PRODUCTION FUNCTIONS

Production: Any activity that creates value is production. It is a series of activities by which resource inputs (raw materials, labour, capital, land utilisation and managerial talents) are transformed over time into outputs of goods or services.

Production function:

$$Q = f(X_a, X_b, \text{----------}X_n)$$

Q = Quantity of output obtainable per period of time.
X_a, X_b, ----------X_n = Quantities of various types of inputs

SOME DEFINITIONS

Fixed and variable inputs

Fixed input – cannot be readily changed in short-run in response to desire to alter a firm's rate of output (e.g. major pieces of equipment and machinery, space available for productive activity (buildings, plant size) and key managerial personnel.

Variable input – Usage rate can be altered easily in desire to lower or increase volume of output (e.g. electric power, most raw materials, labour. etc).

Short-run – Time period so short that the firm is constrained from varying the quantity of its fixed inputs.

Long-run – Time period sufficiently long to allow all inputs to be varied – no inputs are fixed including technology.

Short-run production function:

$$Q = f(X_a, X_b / X_c \text{-------}X_n)$$

Variable Inputs | fixed inputs

Cost functions – in short-run

- **Variable costs:** Costs vary with level of production (e.g. labour, materials).
- **Fixed costs:** (Also referred to as overhead costs) remain relatively constant regardless of the level of production activity. They tend to be proportional to time and independent of the number of units produced (e.g. rent, licence fees, R&D, insurance).

TC = TFC + TVC
TC = TFC + QVC
Function can be linear or non linear

Where,

$$\mathbf{TFC} = \sum_{i=1}^{n} \mathbf{P_i \,.\, X_i}$$

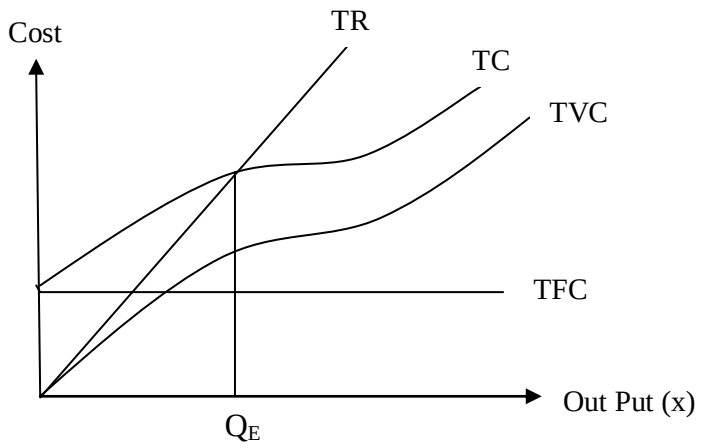
Where, P_i = price of a specified fixed input
X_i = quantity of the specified fixed input
n = number of various fixed inputs

$$\mathbf{TVC} = \sum_{j=1}^{n} \mathbf{P_j \,.\, Q_j}$$

Where, P_j = price of a specified variable input
Q_j = quantity of the specified variable input
n = number of various variable inputs

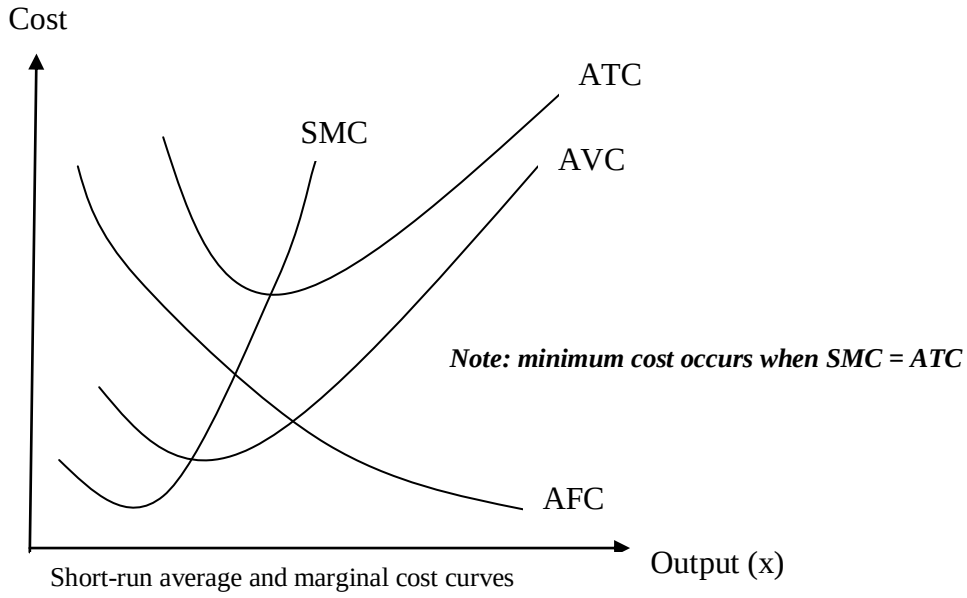
Thus:

$$\mathbf{TC} = \sum_{i=1}^{n} \mathbf{P_i \,.\, X_i} + \sum_{j=1}^{n} \mathbf{P_j \,.\, Q_j}$$



Q_E = Breakeven point (TC=TR)

SHORT- RUN COST CURVES



Short-run average and marginal cost curves

1. **Average Fixed Cost**

   AFC = TFC/Q

   AFC curve falls steadily as output increases. Why? Because you are spreading the fixed cost over an increasing output.

2. **Average variable cost**

   AVC = TVC/Q

AVC will generally fall as output increases from zero to the normal capacity (using all fixed inputs) due to occurrence of increasing returns. But beyond the normal capacity output, ***AVC will rise steeply due to diminishing returns setting in****.*

3. **Average total cost**

ATC = Total cost/output

ATC = TC/Q
= TVC/Q + TFC/Q
= AVC + AFC

ATC is the sum of AVC and AFC. Therefore, as output increases and AFC becomes smaller and smaller, the vertical distance between the ATC curve and ACV curve goes on declining.

Definitions:

- **Totals:** Total (costs, revenues, profits) as a function of output.
- **Averages:** (costs, revenues, profits) at a given output level.
- **Marginals:** Amount of (cost, revenue, profit) added to the total amount by each additional unit of output, at a given level of output.

Equations:

Totals	**Averages**	**Marginals**
TC = f(Q)	AC = TC/Q	MC = dTC/dQ
TR = f(Q)	AR = TR/Q	MR = dTR/dQ
TP = TR – TC	AP = TP/Q	MP= dTP/dQ

Where, Q = output

Break-even occurs when TR = TC

Law of Diminishing Returns

Law of variable proportions occupies an important place in economic theory. This law examines the production function with one factor variable, keeping the quantities of other factors fixed. Since under this law we study the effects on

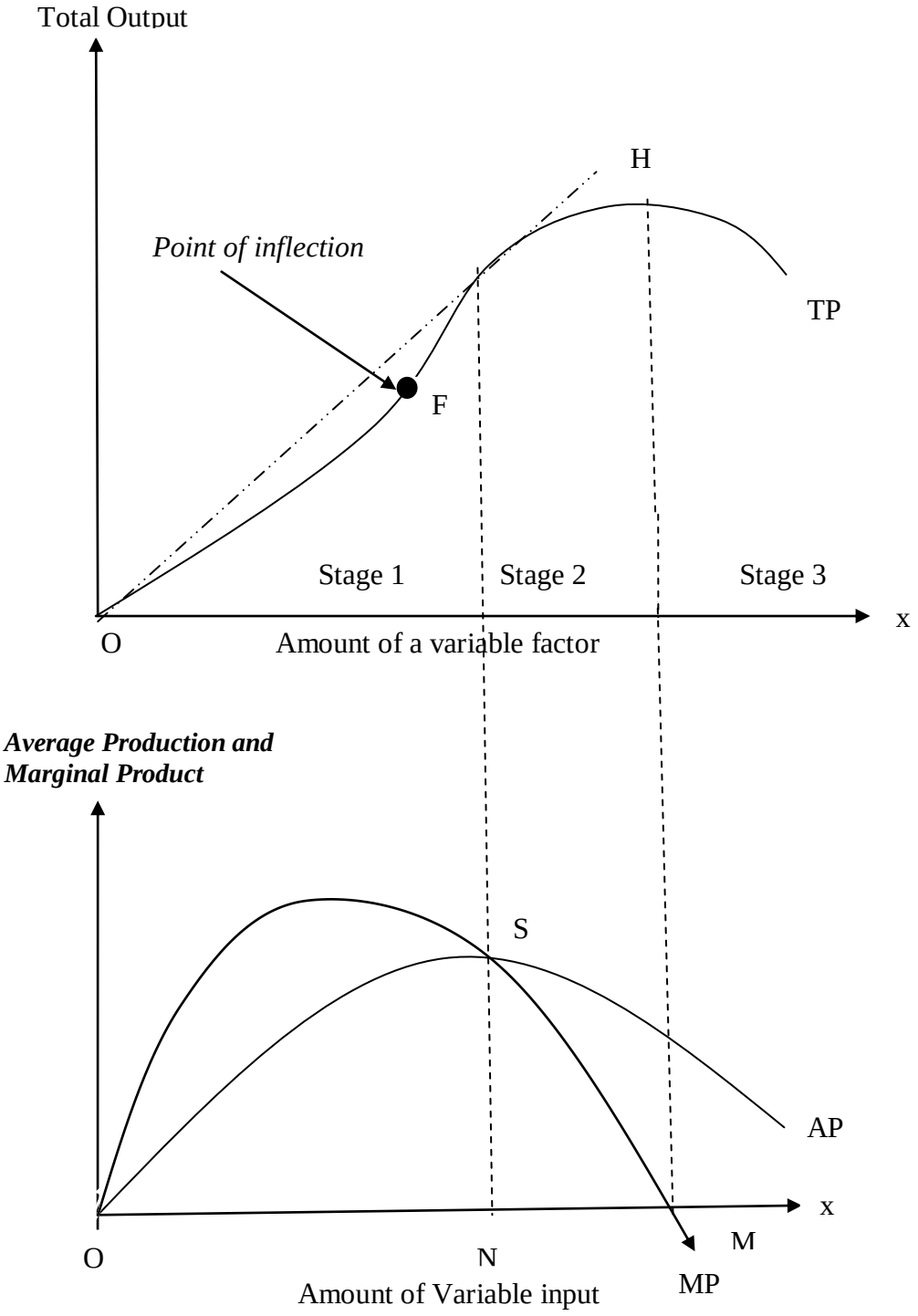
output of variation in factor proportions, this is also known as the ***law of variable proportions***. Thus law of variable proportions is the new name for the famous ***Law of Diminishing Returns*** of classical economics

The law holds that we will get less and less output when we add successive doses of an input while *holding other inputs constant*.

Marginal product: this is the additional output produced by 1 extra unit of input.

According to the law of diminishing returns, the marginal product of each unit of input will decline as the amount of that input increases, holding all other inputs constant.

Three Stages of the Law of Variable Proportions



Stage 1: Increasing returns

In this stage, total product (TP) to a point increases at an increasing rate up to point F in Figure above. Also the marginal product (MP) is rising. From point F onwards during Stage 1, the TP curve goes on rising but its slope is declining

which means that from point F onwards the TP increases at a declining rate until it reaches the ***point of inflection (F).***

This Stage ends where the average product (AP) curve reaches its highest point. Stage one is known as the stage of increasing returns because AP of the variable factor increases throughout this stage.

Stage 2: Stage of Diminishing Returns

In Stage 2, the TP continues to increase at a diminishing rate until it reaches its maximum at point H where the second stage ends. In this stage both the MP and the AP of the variable factor are diminishing but remain positive.

At the end of the second stage, that is, at point MP of the variable factor is zero (corresponding to the highest point H of the TP curve. Thus TP is maximum when MP = 0 or when $\frac{dTP}{dx} = 0$.

Stage 2 is very crucial because the firm will seek to produce in its range. This stage is known as the stage of ***diminishing returns*** as both the AP and the MP of the variable factor continuously fall during this stage.

Stage 3: Stage of Negative Returns

With increase in variable factor, the TP declines and therefore the TP curve slopes downward. As a result, MP is negative and the MP curve goes below the x- axis. In this stage the variable factor is too much relative to the fixed factor. This stage is called the ***stage of negative returns*** since the MP of the variable factor is negative during this stage.

The Stage of Operation

Now, an important question is in which stage a rational producer will seek to produce. A rational producer will never choose stage 3 where MP is negative. A rational producer will also not choose to produce in stage 1 where the MP is negative. A producer producing in stage 1 means he will not be utilising fully the opportunities of increasing production by increasing quantity of the variable factor whose AP continues to rise throughout stage 1. Thus a rational entrepreneur will not stop in stage one but will expand further until at point N.

Example:

Law of Diminishing Returns		
Units of labour (person-years)	Total output (tonnes)	Extra output added by additional unit of labour (tonnes per person-year)
0	0	2,000
1	2,000	1,000
2	3,000	500
3	3,500	100
4	3,800	
5	3,900	
Diminishing returns is a fundamental law of economics and technology. **Determine the marginal output by increasing units of labour from 3 to 4 person-years*		

LONG-RUN AVERAGE COST CURVE (LRAC)

Note there are NO fixed inputs in the long-run. All inputs can be altered.

The LRAC of production is the least possible average cost of production at any given level of output when all inputs are variable, *including of course the size of the plant.* The LRAC is the long-run total cost divided by the level of output.

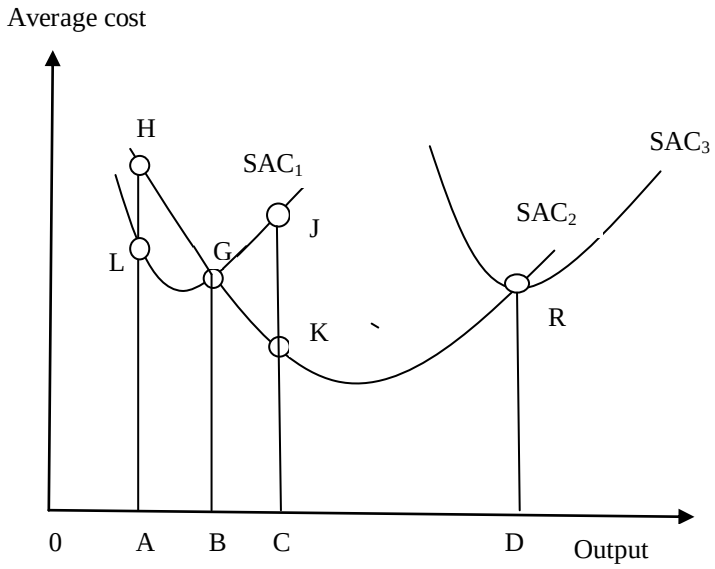
In order to understand the how the LRAC is derived, consider the three short-run average curves as shown below. These short-run average curves are also called *plant curves,* since in the short-run plant is fixed and each of the short-run average cost curves corresponds to a particular plant.

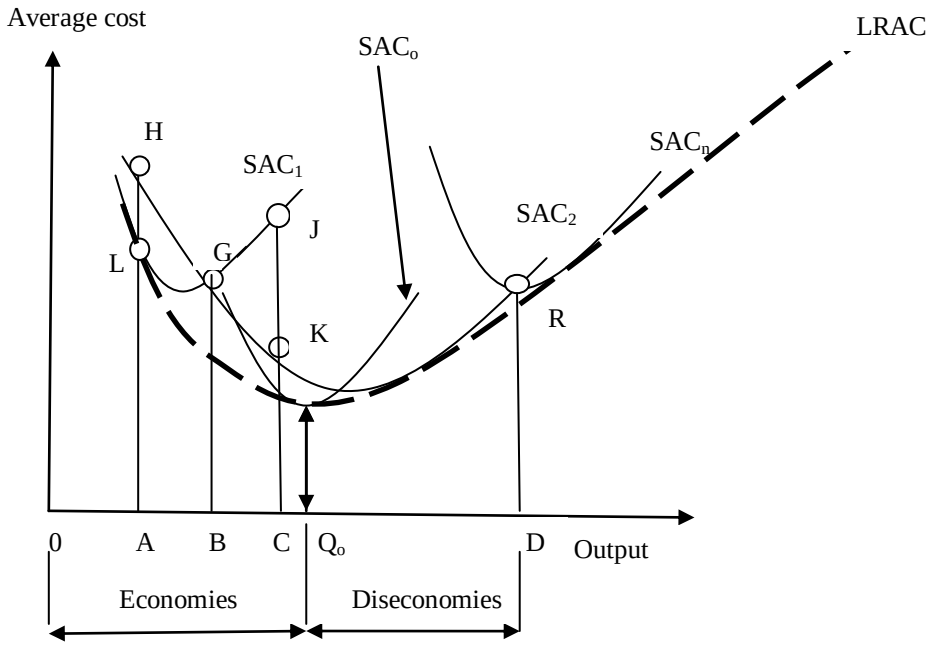
In the short-run, the firm can operate on any short-run cost curve, given the size of the plant. Suppose there are only three technically possible sizes of plants, and no other size of the plant can be built. Given a size of the plant or short-run average cost, the firm will increase or decrease its output by varying the amount of the variable inputs. But, in the long run, the firm can choose among the three possible sizes of plant as depicted by short-run average cost curves SAC_1, SAC_2 and SAC_3. In the long run the firm will decide about with which size of plant or on which short-run average cost curve to operate to produce a given level of output at the minimum cost.

From the figure, it can be observed that up to output OB, the firm will operate on the short-run average cost curve SAC_1, though it could also produce with short-run cost curve SAC_2 because up to OB amount of output, production on SAC_1 curve entails lower cost than on SAC_2. For instance, if the level of output OA is produced with SAC_1, it will cost AL per unit and if it is produced with SAC_2 it will cost AH per unit. AL is smaller than AH. Therefore, at all output levels up to OB can produce more economically with smaller plant SAC_1 than

with the larger plant SAC_2. It is thus clear that in the long-run the firm will produce any output up to OB on SAC_1.

If the firm wants to produce an output which is larger than OB (but less than OD), then it will not be economical to produce on SAC_1. It can be observed that the output larger than OB but less than OD, can be produced at a lower cost per unit on SAC_2 than on SAC_1. Thus, the output OC if produced on SAC_2 costs CK per unit which is lower than CJ which is the cost incurred when produced on SAC_1.





Note: SAC_o is optimal plan design in long-run

Therefore, if the firm plans to produce between output OB and OD, it will employ the plant corresponding to short-run average cost curve SAC_2. If the firm plans to produce an output larger than OD, then the cost per unit will be lower on SAC_3 than on SAC_2. Therefore, for output larger than OD, the firm will employ plant corresponding to the short-run average cost curve SAC_3.

The LRAC depicts the least possible average cost for producing various levels of output when all factors including the size of the plant have been adjusted.

Suppose now that the size of the plant can be varied infinitely small gradations so that there are infinite number of plants corresponding to which there are numerous short-run average costs curves. There will be infinite short-run cost curves in such a case. In this case, the LRAC will be a smooth continuous curve without any scallops as shown in the second figure below. Infact, *the LRAC is nothing else but the locus of all these tangency points with short-run average cost curves.*

Reasons for Economies of Scale

Three main reasons have been given for the economies of scale which accrue to the firm due to which cost per unit falls in the beginning:

1. As the firm increases its scale of operations, *it becomes possible to use more specialized and efficient form of all factors, especially capital equipment and machinery.* For producing higher levels of output, there is generally available a more efficient machinery which when employed to produce a large output yields a lower cost per unit of output.

2. When the scale of operations is increased and the amount of labour and other factors become larger, *introduction of a great degree of division of labour or specialization* becomes possible and as a result the long-run cost per unit declines. Decline in long-run average cost is thus due to use of more efficient forms of machinery and other factors and to the introduction of a greater degree of division of labour in the productive process.

3. ***Indivisibility of factors***: Economies of scale have also been attributed to from the imperfect divisibility of factors. In other words, the factors are considered to be ‘lumpy”, that is, they are available in *large indivisible units* and can therefore yield lower cost of production when they are used to produce a larger output. If a small output is produced

with these costly indivisible units of the factors, the average cost of production will naturally be high.

Reasons for Diseconomies of Scale

1. When the firm has reached a size large enough to allow the utilization of almost all the possibilities of division of labour and the employment of more efficient machinery, further increases in the size of the plant will entail higher long-run unit cost because of ***difficulties of management***. When the scale of operations exceeds a certain limit, the management may not be as efficient as when the scale of operations is relatively small. After a certain sufficiently large size these inefficiencies of management more than outweigh the economies of scale and thereby bring about an increase in the long-run average cost.

2. The second view considers the entrepreneur to be a ***fixed indivisible factor.*** In this view, though all other factors can be increased, the entrepreneur cannot. The entrepreneur and his functions of decision making and ultimate control are indivisible and cannot be increased. There is a certain optimum proportion between an entrepreneur and other inputs. Further increases in the other inputs will result in increasing costs.

Technology Change

Refers to changes in technology – invention of new products, improvements in old products, or changes in the process for producing goods and services.

Technological change occurs when new engineering and technical knowledge allows more output to be produced with the same inputs, or when the same output can be produced with fewer inputs.