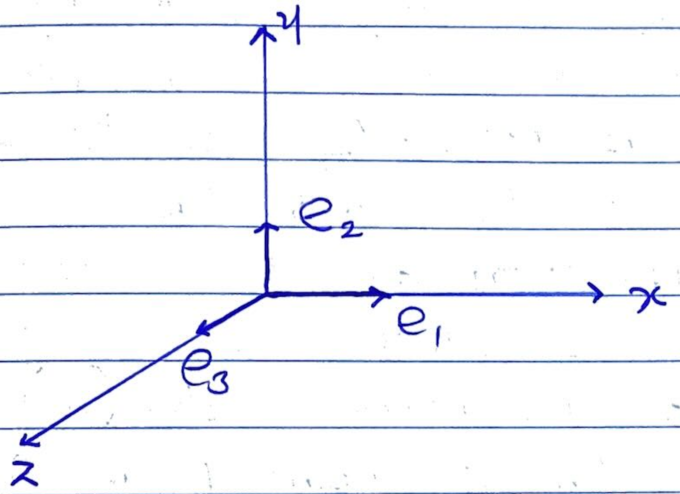
$$\sigma = \frac{F_n}{A} \qquad \tau = \frac{F_s}{A}.$$

STRESS ANALYSIS

1 Cauchy Stress Principal
2 State of Stress at Point
3 State of Stress On an inclined plane
4 Force and moment equilibrium.
5 Stress transformation law
6 Normal & shear stress on plane and On an inclined plane
7 Principal stress
8 Stress Decomposition.
9 Octahedral stress.

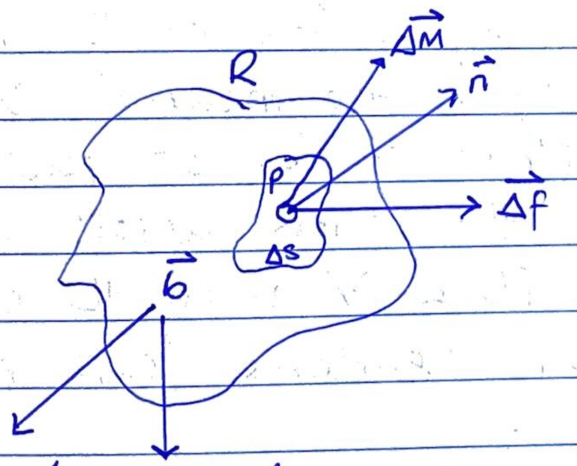
# Cauchy Stress Principal

lets Consider for instance, the Countinuum Shown below occupying a region $R$ of Space and Subjected to body forces $b$ (per unit of mass) and Surface force $f_s$ (fractions) let $x, y, z$ be Cartesian Coordinates System with unit $e_1$ $e_2$ $e_3$ Parallel to $x$, $y$, $z$ directions



$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$



Consider a volume $V$ on the Continuum an infinitesimal Surface element $\Delta s$ Located on the Surface $S$ of $V$, a Point $P$ located on $\Delta s$, and a unit vector $n$ normal at $P$. Under the effect of the body and Surface forces, the material within the volume $V$ intercuts with the material Outside $V$ let $\Delta f$ and $\Delta M$ be respectively the resultant force and momentum exerted across $\Delta s$ by the material Outside of $V$ upon the material with $V$. the Cauchy Stress Principal asserts that the average force per unit area

$\Delta f / \Delta s$ tends to limit $df/ds$ as $\Delta s$ tends to approach Zero, where $\Delta m$ Vanishes in the limitting Process. This limit is called the Stress Vector $t(n)$ i.e

$$t_n = \lim_{\Delta s \to 0} \frac{\Delta f}{\Delta s} = \frac{df}{ds} \quad \text{——} \quad (1)$$

The Stress vector has three components in the x, y, z Coordinate System which are expressed in units of force per unit (MPa). It is noteworthy that the ~~compression~~ Components of the Stress tensor depends on the orientation of the surface element $\Delta s$ which is defined by the coordinates System of its normal vector (n). The Stress vector $t_n$ at Point P in figure 1b is associated by action of the material outside of V upon the material within $t(n)$ be the stress vector at point f Corresponding to the action across $\Delta s$ of material within V upon the material Outside V by Newton's laws of action and reaction.

$$t_n + t_{(-n)} = 0 \; \text{- - - -} \; (2)$$

Equation (2) implies that the stress vectors acting on opposite sides of the same surface are equal in magnitude but Opposite in direction.

State of stress at a point.
State of stress at a point represented by what is Known as a stress tensor.

MATRICES.

- fundamental mathematics for stress analysis
- Stress state at point.
- Stress at an inclined plane.
- Stress transformation.
- Principal / stress decomposition.

Introduction to matrices.

- Defination.

A matrix is defined as rectangular arrangments of mn numbers, in m row and n columns and enclosed within brackets.

- In most cases, we denote matrices by capital letters A, B, C E etc.

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

A is of order $m \times n$ $i^{th}$ row $J^{th}$ column element of the matrix denoted by a

for example

$$A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

# Types of Matrices.

1 Square matrix

A matrix in which the number of rows are equal to the number of Columns.

Examples include the following

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{34} \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

2 Diagonal matrix

A Square matrix $A = (a_{ij})_{m \times n}$ is called a diagonal matrix if each of its non-diagonal element is Zero.

That is $a_{ij} = 0$ if $i \times j$ and atleast one element $a_{ii} \neq 0$

example

$$A = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} \quad B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 9 \end{pmatrix}.$$

3 Identity matrix.

An identity matrix is a diagonal matrix whose diagonal elements are equal to 1

That is $a_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$

Examples include

$$I_n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The identity matrix is Commonly denoted by $I_n$.

4 Upper Triangular matrix.

A square matrix in which the following condition is upheld.

$$a_{ij} = 0 \text{ if } i > j$$

Example.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}$$

5 Lower Triangular matrix.

A square matrix in which the following condition is upheld.

$$a_{ij} = 0 \text{ if } i < j$$

Example

$$A = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

6 Symmetric Matrix.

A square matrix $A = (a_{ij})_{nm}$ said to be a symmetric if $a_{ij} = a_{ji}$ for all $i$ and $j$

Example include.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad B = \begin{pmatrix} 8 & -2 & 7 \\ -2 & -9 & 3 \\ 7 & 3 & 5 \end{pmatrix}$$

7 Skilled - Symetric Matrix.

A Square matrix $A = (a_{ij})_{n \times m}$ Said to be Skilled - Symetric

If $a_{ij} = a_{ji}$ for all $i$ and $j$

An example of Skilled - Symetric matrix is a follows.

$$B = \begin{pmatrix} 8 & -2 & 7 \\ 2 & -9 & 3 \\ -2 & -3 & 5 \end{pmatrix}$$

8 Zero Matrix

A matrix whee all elements are Zero is Called a Zero matrix. A Zero matrix is Commonly denoted as $O_{n \times m}$

An example of the Zero matrix is as follows

$$O_{3 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

9 Row matrix

A matrix that Consists of a single row

Example include

$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \end{pmatrix}$ $B = \begin{pmatrix} 7 & 4 & -3 \end{pmatrix}$

10 Column matrix

A matrix that Consist of a Single Column

Example include.

$A = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}$ $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

## Matrix algebra

### Equality of two matrices.

Two matrices A and B are said to be equal if

(i) They must have the same order.

(ii) There Corresponding elements must be equal.

That is if $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$

then $a_{ij} = b_{ij}$ for all $i$ and $j$

### Scalar multiple of a matrix

let $K$ be a scalar, then scalar product of matrix $A = (a_{ij})_{m \times n}$ given denoted by $KA$ and given by

$$KA = (Ka_{ij})_{m \times n}$$

$$KA = \begin{pmatrix} Ka_{11} & Ka_{12} & \cdots & Ka_{1n} \\ Ka_{21} & Ka_{22} & \cdots & Ka_{2n} \end{pmatrix}$$

### Addition of two matrices.

Two matrices A and B ae said to be

let $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$ the two matrices with same order then the sum of the two matrices is given by.

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n}$$
$$= (a_{ij} + b_{ij})_{m \times n}.$$

### Exercise

Let $\begin{pmatrix} 1 & -2 & 3 \\ 4 & 5 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 & 2 \\ -1 & 1 & 8 \end{pmatrix}$

Calculate;

(i) $5B$ (ii) $A + B$ (iii) $4A - 2B$ (iv) $0A$.

Answers.

(a) $5B$

$$5 \times \begin{pmatrix} 3 & 0 & 2 \\ -1 & 1 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 15 & 0 & 10 \\ -5 & 5 & 40 \end{pmatrix}$$

(b) $A + B$

$$\begin{pmatrix} 1 & -2 & 3 \\ 4 & 5 & -4 \end{pmatrix} + \begin{pmatrix} 3 & 0 & 2 \\ -1 & 1 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -2 & 5 \\ 3 & 5 & 4 \end{pmatrix}$$

(c) $4A - 2B$

$$4\begin{pmatrix} 1 & -2 & 3 \\ 4 & 5 & -4 \end{pmatrix} - 2\begin{pmatrix} 3 & 0 & 2 \\ -1 & 1 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -8 & 12 \\ 16 & 20 & -16 \end{pmatrix} - \begin{pmatrix} 6 & 0 & 4 \\ -2 & 2 & 16 \end{pmatrix}$$

$$\begin{pmatrix} -2 & -8 & 8 \\ 18 & 14 & 0 \end{pmatrix}$$

(d) $0A$

$$0\begin{pmatrix} 1 & -2 & 3 \\ 4 & 5 & -4 \end{pmatrix}$$

$$= 0$$

## Multiplication of two matrices.

Two matrices A and B are conformable for product AB if the number of column in A equals the number of ~~columns~~ rows in matrix B.

let $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{m\times n}$ be two matrices the product matrix $C = AB$ is matrix of order $m \times r$

Where

$$C_{ij} = \sum_{k=1} a_{ik} \times b_{kj} - a_{i1}b_{1j} + a_{i2}b_{2j} + \dots a_{in}b_{nj}$$

### Exercise

Given the following

$$A = \begin{pmatrix} 1 & 2 & -3 & 4 \\ 0 & -5 & -1 & 1 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 2 & 3 \\ -5 & 0 \\ 6 & -2 \\ -4 & -3 \end{pmatrix}$$

Calculate;

(i) AB

(ii) BA

(iii) is AB = BA ?

# Integral power of Matrices.

Let $A$ be a square matrix of order $n$ and $m$ be positive integer then we define

$$A^m = A \times A \times \dots \times A \text{ (m time multiplication)}$$

## Properties of matrices.

Let $A$, $B$ and $C$ are three matrices and $\mu$ and $\gamma$ are scalar quantities, then following must hold;

(i) $A+(B+C) = (A+B)+C$ Associative law.
(ii) $\gamma(A+B) = \lambda A + \lambda B$ Distributive law.
(iii) $\gamma(\mu A) = (\gamma\mu)A$ Associative law.
(iv) $(\gamma A)B = \gamma(AB)$ Associative law.
(v) $A(BC) = (AB)C$ Associative law.
(vi) $A(B+C) = AB+AC$ Distributive law.

# Transpose

The transpose of matrix $A = (a_{ij})_{m\times n}$, written $A^t$ or $A^T$ is a matrix obtained by writting the rows of $A$ in order as colums. That is $A^t = (a_{ji})_{n\times m}$

$$A = \begin{pmatrix} 2 & 3 & 7 \\ 4 & 5 & 6 \\ 8 & 4 & 9 \end{pmatrix} \quad A^T = \begin{pmatrix} 2 & 4 & 8 \\ 3 & 5 & 4 \\ 7 & 6 & 9 \end{pmatrix}$$

## Properties

(i) $(A+B)^t = (A^t + B^t)$
(ii) $(A^t)^t = A$
(iii) $KA^t = KA^t$ for scalar $K$.
(iv) $(AB)^t = B^t A^t$.

# Determinants, Minor and adjoint matrices.

## Defination of determinant.

let $A = (a_{ij})_{n\times m}$ be a squae matrix of $n$, then the nember $|A|$ called determinant of matrix $A$

(i) Determinant of $2\times 2$ matrix

let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ then $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

(ii) Determinant of $3\times 3$ matrix.

let $B = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

then $|B| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31})$
$+ a_{13}(a_{21}a_{32} - a_{22}a_{31})$.

## Exercice

let $A = \begin{pmatrix} 1 & 5 & 4 \\ 2 & 6 & 8 \\ 1 & 9 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{pmatrix}$

let $A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 6 & 8 \\ 1 & 9 & 5 \end{pmatrix}$

$$|A| = 1\big(6(5) - 9(8)\big) - 3\big(2(5) - 8(1)\big) + 4\big(2(9) - 6(1)\big)$$
$$= -42 - 3(2) + 48$$
$$= -42 - 6 + 48$$
$$= 0$$

$B = \begin{pmatrix} 2 & -3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{pmatrix}$

$$|B| = 2\big(6(1) - 7(9)\big) + 3\big(5(1) - 7(8)\big) + 4\big(5(9) - 8(6)\big)$$
$$= -114 - 153 - 12.$$
$$= -279.$$

## Properties of the Determinant.

(a) The determinant of a matrix $A$ and it transpose $A^t$ are equal.

$$|A| = |A^t|$$

(b) let $A$ be a Square matrix

(i) if $A$ has a row / Column of Zeroes then $|A| = 0$

(ii) if $A$ has two identical rows / column then $|A|^t = 0$

(c) (iii) If $A$ is a triangular matrix then $|A|$ is the Product of the diagonal elements.

(d) If $A$ is a Square matrix of order $n$ and $K$ is Scalar then $|KA| = K^n |A|$

## Singular matrix

if $A$ is a square matrix of order $n$, then $A$ is called singular matrix when $|A| = 0$ and non-singular otherwise.

## Minor and Cofactor.

Let $A = (a_{ij})_{m \times n}$ is square matrix. Then $M_{ij}$ denoted a sub-matrix $A$ with order $(m-1) \times (m-1)$ obtained by deleting the $i^{th}$-row and $j^{th}$-column. The determinant $|M_{ij}|$ is called the minor of the elements $a_{ij}$ of $A$.

## The Co-factor

The cofactor of $a_{ij}$ denoted by $A_{ij} = (-1)^{i+j}|M_{ij}|$

Exercise

Given $A = \begin{pmatrix} 5 & 4 & 2 \\ 2 & 3 & 1 \\ 3 & -2 & 1 \end{pmatrix}$

(i) find the cofactor.

$A_{11} = (-1)^{1+1}|M_{j}|$

$M_{11} = \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix}$

$|M_{11}| = (3 \times 1) - (-2 \times 1)$

$= 5.$

$A_{11} = (-1)^2 |5|$

$= 5$

## Adjoin matrix

- An adjoin matrix is the transpose of the matrix of Cofactors of the elements $a_{ij}$ of A
- The adjoin matrix is denoted by adj A.