Introduction to Mineral Economics

TIME VALUE OF MONEY

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Time Value of Money

- If you're like most people, you would choose to receive the $10,000 now than 3 years from now.
- After all, three years is a long time to wait.
- Why would any rational person defer payment into the future when he or she could have the same amount of money now?
- For most of us, taking the money in the present is natural.

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Cont …

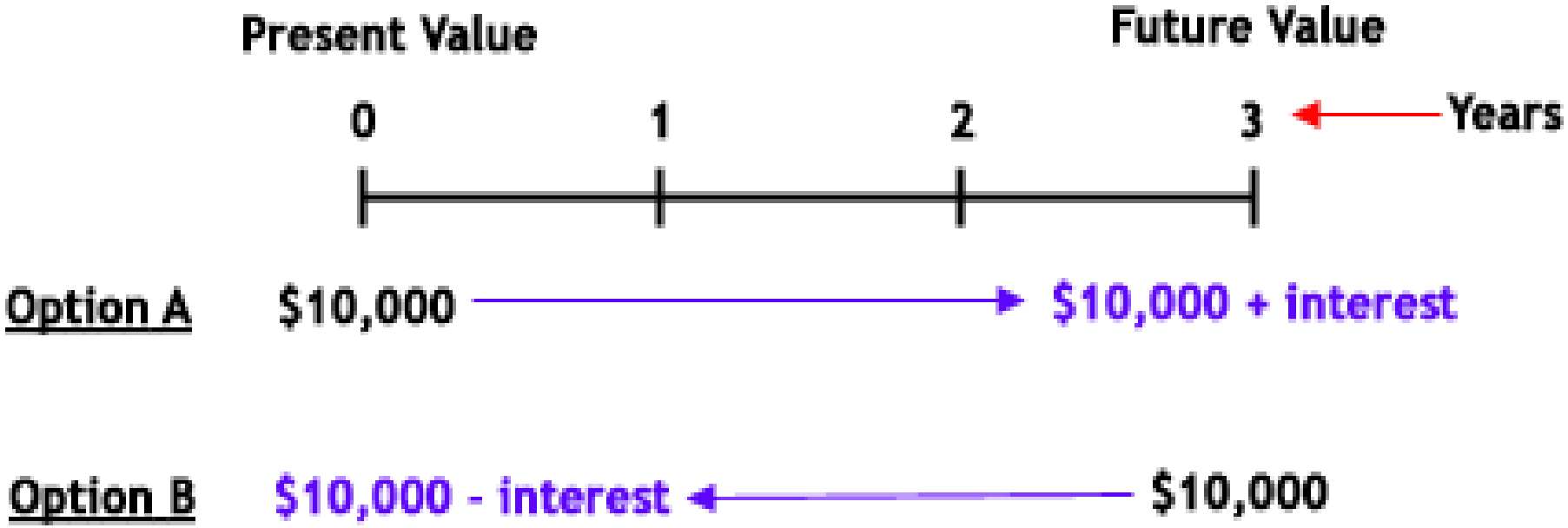
- So the time value of money demonstrates that it is better to have money now rather than later.
- But why is this?
- A $100 bill has the same value as a $100 bill one year from now.
- Although the bill is the same, you can do much more with the money if you have it now.
- Therefore, over time you can earn interest on your money.

Cont …

- If it were not for the existence of interest, the analysis of investment opportunities would be greatly simplified.
- In the absence of interest, investors would be indifferent as to when cash outlays are made or cash benefits are received.
- It does make a considerable difference whether, a firm receives $1 million now or five years from now.
- The reason is that money does have a value which is a function of time.
- Interest is how this time value is measured.

Cont …

- Therefore, by receiving $10,000 today, you are poised to increase the future value of your money;
- This is done by investing the money and gaining interest over a period of time.
- For option B, you don't have time on your side;
- Thus, the payment received in three years would be your future value.


Present Value
Future Value
0
1
2
3
Years
Option A
$10,000
$10,000 + interest
Option B
$10,000 - interest
$10,000

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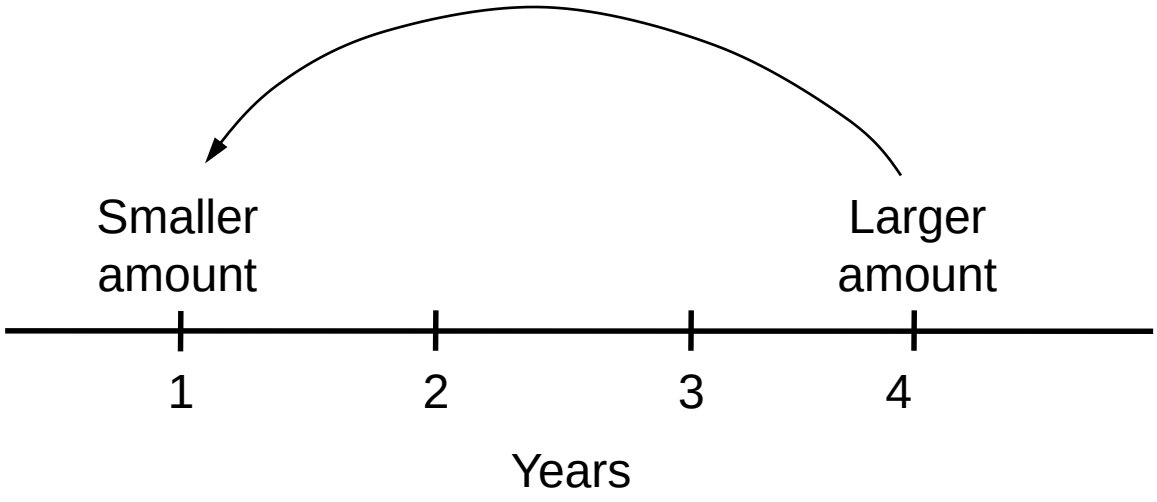
Cont …

- If you are choosing option A, your future value will be $10,000 plus any interest acquired over the three years.
- The future value for option B, on the other hand, would only be $10,000.

Meaning of discounting

- A larger future amount of cash is reduced to its smaller current equivalent.



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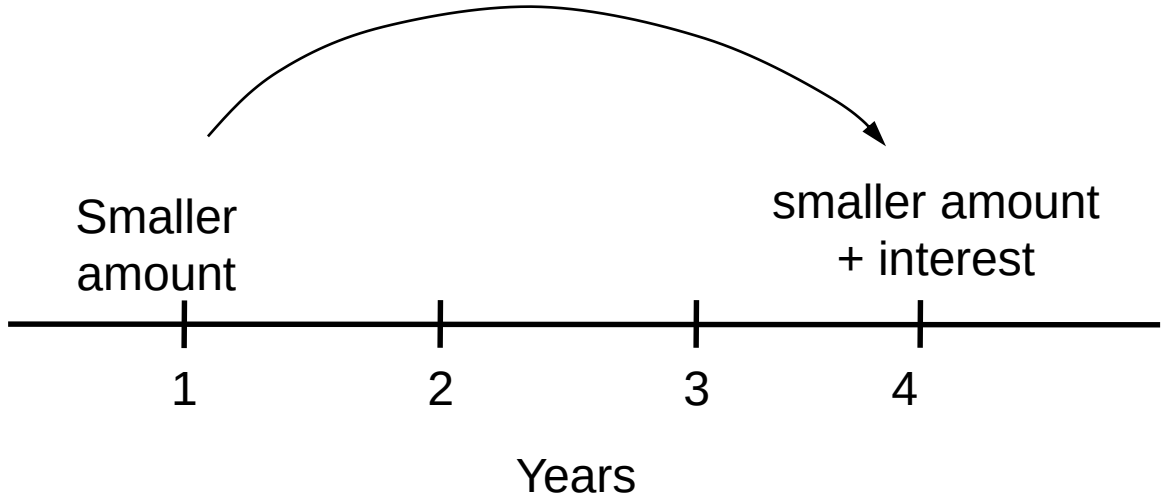
Meaning of compounding

- A smaller amount of cash earns interest and accumulates to a larger amount in the future.



TIME DIAGRAM

- **Given the following;**
- **P** = Present single sum of money (at time zero);
- **F** = Future single sum of money;
- **A** = Amount of each payment (in a uniform series of equal payment) made at the end of each period;
- **n** = Number of interest compounding periods;
- **i** = Period compound interest rate;
- **C** = Capital investment;
- **L** = Salvage value;
- **I** = Period income;

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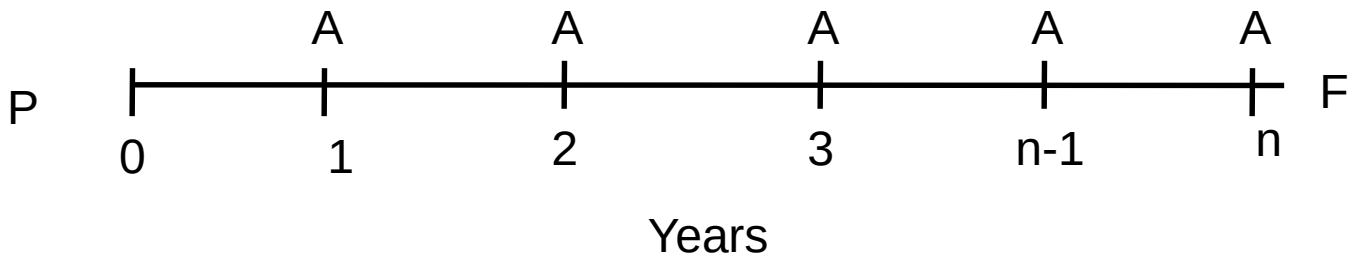

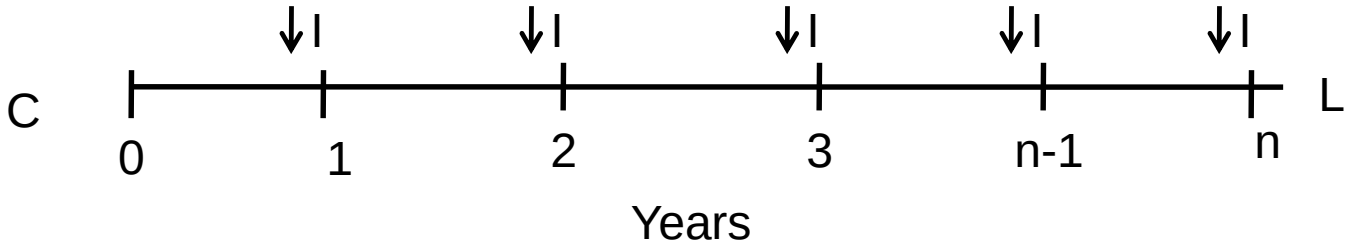

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Interest

- Interest is generally defined as money paid for the use of borrowed money.
- It is the rent paid for the use of someone else's money.
- It is a rental charge for using an asset over some specific time period.
- So, what is interest rate?
- This is the ratio of the interest chargeable at the end of a specific period of time to the money owed, or borrowed, at the beginning of that period.

Cont …

- In order to understand why interest exists, it is necessary to take the lender's viewpoint.
- The lender can justify charging interest for several reasons:-
- **Risk:** The lender is faced with the possibility that the borrower will be unable to repay the loan.
- **Inflation:** Money repaid in the future will be in units of lower value due to inflation.

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Cont …

- **Transaction costs:** There will be expenses incurred in preparing the loan agreement, recording payments, collecting the loan, and other administrative tasks.
- **Opportunity costs:** By committing limited funds to one borrower, a lender is unable to take advantage of other available opportunities.
- **Postponement of pleasure:** By lending money, an individual (or organization) is postponing the pleasure which that money could purchase.

Simple Interest

- This is often referred to as the nominal rate of interest;
- The nominal rate of interest is the annual interest rate without including effects of any compounding during the year.
- It is defined as the annualized percentage of amount borrowed (principal) to the amount paid for the use of the money for the same period of time.
- **Simple Interest = P.i.n**

  Where

  P = Present single sum of money (at time zero)

  i = Interest rate

  n = Number of interest periods

Example

- Find the simple interest on $1000 principle at simple interest of 6% per annum for 120 days;

- **Solution**
- **Simple Interest = P.i.n**
- Simple Interest = 1000 x 0.06 x (120/365)

$$\approx \$20$$

Compound Interest

- If the interest is NOT paid at the end of the period in which it is due but is added to, and becomes part of the principal upon which interest is calculated, the interest is compounded;
- Assume $100 is invested at 10% a year for 3 years;
- What will be the interest at the end of year 3 compounded annually?

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Solution

- Interest in 1st year @ 10%

  = P.i.n

  = 100 x 10/100 x 1 = $10

- Value at end of 1st year = $100 + $10

  = $110

- Interest in 2nd year @ 10%

  = $110 x 10/100 x 1

  = $11

Cont ..

Value at end of 2nd year = $110 + $11
= $121

- Interest at end of 3rd year @ 10%
= 121 x 10/100 x 1
= $12.1

- Value at end of 3rd year
= $121 + $12.1
= $133.1

- Present Value = $100
- Future Value = $133.1

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CONT …

- The compound interest relationship is generally expressed as follows;

$$F = P(1+i)^n$$

Where

F= Future single sum of money
P= Sum of money at present time
i = Annual rate of interest
n= Number of years

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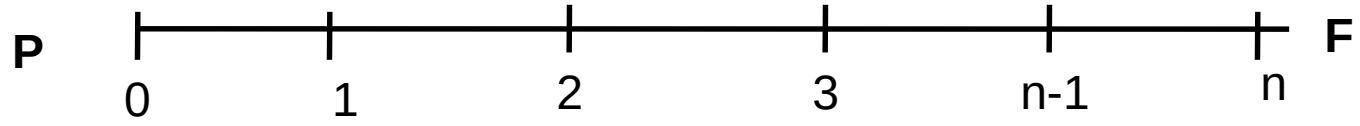
CONT …



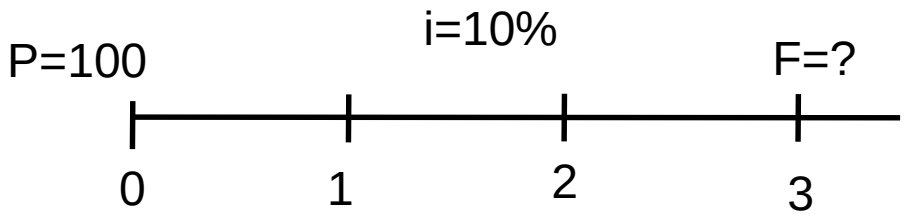
$$F = P(1+i)^n$$

$$(F/P_{i,n}) = (1 + i)^n$$

- The $\mathbf{F/P_{i,n}}$ factor is found in tables or can be calculated mathematically using the above formula.
- i = 0 – 100%; n = 1 – 50 years

EXAMPLE

- Calculate the future worth that $100 today will have three years from now if interest is 10% per year compounded annually.



- $\mathbf{F = P(1 + i)^n}$

$= 100\ (1 + 0.1)^3$

$= \$133.1$

USING TABLES

- $F = P.\ (F/P_{i,n})$
- $F = P.\ (F/P_{10,3})$
- $F = 100\ X\ 1.3310$
- **$F = \$133.1$**

Equivalence Factors

- [$\mathbf{F/P_{i,n}}$] = Used to find future value F after n periods given present value P and discount rate i.
- [$\mathbf{P/F_{i,n}}$] = Used to find present value P given future value F, i and n.
- [$\mathbf{F/A_{i,n}}$] = This is “uniform series compound amount factor”. Finds future value F given “uniform series compound amount A”, i and n.
- [$\mathbf{A/F_{i,n}}$] = This is "sinking fund payment factor“ Used to find A, given F, i and n.

Cont …

- [$\mathbf{A/P_{i,n}}$] = “Capital Recovery Factor“. Given P, i and n, find A.
- [$\mathbf{P/A_{i,n}}$] = This is "uniform series present worth factor“ Given A, i and n, find P?

Single payment present – worth factor ($P/F_{I,N}$)

- This is a factor which will discount future value of an investment opportunity to the current values (what you are supposed to invest now);
- The Present value (P) simply represents an amount of money at present time (t=0);
- This is equivalent to some future cash flows discounted at a specified interest rate.

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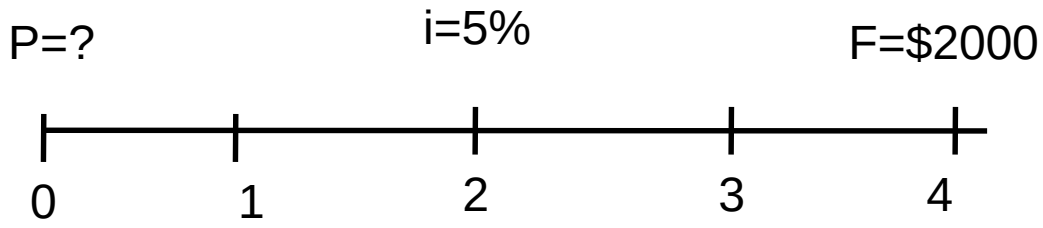
Example

- Calculate the present value of a $2000 payment to be received four years from now if interest is 5% per year compounded annually.



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Solution

$$P = F.\frac{1}{(1+i)^n}$$

$$P = 2000.\frac{1}{(1+(5/100))^4}$$

- P= $1,645.4

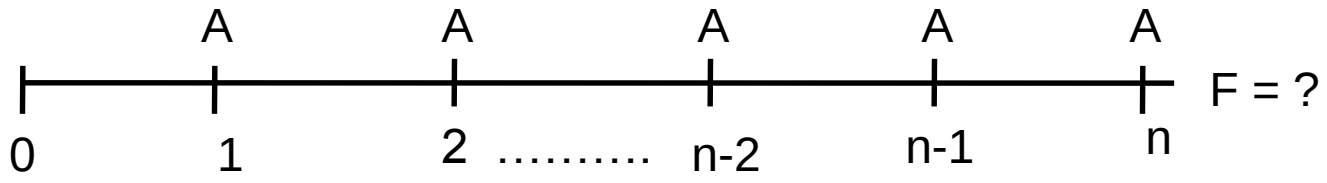
- The result shows that $1,645.40 invested today at 5% interest per year would grow to $2000 in four years.

USING TABLES

- $P = F.(P/F_{i,n})$
- $P = F.(P/F_{5,4})$
- $P = 2000 \text{ X } 0.8227$
- **$P = \$1,645.4$**

Uniform series compound-amount factor ($F/A_{I,N}$)

- Given annual uniform series investment made at the end of each year, the future value after **n** years can be determined.



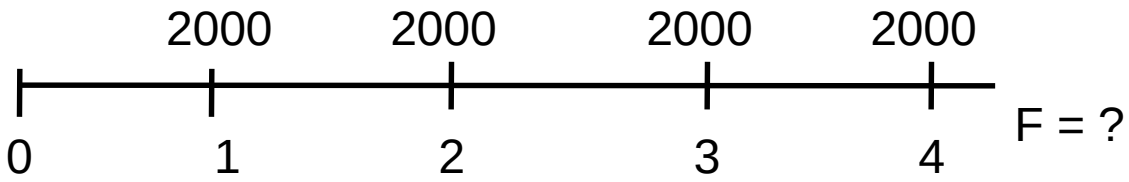
- $F = A(1)^0 + A(1+i)^1 + A(1+i)^2 + \ldots + A(1+i)^{n-1}$

CONT …

$$F = \frac{A[(1+i)^n - 1]}{i} \qquad (F/A_{i,n}) = \frac{[(1+i)^n - 1]}{i}$$

- Example
- Calculate the future value four years from now of a uniform series of $2000 investments made at the end of each year for the next four years if the interest rate is 5% per year compounded annually.

Solution



$$F = \frac{A[(1+i)^n - 1]}{i}$$

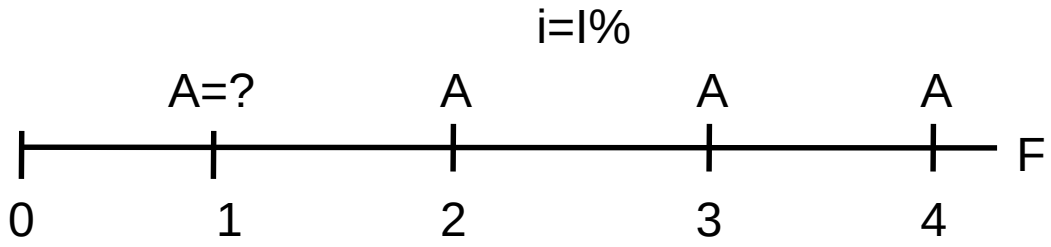
$$F = \frac{2000[(1+0.05)^4 - 1]}{0.05} = \$8{,}620.2$$

USING TABLES

- $F = A.\ (F/A_{i,n})$
- $F = A.\ (F/A_{5,4})$
- $F = 2000\ X\ 4.3101$
- **$F = \$8,620.2$**

Sinking fund factor ($A/F_{I,N}$)

- This determine the amount of money "A" that must be sunk into a fund at the end of each period for n periods at i% interest period to accumulate "F" dollars.

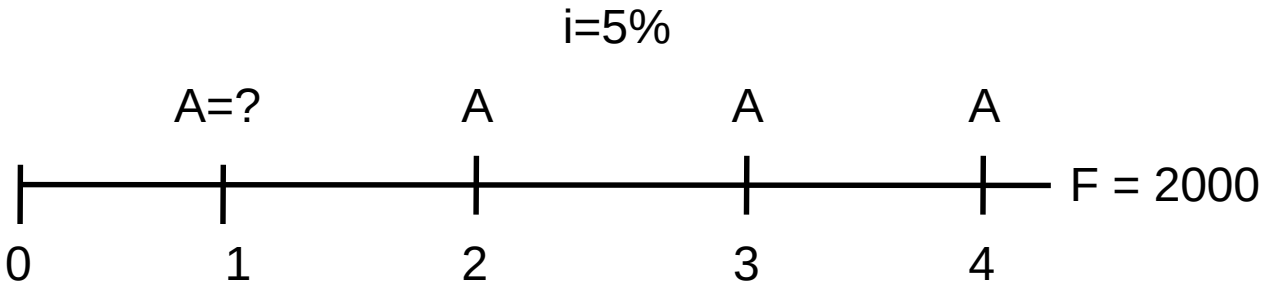


$$A = F\underbrace{\left[\frac{i}{[(1+i)^n - 1]}\right]}_{(A/F_{i,n})}$$

Example

- Calculate the uniform series of equal payments made at the end of each year for the next four years that are equivalent to a $2000 payment four years from now if the interest is 5% per year compounded annually.

Solution



$$A = F\left[\frac{i}{[(1+i)^n - 1]}\right]$$

$$A = 2000\left[\frac{0.05}{[(1+0.05)^4 - 1]}\right] = \$464.02$$

USING TABLES

- $A = F.\ (A/F_{i,n})$
- $A = F.\ (A/F_{5,4})$
- $A = 2000\ X\ 0.23201$
- **$A = \$464.02$**

Capital Recovery Factor ($A/P_{I,N}$)

- CRF is used to determine Equivalent Annual Value (A) of an investment opportunity.
- If we know P, i and n, the uniform series at end of period payment "A" can be determine.
- Equivalent Annual values "A" are annual cash flows which are used to measure the economic attractiveness of an investment opportunity.
- These are the anticipated cash flows per year for an investment opportunity.

$$A = P\underbrace{\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]}_{(A/P_{i,n})}$$

- Example
- Calculate the uniform series of equal payments made at the end of each year for four years that are equivalent to present sum of $2000 if interest is 5% per year compounded annually.

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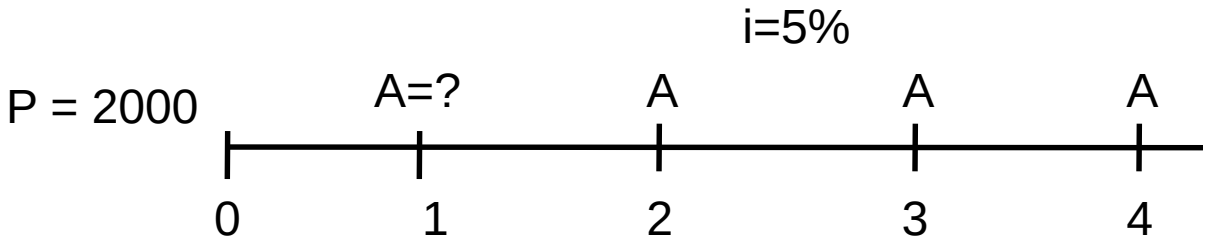
Solution



$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

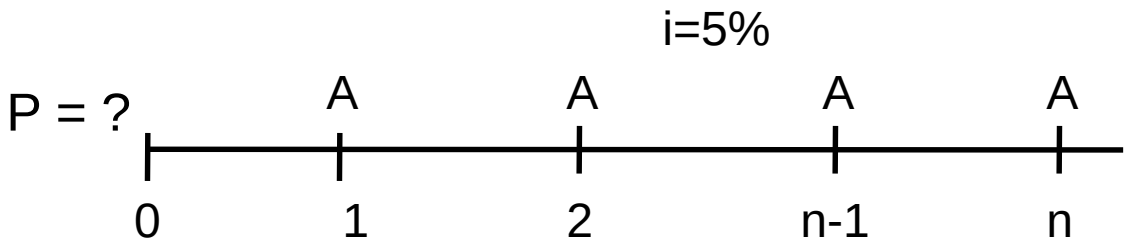
$$A = 2000\left[\frac{0.05(1+0.05)^4}{(1+0.05)^4 - 1}\right] = \$564.02$$

USING TABLES

- $A = P.\ (A/P_{i,n})$
- $A = P.\ (A/P_{5,4})$
- $A = 2000\ X\ 0.28201$
- **$A = \$564.02$**

Uniform series present-worth factor ($P/A_{I,N}$)

- This discounts individual annual cash flows "A" to present values;
- Also known as Cumulative Present Value Factor (CPVF)



$$P = A\underbrace{\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]}_{(P/A_{i,n})}$$

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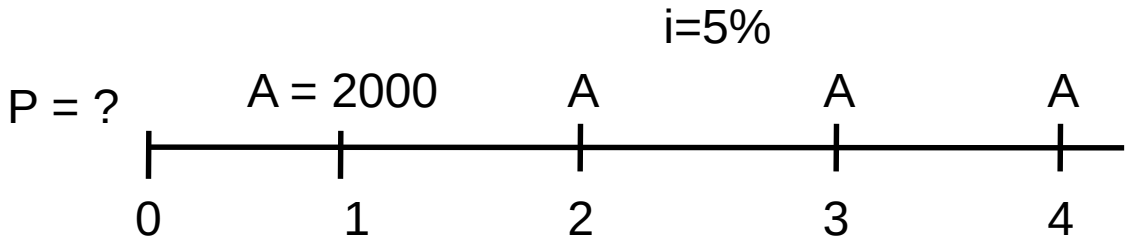
Example 1

- Calculate the present value of a series of $2000 payments to be made at the end of each year for four years if the interest is 5% per year compounded annually.

- Solution

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Cont …



$$P = 2000\left[\frac{(1+0.05)^4 - 1}{0.05(1+0.05)^4}\right] = \$7{,}092$$

Using Tables

- $P = A.\ (P/A_{i,n})$
- $P = A.\ (P/A_{5,4})$
- $P = 2000\ X\ 3.5460$
- **$P = \$7092$**

Example 2

- An investment will yield $610 at the end of each year for 15 years. If interest is 10% what is the maximum purchase price (i.e. present value) for this investment?

- Given
  - A = $610
  - n = 15years
  - i = 10%

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Solution

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$P = 610\left[\frac{(1+0.1)^{15} - 1}{0.1(1+0.1)^{15}}\right] = \$4639.7$$

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USING TABLES

- $P = A.\ (P/A_{i,n})$
- $P = A.\ (P/A_{10,15})$
- $P = 610\ X\ 7.6061$
- **$P = \$4639.72$**

Solving Interest Problems

- Interest problems can usually be simplified if the two steps to be described are followed in order.
- **Abstracting the problem**
- Interest problems are based upon the five variables i.e.
  - Present Value (P)
  - Future Value (F),
  - Annual Values (A),
  - Interest rate (i)
  - Number of interest Periods (n).

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Cont …

- The first step is to determine which three variables are given, and what is the fourth variable to be determined.
- Interest problems become complex with many cash flows of various magnitudes occurring at different points in time.
- It is important in these cases to abstract the problem in terms of the five basic variables before proceeding.

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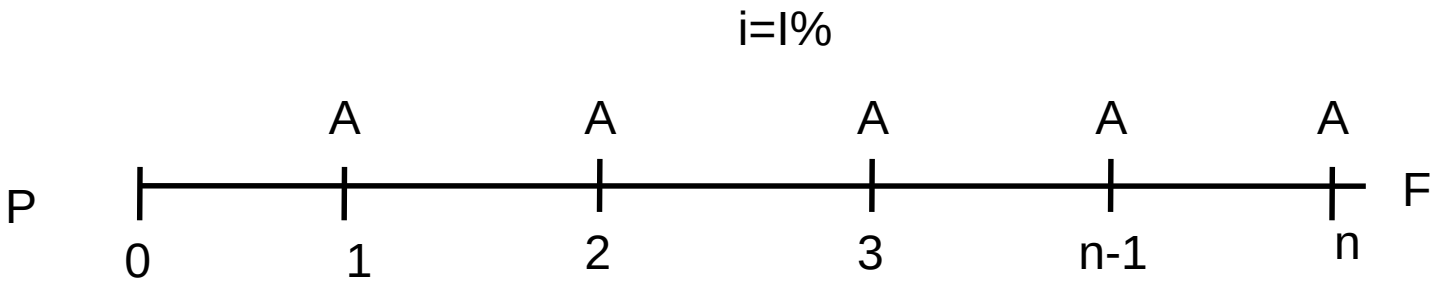
Drawing the Cash Flow Diagram

- It is usually helpful in visualizing the problem to construct a cash flow diagram illustrated below.



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Example 1

- A consulting firm would like to purchase a used testing machine from an independent testing/inspection lab, and it makes two offers:
  - 1) a lump-sum of $40,000 or
  - 2) monthly payments of $1200 over 3 years at a 6% annual interest rate.
- Which option do you think the testing lab would prefer, assuming it has to replace the sold machine?

Example 2

- How much money should you be willing to pay now for a guaranteed $600 per year for 9 years starting next year, at a rate of return of 12% per year?

Example 3

- Consider a new mine equipment alternative where capital expenditure of US$150,000 now would result in annual operating and maintenance costs of US$6,000 over its anticipated 8 year life. If the cost of capital is 10%, find;
- 1. Annual Value of the investment.
- 2. Present value of the investment.
- 3. The future worth of the investment after 5 years.

Discount factors, risk, uncertainty

- Almost all decision making using time-value-of-money concepts revolves around selection of the appropriate discount rate;
- In most mining companies, the rate that the management applies is outside the control of the planning personnel;
- The rate is specified by the board or senior management.
- The elements making up the discount rate are as follows:

Cont …

- 1. The basic interest rate applicable for zero-risk investments in the country;
- 2. Allowances for the cost of capital;
- This comprises both equity funding and debt funding;
- The cost of capital is dependent on the relative mix of the debt and equity;
- 3. Allowances for the finance risk;
- 4. Allowances for the technical risk;

Cont …

- Geotechnical characteristics may cause losses of production or higher costs of production;
- Grades of the orebody may not turn out to be as predicted;
- 5. Allowances for the fact that, the returns for this project must also cover the costs expended in assessment of projects.

Working Capital, Salvage Values and the Timing of Cash Flows

- Most capital expenditure relates to fixed assets;
- These include plant, equipment, buildings, etc.
- The initial purchase of these capital items is a cash outflow, usually occurring at the start of the project;
- Operating expenses such as repair parts are also cash outflow;

Cont …

- Operating expenses must be paid for at the time they are used;
- In practice, there are always timing differences between when costs are incurred and when the spare part is used or production undertaken.

Working Capital

- Working capital represents an operating liquidity available to a business;
- i.e. it is how much, in liquid assets, that a company has on hand;
- A liquid asset is cash or any type of negotiable asset that can be converted quickly and easily into cash;
- e.g. cash, spare parts are examples of working capital;
- Working capital is needed to pay for:

Cont …

- 1. Planned and unexpected expenses;
- 2. Meet the short-term obligations of the business; and
- 3. To build the business;
- Working capital is calculated as follows:
- Working Capital = Current Assets – Current Liabilities

Cont …

- A **current asset** is an asset expected to be sold or used up in the near future, usually within one year;
- e.g. cash, account receivable, inventory, the portion of prepaid accounts which will be used within a year;
- **Current liabilities** are debts, loans or other obligations due for payment within one calendar year;

Salvage Value

- The salvage value is the expected value (after cost of sale has been deducted) realized upon disposal of fixed assets at the end of their useful life;
- It is the total net worth of any asset after it has exhausted its useful life span;
- Salvage value is a cash flow, exactly opposite to capital expenditure;

Timing of cash Flows

- The timing of cash flows is important ;
- Over half of all bankruptcies can be attributed to miscalculation or unexpected change in the timing of cash flows;
- Timing is more critical in highly leveraged projects (e.g., where much of the equipment is hired);
- Timing is also equally vital in three other areas;
- 1. Commodities for which the prices change in cycles;

Cont …

2. Cases where less capital-intensive plans are being upgraded to more capital-intensive schemes;
3. During mine expansion;

DCF RANKING CRITERIA

- How does a company decide whether a project should proceed or not;
- Or whether one project is better than another project?
- The two most common measures are based on;
- 1. Net Present Value (NPV)
- 2. Internal Rate of Return (IRR)
- In either case, higher values are preferable;
- Alternatively, the discount rate can be selected so that the cash outflows exactly balance the cash inflows (in present value terms);

Cont …

- At this rate the NPV will be zero;
- This rate is termed the IRR;
- On the basis of this criterion, a project with an IRR of 19.5% would be better than a project with IRR of 18.5%;
- Either criterion can be used;
- However, the methods do not always yield the same answer;
- A small project with a very high return might have a lower NPV than a larger project with a lower return;

END

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