Chapter 8

TORQUE AND ANGULAR MOMENTUM

Conceptual Questions

1. To maximize the torque, locate it as far as possible from the rotation axis: along the lower edge.

2. The ease of driving a screw into a piece of wood is determined by the magnitude of the torque required to produce the necessary downward force on the screw. The torque produced is equal to the product of the radius of the screwdriver handle and the magnitude of the tangential force applied by the operator's hand. Thus, the larger diameter handle reduces the applied force required to create the necessary torque. The same amount of work is done in driving the screw, but the task is made easier.

3. When you push near the edge, you have a larger moment arm. When you push in the middle, the moment arm is half as much so you need to push with twice the force.

4. Of the three axes, the book has the smallest moment of inertia about the axis along the binding of the book (axis 1). The moments of inertia about the other two axes are larger because the mass of the book is, on average, farther from those axes.

5. For a body to be in equilibrium, both the net force and the net torque acting on it must equal zero. To satisfy the first requirement, the two forces must be equal in magnitude and opposite in direction. To satisfy the second requirement, the two forces must act along the same line—a net torque would otherwise act to rotate the object.

6. When the angular momentum of the main propeller changes, the body of the helicopter would suffer an opposite change in angular momentum if no external torque about a vertical axis acts on the helicopter. The small propeller attached to the tail of the helicopter is used to produce this external torque to keep the helicopter body from rotating. Attaching the propeller at the tail produces the longest lever arm and therefore the greatest torque about the vertical axis through its center of mass.

7. The total kinetic energy of a car is found by summing the translational and rotational kinetic energies of each of the four wheels and the translational kinetic energy of the car's body. The fraction of the car's total kinetic energy due to the rotation of the wheels depends on the ratio of the mass of the car's body to the mass of the car's wheels. Thus, if two cars differ only in the mass of the body (while having wheels of the same mass), the more massive car converts a greater fraction of its gravitational potential energy into translational kinetic energy—the heavier car wins the race.

8. The force due to static friction acting on the barrel produces the torque that makes the barrel roll. If there were no friction acting on the barrel due to the floor, the applied force would make the barrel slide along the floor without rotating.

9. An object's moment of inertia depends on how its mass is distributed with respect to the axis of rotation. The farther the mass is from the axis, the greater the object's moment of inertia. When animals have leg muscles that are concentrated close to the hip joint, their legs have relatively small moments of inertia. This makes it easier for them to rotate their legs, allowing them to run faster.

10. When the triceps muscle connects to the forearm as in Fig. 8.46a, the lever arm for the muscular force remains relatively constant as the angle θ is varied. For an angle of about 90°, the lever arm in Fig. 8.46b is approximately the same as the lever arm in the previous figure. If this angle is increased or decreased however, the lever arm decreases significantly, resulting in a smaller torque about the elbow joint. For this reason, the situation depicted in Fig. 8.46a is much more effective.

11. With the forearm horizontal, the lever arms for the muscular forces on the forearm in Fig. 8.47a and Fig. 8.47b are about the same, so the two arrangements would be about equally effective. However, for large angles (with the arm nearly straight), the arrangement of Fig. 8.47b would have a very small lever arm and thus provide little torque. The primary advantage of the arrangement shown in Fig. 8.47a is that the muscle is concentrated closer to the shoulder, thereby reducing the moment of inertia of the forearm and the arm as a whole. This makes the arm easier to move around.

12. The two forces, $\vec{\mathbf{F}}_{12}$ and $\vec{\mathbf{F}}_{21}$, are not only equal and opposite; they also have the same line of action. Hence they have the same lever arm as well. The torques they produce are therefore equal in magnitude and opposite in direction.

13. The vertical component of the angular momentum of the system (merry-go-round and child) is conserved throughout this process, since there are no external torques about the vertical axis of the merry-go-round. When the child moves out to the rim, the rotational inertia of the system increases, because the child is located farther from the axis. To conserve angular momentum, the angular velocity must therefore decrease. Noting that the rotational kinetic energy can be written as $L^2/(2I)$ and that L remains constant while I increases, we see that the rotational kinetic energy of the system decreases.

14. The center of mass of the toy lies below the wire on which is it balancing. If the toy is pushed slightly off center, the force of gravity acting at the center of mass produces a torque that tends to rotate the toy back toward the center. If the center of mass were above the wire, this situation would be reversed and the toy would be unstable.

15. To knock a person over, their center of gravity must be moved until it is beyond the horizontal extent of their support base. The force of gravity will then produce a net torque about the edge of their support base, and they will topple over. The posture taken by defensive linemen makes them more difficult to push over because they have a larger support base and a lower center of gravity. They must therefore be pushed (rotated) by a greater amount to move their center of gravity beyond the edge of their support base. Four legged animals similarly have a relatively large support base and low center of gravity compared to humans, making them naturally more stable. Consequently, their neurological systems for maintaining balance do not need to be as complex as a human's.

16. The location of the CG below the hips in birds makes them naturally more stable than humans. If the upper body were displaced a little to the side, the torque produced by gravity about an axis through the hips would tend to rotate the upper body back toward the center in birds and farther away from the center in humans.

17. The astronaut and satellite constitute an isolated system. The initial angular momentum of the system is zero. When the astronaut tries to remove the bolt, both he and the satellite will rotate. They will rotate in opposite directions so that the total angular momentum of the system remains zero. To put it another way, when the astronaut applies a torque to a part of the satellite, the satellite applies an equal and opposite torque to him. The astronaut must anchor the satellite and himself somehow before trying to remove the bolt.

18. The best place is as far from the hinge as possible so as to have the greatest possible moment arm for the torque exerted by the stopper on the door. This way, the force required by the stopper to hold the door open will be as small as possible, making the stopper less likely to slip on the floor.

19. Low gears are used for going uphill and high gears are used for downhill. The bicycle gears act like levers. The energy remains the same so that the force you exert on the pedals times the distance the pedals move will equal the force exerted on the rim of the wheel times the distance it moves. A low gear converts the force you exert at the pedals into a lesser force that acts over a slightly longer distance, while a high gear converts the force you exert on the pedals into a much smaller force exerted over a much longer distance. The gear ratio tells how many times the rear bicycle wheel goes around for each time the pedals go around once. In a low gear ratio the bicycle will go a shorter distance for each rotation of the pedals while in a high gear the bicycle goes a long distance for each rotation of the pedals.

20. The motion of the suspended irregular object is influenced by two external forces—the normal force from the nail acting at its point of contact and the force due to the weight of the object acting at its center of mass. No torque is produced by the normal force because it acts along the rotation axis. The net torque on the object is therefore solely a result of the force from its weight. The object will rotate back and forth as determined by the direction of the torque until frictional forces have brought its center of mass to rest directly beneath the rotation axis. At this point, the applied force is parallel to the lever arm and no torque is produced. For each orientation of the object, the line drawn will pass through its center of mass—the intersection of several such lines must therefore occur at the center of mass location.

21. The melting of Earth's polar ice caps would distribute some of its mass from locations near its rotation axis to locations that are on average farther from its rotation axis. The rotational inertia of a sphere is greater if its mass is distributed farther from its axis of rotation—the Earth's moment of inertia would therefore increase. Angular momentum conservation requires that the product of the Earth's rotational inertia and its angular velocity be constant. A larger moment of inertia must be accompanied by a smaller angular velocity—the melting of the caps would therefore increase the length of the day.

Problems

1. **Strategy and Solution** I has units $\text{kg}\cdot\text{m}^2$. ω^2 has units $(\text{rad/s})^2$. So, $\frac{1}{2}I\omega^2$ has units $\text{kg}\cdot\text{m}^2\cdot\text{rad}^2/\text{s}^2 = \text{kg}\cdot\text{m}^2/\text{s}^2 = \text{J}$, which is a unit of energy.

2. **Strategy** The rotational inertia of a solid disk is $I = \frac{1}{2}MR^2$.

Solution Find the rotational inertial of the solid iron disk.

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(49\text{ kg})(0.200\text{ m})^2 = \boxed{0.98\text{ kg}\cdot\text{m}^2}$$

3. **Strategy** $I = \frac{2}{5}MR^2$ for a solid sphere and mass density is $\rho = M/V$.

Solution

(a) $M = \rho V = \rho\frac{4}{3}\pi R^3$ for a solid sphere. Form a proportion.

$$\frac{M_{\text{child}}}{M_{\text{adult}}} = \left(\frac{R_{\text{child}}}{R_{\text{adult}}}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8},$$ so the mass is $\boxed{\text{reduced by a factor of 8}}$.

(b) Form a proportion.

$$\frac{I_{\text{child}}}{I_{\text{adult}}} = \frac{1}{8}\left(\frac{R_{\text{child}}}{R_{\text{adult}}}\right)^2 = \frac{1}{8}\left(\frac{1}{2}\right)^2 = \frac{1}{32}$$

The rotational inertia is $\boxed{\text{reduced by a factor of 32}}$.

4. Strategy Use Eq. (8-2) to find the rotational inertia. Use Eq. (7-9) for the center of mass.

Solution

(a) $I_x = \sum_{i=A}^{C} m_i r_i^2 = (200\text{ g})(5.0\text{ cm})^2 + (300\text{ g})(0\text{ cm})^2 + (500\text{ g})(4.0\text{ cm})^2 = \boxed{13{,}000\text{ g}\cdot\text{cm}^2}$

(b) $I_y = \sum_{i=A}^{C} m_i r_i^2 = (200\text{ g})(3.0\text{ cm})^2 + (300\text{ g})(6.0\text{ cm})^2 + (500\text{ g})(5.0\text{ cm})^2 = \boxed{25{,}000\text{ g}\cdot\text{cm}^2}$

(c) $I_z = \sum_{i=A}^{C} m_i r_i^2 = (200\text{ g})[(3.0\text{ cm})^2 + (5.0\text{ cm})^2] + (300\text{ g})(6.0\text{ cm})^2 + (500\text{ g})[(5.0\text{ cm})^2 + (4.0\text{ cm})^2]$
$= \boxed{38{,}000\text{ g}\cdot\text{cm}^2}$

(d) $x_{\text{CM}} = \dfrac{(200\text{ g})(-3.0\text{ cm}) + (300\text{ g})(6.0\text{ cm}) + (500\text{ g})(-5.0\text{ cm})}{200\text{ g} + 300\text{ g} + 500\text{ g}} = \boxed{-1.3\text{ cm}}$

$y_{\text{CM}} = \dfrac{(200\text{ g})(5.0\text{ cm}) + (300\text{ g})(0\text{ cm}) + (500\text{ g})(-4.0\text{ cm})}{200\text{ g} + 300\text{ g} + 500\text{ g}} = \boxed{-1.0\text{ cm}}$

5. Strategy Find the rotational inertia in each case by using Eq. (8-2).

Solution

(a) $I = m(r^2 + 0^2 + 0^2 + r^2) = 2mr^2 = 2(3.0\text{ kg})(0.50\text{ m})^2 = \boxed{1.5\text{ kg}\cdot\text{m}^2}$

(b) $I = m(0^2 + r^2 + 0^2 + r^2) = 2mr^2 = 2(3.0\text{ kg})(0.50\text{ m}/\sqrt{2})^2 = \boxed{0.75\text{ kg}\cdot\text{m}^2}$

(c) $I = m(r^2 + r^2 + r^2 + r^2) = 4mr^2 = 4(3.0\text{ kg})(0.50\text{ m}/\sqrt{2})^2 = \boxed{1.5\text{ kg}\cdot\text{m}^2}$

6. Strategy The rotational inertia of a solid disk is $I = \frac{1}{2}MR^2$. Use the work-kinetic energy theorem.

Solution Find the work done to spin the CD.

$$W = \Delta K = \frac{1}{2}I(\omega_\text{f}^2 - \omega_\text{i}^2) = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left[\left(\frac{v_\text{f}}{r}\right)^2 - \left(\frac{v_\text{i}}{r}\right)^2\right] = \frac{1}{4}M\left(\frac{R}{r}\right)^2(v_\text{f}^2 - v_\text{i}^2)$$
$$= \frac{1}{4}(0.0158\text{ kg})\left[\frac{(0.120\text{ m})/2}{0.0200\text{ m}}\right]^2[(1.20\text{ m/s})^2 - 0] = \boxed{0.0512\text{ J}}$$

7. Strategy $I = \frac{2}{5}MR^2$ for a solid sphere and $I = MR^2$ for the Earth about the Sun.

Solution Form a proportion.

$$\frac{I_\text{axis}}{I_\text{Sun}} = \frac{\frac{2}{5}MR_\text{E}^2}{MR_\text{o}^2} = \frac{2R_\text{E}^2}{5R_\text{o}^2}$$

So, $\dfrac{I_\text{axis}}{I_\text{Sun}} = \boxed{\dfrac{2}{5}\dfrac{R_\text{E}^2}{R_\text{o}^2}\text{, where } R_\text{E} \text{ is the Earth's radius and } R_\text{o} \text{ is Earth's orbital radius about the Sun.}}$

8. Strategy Use Eq. (8-1) and form a proportion.

Solution Find the fraction of the total kinetic energy that is rotational.

$$\frac{K_{\text{rot}}}{K_{\text{total}}}=\frac{2\left(\frac{1}{2}I\omega^2\right)}{2\left(\frac{1}{2}I\omega^2\right)+\frac{1}{2}Mv^2}=\frac{2}{2+\frac{Mv^2}{I\omega^2}}=\frac{2}{2+\frac{Mv^2}{I(v^2/R^2)}}=\frac{2}{2+\frac{MR^2}{I}}=\frac{2}{2+\frac{(79\text{ kg})(0.32\text{ m})^2}{0.080\text{ kg}\cdot\text{m}^2}}=\boxed{0.019}$$

9. (a) Strategy and Solution Since a significant fraction of the wheel's kinetic energy is rotational, to model it as if it were sliding without friction would be unjustified. So, the answer is $\boxed{\text{no}}$.

(b) Strategy Use Eq. (8-1) and form a proportion.

Solution Find the fraction of the total kinetic energy that is rotational.

$$\frac{K_{\text{rot}}}{K_{\text{total}}}=\frac{4\left(\frac{1}{2}I\omega^2\right)}{\frac{1}{2}Mv^2+4\left(\frac{1}{2}I\omega^2\right)}=\frac{1}{\frac{Mv^2}{4I\omega^2}+1}=\frac{1}{1+\frac{Mv^2}{4I(v^2/R^2)}}=\frac{1}{1+\frac{MR^2}{4I}}=\frac{1}{1+\frac{(1300\text{ kg})(0.35\text{ m})^2}{4(0.705\text{ kg}\cdot\text{m}^2)}}=\boxed{0.017}$$

10. Strategy The total energy required to bring the centrifuge from rest to 420 rad/s is equal to the kinetic energy when it rotates at $\omega = 420$ rad/s. Use Eq. (8-1).

Solution Find the energy required to spin the centrifuge.

$$K=\frac{1}{2}I\omega^2=\frac{1}{2}(6.5\times10^{-3}\text{ kg}\cdot\text{m}^2)(420\text{ rad/s})^2=\boxed{570\text{ J}}$$

11. Strategy Use Eq. (8-3).

Solution Find the magnitude of the torque applied to the wrench.

$$|\tau|=rF_\perp=(0.16\text{ m})(25\text{ N})=\boxed{4.0\text{ N}\cdot\text{m}}$$

12. Strategy Use Eq. (8-3).

Solution Find the magnitude of the torque applied to the drum.

$$|\tau|=rF_\perp=(0.0600\text{ m})(75\text{ N})=\boxed{4.5\text{ N}\cdot\text{m}}$$

13. Strategy Use Eq. (8-3).

Solution Find the magnitude of the torque.

$$|\tau|=F_\perp r=mgr=(40.0\text{ kg})(9.80\text{ N/kg})(2.0\text{ m})=\boxed{780\text{ N}\cdot\text{m}}$$

14. Strategy Use Eq. (8-3).

Solution Find the magnitude of the torque.

$$|\tau|=F_\perp r=mgr=(0.124\text{ kg})(9.80\text{ N/kg})(0.25\text{ m})=\boxed{0.30\text{ N}\cdot\text{m}}$$

15. Strategy The point of application of the force of gravity is at the geometrical center of the door, so $r_\perp=(1.0\text{ m})/2$. The force is equal to the weight of the door. Use Eq. (8-4).

Solution Find the magnitude of the torque.

$$|\tau|=r_\perp F=r_\perp mg=[(1.0\text{ m})/2](50.0\text{ N})=\boxed{25\text{ N}\cdot\text{m}}$$

16. Strategy Use Eqs. (8-3) and (8-4).

Solution

(a) The force is parallel to the lever arm at noon.
$\tau = Fr_{\perp} = F(0) = \boxed{0}$

(b) The torque is CCW (positive). The center of mass is $(2.7\text{ m})/2$ from the axis.
$\tau = F_{\perp}r = mgr = (60.0\text{ kg})(9.80\text{ N/kg})[(2.7\text{ m})/2] = \boxed{790\text{ N}\cdot\text{m}}$

17. Strategy Use Eq. (8-4).

Solution Find the net torque in each case.

(a) $\Sigma\tau = F(r_{2\perp} - r_{1\perp}) = Fx_2 - Fx_1 = F(x_2 - x_1) = Fd,$ since $d = x_2 - x_1.$

(b) $\Sigma\tau = F(r_{2\perp} - r_{1\perp}) = F(r_2 \sin\theta_2) - F(r_1 \sin\theta_1) = Fx_2 - Fx_1 = F(x_2 - x_1) = Fd$

18. Strategy Use Eq. (8-3) to compute the torque in each case.

Solution

(a) The force is applied perpendicularly to the door, so $\tau = rF = (1.26\text{ m})(46.4\text{ N}) = \boxed{58.5\text{ N}\cdot\text{m}}.$

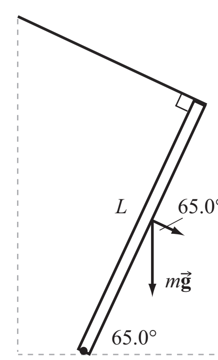
(b) The force is applied at $43.0°$ from the door's surface, so
$|\tau| = rF_{\perp} = rF\sin\theta = (1.26\text{ m})(46.4\text{ N})\sin 43.0° = \boxed{39.9\text{ N}\cdot\text{m}}.$

(c) Since the force is applied such that its line of action passes through the axis of the door hinges—the axis of rotation—there is no perpendicular component of the force and the torque is $\boxed{0}$.

19. Strategy Use Eq. (8-3) to find the torque.

Solution Let the axis of rotation be a the hinge of the trap door. Since the door is in equilibrium, the magnitude of the torque exerted on the door by the rope is the same as that exerted by gravity. Compute the torque due to the rope.

$$\begin{aligned}\tau &= rF_{\perp}\\ &= \frac{L}{2}mg\cos 65.0°\\ &= \frac{1.65\text{ m}}{2}(16.8\text{ kg})(9.80\text{ m/s}^2)\cos 65.0°\\ &= \boxed{57.4\text{ N}\cdot\text{m}}\end{aligned}$$



20. Strategy The center of gravity is located at the center of mass.

Solution Find the center of gravity.

$$x_{CG} = x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M} = \frac{(5.0\text{ kg})(0.0) + (15.0\text{ kg})(5.0\text{ m}) + (10.0\text{ kg})(10.0\text{ m})}{5.0\text{ kg} + 15.0\text{ kg} + 10.0\text{ kg}} = \boxed{5.83\text{ m}}$$

21. Strategy The center of gravity is located at the center of mass. Let the origin be at the center of the door.

Solution Due to symmetry, $y_{CM} = 0$.

$$x_{CM} = \frac{m_1x_1 + m_2x_2}{M} = \frac{m_1(0) + m_2(x)}{M} = \frac{W_2x}{W} = \frac{(5.0\text{ N})(-0.75\text{ m})}{5.0\text{ N} + 300.0\text{ N}} = -0.012\text{ m}$$

The center of gravity is located $\boxed{\text{1.2 cm toward the doorknob as measured from the center of the door.}}$

22. Strategy The center of gravity is at the center of mass of the plate. Imagine that the plate consists of a rectangular plate (on the left) and a square (on the right). The mass is proportional to the area for a uniform distribution.

Solution Find the center of gravity.

$$x_{CM} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{0.50s^2\left(\frac{0.50s}{2}\right) + 0.50^2s^2\left(0.50s + \frac{0.50s}{2}\right)}{0.50s^2 + 0.50^2s^2} = 0.42s$$

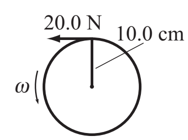
$$y_{CM} = \frac{0.50s^2(0.50s) + 0.50^2s^2\left(0.50s + \frac{0.50s}{2}\right)}{0.50s^2 + 0.50^2s^2} = 0.58s$$

So, the center of gravity is located at $\boxed{(0.42s, 0.58s)}$.

23. Strategy Use Eqs. (8-6) and (8-4).

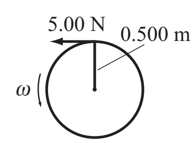
Solution Compute the work done on the stone.

$$W = \tau\Delta\theta = r_\perp F\Delta\theta = (0.100\text{ m})(20.0\text{ N})(12\text{ rev})(2\pi\text{ rad/rev}) = \boxed{150\text{ J}}$$



24. (a) Strategy and Solution One revolution is equal to the circumference of the wheel, so the rope unwinds

$$C = 2\pi r = 2\pi(0.500\text{ m}) = \boxed{3.14\text{ m}}.$$



(b) Strategy The work done by the rope on the wheel is equal to the force times the distance.

Solution

$$W = Fd = (5.00\text{ N})(3.14\text{ m}) = \boxed{15.7\text{ J}}$$

(c) Strategy Use Eq. (8-4).

Solution Find the torque on the wheel due to the rope.

$$\tau = r_\perp F = (0.500\text{ m})(5.00\text{ N}) = \boxed{2.50\text{ N}\cdot\text{m}}$$

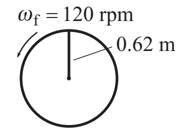
(d) Strategy and Solution There are 2π rad per revolution, so the angular displacement is

$$\Delta\theta = (1.00\text{ rev})(2\pi\text{ rad/rev}) = \boxed{6.28\text{ rad}}.$$

(e) Strategy and Solution $\tau\Delta\theta = (2.50\text{ N}\cdot\text{m})(6.28\text{ rad}) = 15.7\text{ J} = W$

25. (a) Strategy Use the work-kinetic energy theorem.

Solution Find the work done spinning up the wheel.



$$W = \Delta K = \frac{1}{2}I\omega_f^2 = \frac{1}{2}(MR^2)\omega_f^2$$

$$= \frac{1}{2}(182 \text{ kg})(0.62 \text{ m})^2[(120 \text{ rev/min})(1/60 \text{ min/s})(2\pi \text{ rad/rev})]^2 = \boxed{5.5 \text{ kJ}}$$

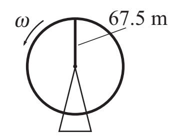
(b) Strategy Use the equations for rotational motion with constant acceleration and the relationship between work, torque, and angular displacement.

Solution Find the torque.

$$W = \tau\Delta\theta = \tau(\omega_{av}\Delta t), \text{ so } \tau = \frac{W}{\omega_{av}\Delta t} = \frac{5.5\times10^3 \text{ J}}{(120 \text{ rev/min})(1/60 \text{ min/s})(2\pi \text{ rad/rev})(30.0 \text{ s})/2} = \boxed{29 \text{ N}\cdot\text{m}}.$$

26. (a) Strategy The rotational inertia of a hoop is MR^2. Use the work-kinetic energy theorem and Eq. (8-1).

Solution Find the work.



$$W = \Delta K = \frac{1}{2}I(\omega_f^2 - \omega_i^2) = \frac{1}{2}(MR^2)(\omega_f^2 - 0)$$

$$= \frac{1}{2}(1.90\times10^6 \text{ kg})(67.5 \text{ m})^2(3.50\times10^{-3} \text{ rad/s})^2 = \boxed{53.0 \text{ kJ}}$$

(b) Strategy Constant torque implies constant angular acceleration, so $\Delta\theta = \omega_{av}\Delta t$. Use Eq. (8-6).

Solution Find the torque.

$$W = \tau\Delta\theta = \tau\omega_{av}\Delta t = \tau\left(\frac{\omega_f + \omega_i}{2}\right)\Delta t = \tau\left(\frac{\omega_f + 0}{2}\right)\Delta t, \text{ so}$$

$$\tau = \frac{2W}{\omega_f\Delta t} = \frac{2(53.0\times10^3 \text{ J})}{(3.50\times10^{-3} \text{ rad/s})(20.0 \text{ s})} = \boxed{1.51 \text{ MN}\cdot\text{m}}.$$

27. Strategy Choose the axis of rotation at the fulcrum. Use Eqs. (8-8).

Solution Find the force required to lift the load.

$$\Sigma\tau = 0 = -F_A\cos\theta(2.4 \text{ m}) + F_{load}\cos\theta(1.2 \text{ m}), \text{ so}$$

$$F_A = \frac{1.2 \text{ m}}{2.4 \text{ m}}F_{load} = 0.50mg = 0.50(20.0 \text{ kg})(9.80 \text{ N/kg}) = \boxed{98 \text{ N}}.$$

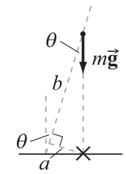
28. Strategy Choose the axis of rotation at the fulcrum. Use Eqs. (8-8).

Solution Find F.

$$\Sigma\tau = 0 = -F(3.0 \text{ m}) + (1200 \text{ N})(0.50 \text{ m}), \text{ so } F = \frac{(1200 \text{ N})(0.50 \text{ m})}{3.0 \text{ m}} = \boxed{200 \text{ N}}.$$

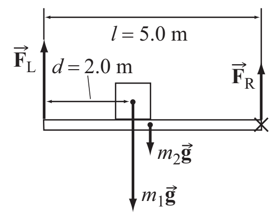
29. Strategy Choose the rotation axis at the edge of the base of the sculpture that is in contact with the floor as it is tipped. The angle that the base makes with the floor is the same angle that the force due to gravity makes with the vertical axis of the sculpture.

Solution Set the net torque equal to zero at the equilibrium point to find the maximum angle.
$\Sigma\tau = 0 = -mgb\sin\theta + mga\cos\theta$, where $b = 1.80$ m and $a = (1.10 \text{ m})/2 = 0.550$ m.
Solve for the angle.



$b\sin\theta = a\cos\theta$, so $\theta = \tan^{-1}\frac{a}{b} = \tan^{-1}\frac{0.550 \text{ m}}{1.80 \text{ m}} = \boxed{17.0°}$.

30. (a) Strategy Choose the axis of rotation at the point at which the right-hand cable connects to the platform. Let $m_1 = 75$ kg and $m_2 = 20.0$ kg. Let $l = 5.0$ m. The system is in equilibrium.



Solution Find the force exerted by the left-hand cable.

$\Sigma\tau = 0 = -F_L l + m_1 g(l-d) + m_2 g\left(\frac{l}{2}\right)$, so

$$F_L = g\left[m_1\left(1-\frac{d}{l}\right)+\frac{m_2}{2}\right]$$
$$= (9.80 \text{ N/kg})\left[(75 \text{ kg})\left(1-\frac{2.0 \text{ m}}{5.0 \text{ m}}\right)+\frac{20.0 \text{ kg}}{2}\right] = \boxed{540 \text{ N}}.$$

(b) Strategy Use Newton's second law.

Solution Find the force exerted by the right-hand cable.
$\Sigma F = 0 = -m_1 g - m_2 g + F_L + F_R$, so

$F_R = (m_1 + m_2)g - F_L = (75 \text{ kg} + 20.0 \text{ kg})(9.80 \text{ N/kg}) - 539 \text{ N} = \boxed{390 \text{ N}}$.

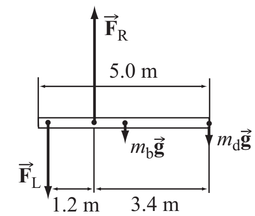
31. Strategy A system balances if its center of mass is above its base of support. Use Eq. (7-9) to find the center of mass of the metersticks.

Solution Let the left end of the lowest meterstick be the origin.

$$x_{CM} = \frac{mx_1 + mx_2 + mx_3 + mx_4}{4m} = \frac{x_1 + x_2 + x_3 + x_4}{4}$$
$$= \frac{0.5000 + (0.5000 + 0.3333) + (0.5000 + 0.3333 + 0.1667) + (0.5000 + 0.3333 + 0.1667 + 0.0833)}{4} \text{ m}$$
$$= 0.8542 \text{ m}$$

Since $\boxed{\text{the center of mass} = 0.8542 \text{ m} < 0.8600 \text{ m, so the system balances}}$.

32. Strategy Use Eqs. (8-8).



Solution Find the forces acting on the board.
Left support: Choose the axis of rotation at the top of the right support.
$\Sigma\tau = 0 = F_L(1.2\text{ m}) - m_b g(3.4\text{ m} - 2.5\text{ m}) - m_d g(3.4\text{ m})$, so

$$F_L = \frac{(9.80\text{ N/kg})[(3.4\text{ m})(55\text{ kg} + 65\text{ kg}) - (2.5\text{ m})(55\text{ kg})]}{1.2\text{ m}} = 2.2\text{ kN}.$$

Since $F > 0$, the force is downward (CCW rotation for torque). Thus, $\vec{\mathbf{F}} = \boxed{2.2\text{ kN downward}}$.

Right support: Choose the axis of rotation at the top of the left support.
$\Sigma\tau = 0 = F_R(1.2\text{ m}) - m_b g(1.2\text{ m} + 3.4\text{ m} - 2.5\text{ m}) - m_d g(4.6\text{ m})$, so

$$F_R = \frac{(9.80\text{ N/kg})[(55\text{ kg})(2.1\text{ m}) + (65\text{ kg})(4.6\text{ m})]}{1.2\text{ m}} = 3.4\text{ kN}.$$

Since $F > 0$, the force is upward (CCW rotation for torque). Thus, $\vec{\mathbf{F}} = \boxed{3.4\text{ kN upward}}$.

33. Strategy Use Eqs. (8-8).

Solution Choose the axis of rotation at the point of contact between the driveway and the ladder.
$\Sigma F_x = 0 = f - N_w$, so $f = N_w$.

$$\Sigma\tau = 0 = N_w(4.7\text{ m}) - W_l(2.5\text{ m})\cos\theta - W_p\left(\frac{3.0\text{ m}}{4.7\text{ m}}\right)(5.0\text{ m})\cos\theta, \text{ so } N_w = \frac{\cos\theta}{4.7\text{ m}}\left[W_l(2.5\text{ m}) + W_p\left(\frac{15\text{ m}}{4.7}\right)\right].$$

Find θ.

$$4.7\text{ m} = (5.0\text{ m})\sin\theta, \text{ so } \theta = \sin^{-1}\frac{4.7}{5.0}.$$

Calculate *f*.

$$f = N_w = \frac{\cos\sin^{-1}\frac{4.7}{5.0}}{4.7\text{ m}}\left[(120\text{ N})(2.5\text{ m}) + (680\text{ N})\left(\frac{15\text{ m}}{4.7}\right)\right] = 180\text{ N}$$

So, the force of friction is $\boxed{180\text{ N toward the wall}}$.

34. Strategy Use Eqs. (8-8).

Solution

(a) Choose the axis of rotation at the point of contact between the vertical wall and the climber's feet.

$$\Sigma\tau = T\cos\theta(1.06\text{ m}) - W_c(0.91\text{ m}) = 0, \text{ so } T = \frac{(0.91\text{ m})W_c}{(1.06\text{ m})\cos\theta} = \frac{(0.91\text{ m})(770\text{ N})}{(1.06\text{ m})\cos 25°} = \boxed{730\text{ N}}.$$

(b) $\Sigma F_x = 0 = F_x - T\sin\theta$ and $\Sigma F_y = 0 = F_y + T\cos\theta - W_c$, so $F_x = T\sin\theta$ and $F_y = W_c - T\cos\theta$.
Find the magnitude of the force.

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{T^2\sin^2\theta + W_c^2 + T^2\cos^2\theta - 2W_cT\cos\theta} = \sqrt{T^2 + W_c^2 - 2W_cT\cos\theta}$$
$$= \sqrt{(730\text{ N})^2 + (770\text{ N})^2 - 2(770\text{ N})(730\text{ N})\cos 25°} = 330\text{ N}$$

Find the direction.

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{W_c - T\cos\theta}{T\sin\theta} = \tan^{-1}\frac{770\text{ N} - (730\text{ N})\cos 25°}{(730\text{ N})\sin 25°} = 19°$$

Thus, $\vec{\mathbf{F}} = \boxed{330\text{ N at 19° above the horizontal}}$.

35. Strategy Use Eqs. (8-8).

Solution Choose the axis of rotation at the hinge.

$\Sigma\tau = 0 = T(2.38\text{ m})\sin 35° - (80.0\text{ N})(1.50\text{ m}) - (120.0\text{ N})(3.00\text{ m})$, so

$T = \dfrac{(80.0\text{ N})(1.50\text{ m}) + (120.0\text{ N})(3.00\text{ m})}{(2.38\text{ m})\sin 35°} = \boxed{350\text{ N}}$.

Find F_x and F_y.

$\Sigma F_x = 0 = -T\cos 35° + F_x$ and $\Sigma F_y = 0 = F_y + T\sin 35° - 80.0\text{ N} - 120.0\text{ N}$, so

$F_x = T\cos 35° = (350\text{ N})\cos 35° = \boxed{290\text{ N}}$ and $F_y = -(351.6\text{ N})\sin 35° + 80.0\text{ N} + 120.0\text{ N} = \boxed{-2\text{ N}}$.

$\boxed{\text{The magnitude of } F_y \text{ is small}}$ compared to that of F_x and T.

36. Strategy Use Eqs. (8-8).

Solution Choose the axis of rotation at the hinge.

$\Sigma\tau = 0 = Wl\cos\theta - Tl\sin\theta + mg\dfrac{l}{2}\cos\theta$, so $T = \boxed{\dfrac{mg/2 + W}{\tan\theta}}$.

$\boxed{\text{For } \theta = 0, T \to \infty, \text{ and for } \theta = 90°, T \to 0.}$

37. Strategy Use Eqs. (8-8). Choose the axis of rotation at the point where the beam meets the store.

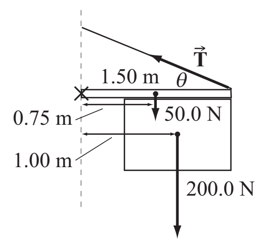
Solution The tension in the cable cannot exceed 417 N. Sum the torques.

$\Sigma\tau = 0 = T\sin\theta(1.50\text{ m}) - (50.0\text{ N})(0.75\text{ m}) - (200.0\text{ N})(1.00\text{ m})$

Solve for θ and substitute 417 N (the breaking strength) for T.

$\theta = \sin^{-1}\dfrac{(50.0\text{ N})(0.75\text{ m}) + (200.0\text{ N})(1.00\text{ m})}{(417\text{ N})(1.50\text{ m})} = 22.3°$

The minimum angle is $\boxed{22.3°}$.



38. Strategy Use the results from Problem 37.

Solution We must add one term (for the cat) and substitute for the angle in the summation of the torques. Let d be the distance between the store and the center of mass of the cat.

$\Sigma\tau = 0 = (417\text{ N})\sin 33.8°(1.50\text{ m}) - (50.0\text{ N})(0.75\text{ m}) - (200.0\text{ N})(1.00\text{ m}) - (8.7\text{ kg})(9.80\text{ m/s}^2)d$, so

$d = \dfrac{(417\text{ N})\sin 33.8°(1.50\text{ m}) - (50.0\text{ N})(0.75\text{ m}) - (200.0\text{ N})(1.00\text{ m})}{(8.7\text{ kg})(9.80\text{ m/s}^2)} = \boxed{1.3\text{ m}}$.

39. Strategy Use Eqs. (8-8). Choose the axis of rotation at the point of contact between the floor and the man's feet.

Solution Find the forces exerted by the floor.

Palms:

$\Sigma\tau = 0 = -F(1.70\text{ m}) + mg(1.00\text{ m})$, so $F = \dfrac{mg(1.00\text{ m})}{1.70\text{ m}} = \dfrac{(68\text{ kg})(9.80\text{ N/kg})(1.00\text{ m})}{1.70\text{ m}} = \boxed{390\text{ N}}$.

Feet:

$\Sigma F_y = 0 = F_\text{p} + F_\text{f} - mg$, so $F_\text{f} = mg - F_\text{p} = (68\text{ kg})(9.80\text{ N/kg}) - 392\text{ N} = \boxed{270\text{ N}}$.

40. Strategy Assuming F_b is (nearly) straight down, F_s is simply equal to the magnitude of the sum of the forces due to gravity on your friend and the package.

Solution Find F_s.

$F_s = Mg + mg = (M + m)g = (55\text{ kg} + 10\text{ kg})(9.80\text{ N/kg}) = \boxed{640\text{ N}}$.

41. Strategy Use Eqs. (8-8). Choose the axis of rotation at the point of contact of the normal force.

Solution Find the tension in the Achilles tendon, F_A.

$\Sigma\tau = 0 = -F_A(4.60\text{ cm} + 12.8\text{ cm}) + F_T(12.8\text{ cm})$ and $\Sigma F_y = 0 = N + F_A - F_T$, or $F_T = N + F_A$, so

$-F_A(17.4\text{ cm}) + (N + F_A)(12.8\text{ cm}) = 0$, or $F_A = \dfrac{(12.8\text{ cm})(750\text{ N})}{4.6\text{ cm}} = 2100\text{ N}.$

Find the force that the tibia exerts on the ankle joint, F_T.

$F_T = N + F_A = 750\text{ N} + \dfrac{12.8}{4.6}(750\text{ N}) = 2800\text{ N}$

The forces are: $\boxed{\text{tendon, 2100 N upward and tibia, 2800 N downward}}$.

42. Strategy Use Eqs. (8-8). Choose the axis of rotation at the shoulder joint. One arm supports half of the person's weight, so $F_p = \frac{1}{2}(700\text{ N}) = 350\text{ N}$.

Solution Find the force each muscle exerts.

$\Sigma\tau = 0 = F_m(12\text{ cm})\sin 15° - F_g(27.5\text{ cm}) - F_p(60\text{ cm})$, so

$$F_m = \frac{F_g(27.5\text{ cm}) + F_p(60\text{ cm})}{(12\text{ cm})\sin 15°} = \frac{(30.0\text{ N})(27.5\text{ cm}) + (350\text{ N})(60\text{ cm})}{(12\text{ cm})\sin 15°} = \boxed{7.0\text{ kN}}.$$

43. Strategy Use Eqs. (8-8). Choose the axis of rotation at the elbow.

Solution Find the force exerted by the biceps muscle.

$\Sigma\tau = 0 = -W_m(35.0\text{ cm}) - W_a(16.5\text{ cm}) + F_b(5.00\text{ cm})\sin\theta$, so

$$F_b = \frac{W_m(35.0\text{ cm}) + W_a(16.5\text{ cm})}{(5.00\text{ cm})\sin\theta} = \frac{(9.9\text{ N})(35.0\text{ cm}) + (18.0\text{ N})(16.5\text{ cm})}{(5.00\text{ cm})\frac{30.0\text{ cm}}{\sqrt{(30.0\text{ cm})^2 + (5.00\text{ cm})^2}}} = \boxed{130\text{ N}}.$$

44. Strategy Use Eqs. (8-8). Choose the axis of rotation at the knee.

Solution Find the forces exerted by the patellar tendon.

(a) $\Sigma\tau = 0 = F_p(10.0\text{ cm})\sin 20.0° - F_w(41\text{ cm})\sin 30.0° - F_L(22\text{ cm})\sin 30.0°$, so

$$F_p = \frac{g\sin 30.0°[m_w(41\text{ cm}) + m_L(22\text{ cm})]}{(10.0\text{ cm})\sin 20.0°} = \frac{(9.80\text{ N/kg})\sin 30.0°[(3.0\text{ kg})(41\text{ cm}) + (5.0\text{ kg})(22\text{ cm})]}{(10.0\text{ cm})\sin 20.0°}$$

$$= \boxed{330\text{ N}}.$$

(b) $\Sigma\tau = 0 = F_q(10.0\text{ cm})\sin 20.0° - F_w(41\text{ cm})\sin 90.0° - F_L(22\text{ cm})\sin 90.0°$, so

$$F_q = \frac{g[m_w(41\text{ cm}) + m_L(22\text{ cm})]}{(10.0\text{ cm})\sin 20.0°} = \frac{(9.80\text{ N/kg})[(3.0\text{ kg})(41\text{ cm}) + (5.0\text{ kg})(22\text{ cm})]}{(10.0\text{ cm})\sin 20.0°} = \boxed{670\text{ N}}.$$

45. Strategy Refer to Figure 8.32. First find the magnitude of the force exerted by the back F_b by analyzing the torques about an axis at the sacrum; then, find the horizontal component of the extreme force on the sacrum F_s. Use Eqs. (8-8).

Solution Sum the torques to find F_b.

$\Sigma\tau = 0 = F_b(44\text{ cm})\sin 12° - (10\text{ kg})(9.80\text{ m/s}^2)(76\text{ cm}) - (55\text{ kg})(9.80\text{ m/s}^2)(38\text{ cm}),$ so

$$F_b = \frac{(10\text{ kg})(9.80\text{ m/s}^2)(76\text{ cm}) + (55\text{ kg})(9.80\text{ m/s}^2)(38\text{ cm})}{(44\text{ cm})\sin 12°} = 3053\text{ N}.$$

The only forces with components in the horizontal direction are those due to the back and the sacrum. Find the horizontal component of the extreme force, F_{sx}.

$\Sigma F_x = 0 = F_{sx} - F_b\cos 12°,$ so $F_{sx} = F_b\cos 12° = (3053\text{ N})\cos 12° = \boxed{3.0\text{ kN}}.$

$\frac{(3053\text{ N})\cos 12°}{540\text{ N}} = 5.5,$ so the force is $\boxed{\text{about 5.5 times larger}}$ than that from his torso alone!

46. Strategy Use Eqs. (8-8).

Solution

(a) The torque exerted by the erector spinae muscles must be equal in magnitude and opposite in direction to the torque due to the mass of the upper body and the 60.0-kg mass.

$\Sigma\tau = x_{CM}W_{ub} + mgx = (0.38\text{ m})(455\text{ N}) + (60.0\text{ kg})(9.80\text{ N/kg})(0.76\text{ m}) = \boxed{620\text{ N}\cdot\text{m}}$

(b) $\tau = (F_b\sin\theta)d$ where $d = 0.44$ m, F_b is the magnitude of the force due to the erector spinae muscles, $\theta = 12°,$ and $\tau = 620\text{ N}\cdot\text{m}.$

$$F_b = \frac{\tau}{d\sin\theta} = \frac{620\text{ N}\cdot\text{m}}{(0.44\text{ m})\sin 12°} = 6800\text{ N}$$

The force exerted by the erector spinae muscles is $\boxed{\text{6800 N at 12° above the horizontal}}$.

(c) The component of the force that compresses the spinal column is $F_b\cos\theta = (6780\text{ N})\cos 12° = \boxed{6600\text{ N}}.$

47. Strategy and Solution Torque has units $\text{N}\cdot\text{m} = \text{kg}\cdot\text{m}\cdot\text{s}^{-2}\cdot\text{m} = \text{kg}\cdot\text{m}^2\cdot\text{s}^{-2}.$ Inertia times angular acceleration has units $\text{kg}\cdot\text{m}^2\cdot\text{s}^{-2} = \text{N}\cdot\text{m}.$ Thus, the units are consistent.

48. Strategy Use the rotational form of Newton's second law.

Solution Find the frictional torque.

$$\Sigma\tau = I\alpha = I\frac{\Delta\omega}{\Delta t} = (400.0\text{ kg}\cdot\text{m}^2)\frac{0 - 20.0\text{ rad/s}}{300.0\text{ s}} = -26.7\text{ N}\cdot\text{m}$$

The torque is $\boxed{26.7\text{ N}\cdot\text{m opposite the flywheel's rotation}}$.

49. Strategy Use the rotational form of Newton's second law and Eq. (5-21).

Solution Find the torque that the motor must deliver.

$I = \frac{1}{2}MR^2$ for a uniform disk, so

$$\Sigma\tau = I\alpha = \frac{1}{2}MR^2\left(\frac{\omega_f^2 - \omega_i^2}{2\Delta\theta}\right) = \frac{MR^2\omega_f^2}{4\Delta\theta} = \frac{(0.22\text{ kg})\left(\frac{0.305\text{ m}}{2}\right)^2(3.49\text{ rad/s})^2}{4(2.0\text{ rev})(2\pi\text{ rad/rev})} = \boxed{0.0012\text{ N}\cdot\text{m}}.$$

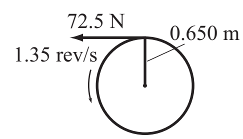
50. Strategy The rotational inertia of the gear is $I = 9.20\times10^{-2}\text{ kg}\cdot\text{m}^2$ and $\alpha = \Delta\omega/\Delta t$. Let $r = 15.0$ cm, the length of each spout, and let F be the force per spout. Use the rotational form of Newton's second law.

Solution Find F.

$$\Sigma\tau = 3Fr = I\alpha = I\frac{\Delta\omega}{\Delta t},\text{ so } F = \frac{I\Delta\omega}{3r\Delta t} = \frac{(9.20\times10^{-2}\text{ kg}\cdot\text{m}^2)(2.2\text{ rev/s})}{3(0.150\text{ m})(3.20\text{ s})}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \boxed{0.88\text{ N}}.$$

51. Strategy The rotational inertia of the gear is $I = \frac{1}{2}MR^2$ and $\alpha = \Delta\omega/\Delta t$. Use the rotational form of Newton's second law.

Solution Find the total frictional torque.



$$\Sigma\tau = TR - \tau_{\text{frictional}} = I\alpha = \frac{1}{2}MR^2\frac{\Delta\omega}{\Delta t},\text{ so}$$

$$\tau_{\text{frictional}} = TR - \frac{1}{2}MR^2\frac{\Delta\omega}{\Delta t}$$

$$= (72.5\text{ N})(0.650\text{ m}) - \frac{1}{2}(40.6\text{ kg})(0.650\text{ m})^2\frac{1.35\text{ rev/s}}{1.70\text{ s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \boxed{4.3\text{ N}\cdot\text{m}}.$$

52. Strategy Use the rotational form of Newton's second law and the definition of rotational inertia.

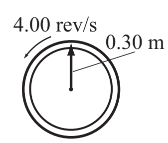
Solution Find the torque required to cause the angular acceleration.

$I = \sum_{i=A}^{D} m_i r_i^2 = (m_A + m_B + m_C + m_D)r^2$, since all four masses are (0.75 m)/2 from the axis.

$$\Sigma\tau = I\alpha = (4.0\text{ kg} + 3.0\text{ kg} + 5.0\text{ kg} + 2.0\text{ kg})[(0.75\text{ m})/2]^2(0.75\text{ rad/s}^2) = \boxed{1.5\text{ N}\cdot\text{m}}$$

53. Strategy The rotational inertia of the wheel is $I = MR^2$. Use the rotational form of Newton's second law.

Solution Find the magnitude of the average torque.



$$|\Sigma\tau_{\text{av}}| = I\alpha = MR^2\left|\frac{\Delta\omega}{\Delta t}\right| = (2\text{ kg})(0.30\text{ m})^2\left(\frac{4.00\text{ rev/s}}{50\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) = \boxed{0.09\text{ N}\cdot\text{m}}$$

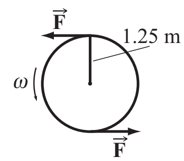
54. (a) Strategy The rotational inertia of the merry-go-round is $I = \frac{1}{2}MR^2$ and that of the children is $I = 2MR^2$. Use the rotational form of Newton's second law.

Solution Find the torque on the merry-go-round.

$$\Sigma\tau = I\alpha = \left(\frac{1}{2}MR^2 + 2mR^2\right)\frac{\Delta\omega}{\Delta t}$$

$$= \left[\frac{1}{2}(350.0\text{ kg})(1.25\text{ m})^2 + 2(30.0\text{ kg})(1.25\text{ m})^2\right]\left(\frac{25\text{ rpm}}{20.0\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{48\text{ N}\cdot\text{m}}$$

(b) Strategy Let F be the magnitude of the tangential force with which each child must push the rim.



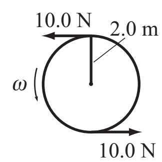
Solution Find F.

$$FR + FR = \Sigma\tau,\text{ so } F = \frac{\Sigma\tau}{2R} = \frac{48\text{ N}\cdot\text{m}}{2(1.25\text{ m})} = \boxed{19\text{ N}}.$$

55. Strategy The rotational inertia is $I = \frac{1}{2}MR^2$. Use the rotational form of Newton's second law and Eq. (5-18).

Solution

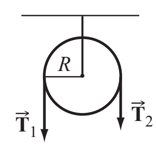
(a) $$\alpha = \frac{\Sigma\tau}{I} = \frac{FR + FR}{\frac{1}{2}MR^2} = \frac{4F}{MR} = \frac{4(10.0\text{ N})}{(180\text{ kg})(2.0\text{ m})} = \boxed{0.11\text{ rad/s}^2}$$



(b) $$\omega_f = \omega_i + \alpha\Delta t = 0 + (0.11\text{ rad/s}^2)(4.0\text{ s}) = \boxed{0.44\text{ rad/s}}$$

56. (a) Strategy and Solution This is just the relation between tangential acceleration and angular acceleration, $\boxed{a = R\alpha}$.

(b) Strategy Use Eqs. (8-8).



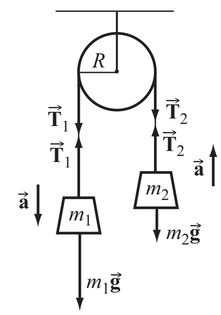
Solution Find the net torque on the pulley about its axis of rotation.

$$\Sigma\tau = T_1R - T_2R = (T_1 - T_2)R$$

The motion is CCW, so $\Sigma\tau = \boxed{(T_1 - T_2)R\text{ CCW}}$.

(c) Strategy and Solution If $m_1 = m_2$, $T_1 = T_2$, so $\Sigma\tau = 0$.

If $m_1 \neq m_2$, the blocks accelerate, so the pulley has an angular acceleration. Since a nonzero net torque is required for the pulley to accelerate, $T_1 - T_2 \neq 0$, thus $T_1 \neq T_2$.



(d) Strategy The rotational inertia of a pulley is $I = \frac{1}{2}MR^2$. Use Eqs. (8-8) and (8-9).

Solution Find the magnitudes of the tensions.

$m_1a = m_1g - T_1 \Rightarrow \boxed{T_1 = m_1(g-a)}$ and $m_2a = T_2 - m_2g \Rightarrow \boxed{T_2 = m_2(g+a)}$.

Find a.

$$\Sigma\tau = (T_1 - T_2)R = (m_1g - m_1a - m_2g - m_2a)R = I\alpha = \frac{1}{2}MR^2\frac{a}{R} = \frac{1}{2}MRa,$$

so $\boxed{a = \dfrac{(m_1 - m_2)g}{\frac{M}{2} + m_1 + m_2}}$.

(e) Strategy Use the result for the speed from Example 8.2.

Solution Check the answer for a.

From Example 8.2, $v = \sqrt{\dfrac{2(m_1 - m_2)gh}{m_1 + m_2 + I/R^2}}$. Find a.

$$2a_y\Delta y = 2ah = v_{fy}^2 - v_{iy}^2 = v^2 - 0 = \frac{2(m_1 - m_2)gh}{m_1 + m_2 + I/R^2}, \text{ so } a = \frac{(m_1 - m_2)g}{m_1 + m_2 + I/R^2}.$$

Now, $I = \frac{1}{2}MR^2$, so $a = \dfrac{(m_1 - m_2)g}{m_1 + m_2 + \frac{1}{2}MR^2/R^2} = \dfrac{(m_1 - m_2)g}{m_1 + m_2 + \frac{M}{2}}$.

The expression for a is the same as that found in part (d).

57. Strategy Follow the steps to derive the rotational from of Newton's second law.

Solution

(a) According to Newton's second law, $F_i = m_ia_i$, so $a_i = F_i/m_i$.

(b) The torque is the product of the perpendicular component of the force and the shortest distance between the rotation axis and the point of application of the force, so $\tau_i = F_ir_i = m_ia_ir_i$.

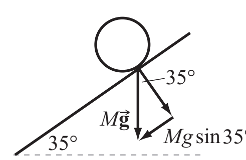
(c) The tangential acceleration is related to the angular acceleration by $a_i = r_i\alpha$, so $\tau_i = m_i(r_i\alpha)r_i = m_ir_i^2\alpha$.

(d) Summing the torques and using the definition of rotational inertia, we have

$$\sum_{i=1}^{N}\tau_i = \sum_{i=1}^{N}m_ir_i^2\alpha = \left(\sum_{i=1}^{N}m_ir_i^2\right)\alpha = I\alpha.$$

58. Strategy The rotational inertia of a uniform solid sphere is $\frac{2}{5}MR^2$. Use the expression for the acceleration found in Example 8.13.

Solution Find the acceleration of the solid sphere.

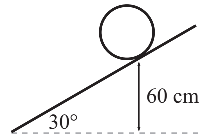


$$a_{CM} = \frac{g\sin\theta}{1 + I/(MR^2)} = \frac{g\sin\theta}{1 + \frac{2}{5}MR^2/(MR^2)} = \frac{g\sin\theta}{1 + 2/5} = \frac{(9.80\text{ m/s}^2)\sin 35°}{1 + 2/5}$$

$$= \boxed{4.0\text{ m/s}^2}$$

59. Strategy Use conservation of energy. The rotational inertia of a uniform solid sphere is $\frac{2}{5}MR^2$.

Solution Find the speed of the sphere.



$$\Delta K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 - 0 = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$
$$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2 = -\Delta U = Mgh, \text{ so}$$
$$v = \sqrt{\frac{10}{7}gh} = \sqrt{\frac{10}{7}(9.80\ \text{m/s}^2)(0.60\ \text{m})} = \boxed{2.9\ \text{m/s}}.$$

60. Strategy Use Eqs. (6-6) and (8-1).

Solution Find the total kinetic energies of each object.
Solid sphere:

$$K_{\text{tr}} + K_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

Solid cylinder:

$$K_{\text{tr}} + K_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

Hollow cylinder:

$$K_{\text{tr}} + K_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$$

In order from smallest to largest, the total kinetic energies are

$$\boxed{\text{solid sphere: } K = \frac{7}{10}mv^2;\ \text{solid cylinder: } K = \frac{3}{4}mv^2;\ \text{hollow cylinder: } K = mv^2}.$$

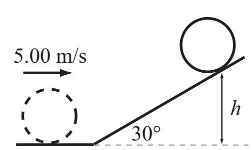
61. Strategy The sphere is rolling on a horizontal surface, so its total energy is equal to its total kinetic energy. Use conservation of energy.

Solution Compute the total energy.

$$E_{\text{total}} = K_{\text{tr}} + K_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$
$$= \frac{7}{10}(0.600\ \text{kg})(5.00\ \text{m/s})^2 = 10.5\ \text{J}$$

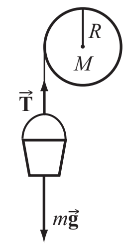
Find the height achieved by the sphere.

$$\Delta U = mgh = -\Delta K = K, \text{ so } h = \frac{K}{mg} = \frac{10.5\ \text{J}}{(0.600\ \text{kg})(9.80\ \text{N/kg})} = \boxed{1.79\ \text{m}}.$$



62. (a) Strategy Use conservation of energy. The rotational inertia of a uniform solid cylinder is $\frac{1}{2}MR^2$.

Solution Let $h = 0.80$ m, m be the mass of the bucket, and M be the mass of the cylinder. The tangential speed of the cylinder is the same as the linear speed of the bucket, since they are attached by a rope.



$$\Delta K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2 = -\Delta U = mgh, \text{ so}$$

$$v = \sqrt{\frac{4mgh}{2m+M}} = \sqrt{\frac{4(2.0\text{ kg})(9.80\text{ m/s}^2)(0.80\text{ m})}{2(2.0\text{ kg}) + 3.0\text{ kg}}} = \boxed{3.0\text{ m/s}}.$$

(b) Strategy Use the work-kinetic energy theorem.

Solution Find the tension T in the rope as the bucket falls a distance h.

$$W_{\text{total}} = \Delta K = \frac{1}{2}mv^2 = W_{\text{rope}} + W_{\text{grav}} = -Th + mgh, \text{ so}$$

$$T = m\left(g - \frac{v^2}{2h}\right) = (2.0\text{ kg})\left[9.80\text{ m/s}^2 - \frac{8.96\text{ m}^2/\text{s}^2}{2(0.80\text{ m})}\right] = \boxed{8.4\text{ N}}.$$

(c) Strategy Use Newton's second law.

Solution Find the acceleration of the bucket as it falls.

$$\Sigma F_y = T - mg = ma_y, \text{ so } a_y = -g + \frac{T}{m} = -9.80\text{ m/s}^2 + \frac{8.4\text{ N}}{2.0\text{ kg}} = -5.6\text{ m/s}^2, \text{ or } \boxed{5.6\text{ m/s}^2\text{ down}}.$$

63. Strategy Let $h = 17.0$ m, m be the mass of the bucket, and M be the mass of the cylinder. The tangential speed of the cylinder is the same as the linear speed of the bucket, since they are attached by a rope. Use conservation of energy. The rotational inertia of a uniform solid cylinder is $\frac{1}{2}MR^2$.

Solution Find the speed of the bucket when it reaches the bottom of the well.

$$\Delta K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2 = -\Delta U = mgh, \text{ so } v = \sqrt{\frac{4mgh}{2m+M}}.$$

Compute how long it will take for the bucket to fall to the bottom of the well.

$$\Delta y = h = \frac{1}{2}(v_{\text{fy}} + v_{\text{iy}})\Delta t, \text{ so}$$

$$\Delta t = \frac{2h}{v_{\text{fy}} + v_{\text{iy}}} = \frac{2h}{\sqrt{\frac{4mgh}{2m+M}} + 0} = \sqrt{\frac{h(2m+M)}{mg}} = \sqrt{\frac{(17.0\text{ m})[2(1.10\text{ kg}) + 2.60\text{ kg}]}{(1.10\text{ kg})(9.80\text{ m/s}^2)}} = \boxed{2.75\text{ s}}.$$

64. (a) Strategy and Solution The drilled cylinder takes more time because it converts a larger fraction of its potential energy to rotational kinetic energy and a smaller fraction to translational kinetic energy than the solid cylinder; the drilled cylinder takes more time because its rotational inertia is larger .

(b) Strategy Use conservation of energy and the result for the acceleration from Example 8.13.

Solution Find the speeds of the solid and drilled cylinders.
Solid cylinder:
Let m = the mass of the solid cylinder; its rotational inertia is $I_s = mR^2/2$, where R is the radius. Let h be the vertical height of the incline and v_s be the speed of the solid cylinder at the bottom. From conservation of energy,

$$mgh = K_{tr} + K_{rot} = \frac{1}{2}mv_s^2 + \frac{1}{2}I_s\omega^2 = \frac{1}{2}mv_s^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v_s^2}{R^2}\right) = \frac{1}{2}mv_s^2 + \frac{1}{4}mv_s^2 = \frac{3}{4}mv_s^2, \text{ so } v_s = 2\sqrt{\frac{gh}{3}}.$$

Drilled cylinder:
Let m' be the mass of the drilled cylinder. It has inner radius $b = R/2$ and outer radius $a = R$. From Table 8.1, its rotational inertia is

$$I_d = \frac{1}{2}m'(a^2 + b^2) = \frac{1}{2}m'\left[R^2 + \left(\frac{R}{2}\right)^2\right] = \frac{5}{8}m'R^2$$

Let v_d be the speed of the drilled cylinder at the bottom. From conservation of energy,

$$m'gh = \frac{1}{2}m'v_d^2 + \frac{1}{2}\left(\frac{5}{8}m'R^2\right)\left(\frac{v_d^2}{R^2}\right) = \frac{1}{2}m'v_d^2 + \frac{5}{16}m'v_d^2 = \frac{13}{16}m'v_d^2, \text{ so } v_d = 4\sqrt{\frac{gh}{13}}.$$

The ratio of the times to move down the incline is the inverse ratio of the final speeds. Why? Both move with constant acceleration starting from rest, so their average velocities are one half of their final velocities. They move the same distance—call it d—along the incline, so

$$d = \frac{1}{2}v_d\Delta t_d = \frac{1}{2}v_s\Delta t_s$$

Then,

$$\frac{\Delta t_d}{\Delta t_s} = \frac{v_s}{v_d} = \frac{2\sqrt{gh/3}}{4\sqrt{gh/13}} = \frac{1}{2}\sqrt{\frac{13}{3}} \approx 1.0408$$

The time for the drilled cylinder to roll down the incline is 4.08% longer than that for the solid cylinder.

65. (a) Strategy Use conservation of energy and the relationship between speed and radial acceleration.

Solution At the top of the loop, the sphere's speed must be at least the speed that results in a radial acceleration of g.

$\frac{v^2}{r} = g,$ so $v^2 = gr.$

The sphere's kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2}mgr,$ and it must equal the potential energy difference

$mgh - mg(2r).$ Thus, $\frac{1}{2}r = h - 2r$ or $h = \boxed{\frac{5}{2}r}.$

(b) Strategy The rotational inertia of a uniform solid sphere is $\frac{2}{5}mr^2.$ Use conservation of energy.

Solution Find the kinetic energy of the sphere.

$K = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v^2}{r^2}\right) = \frac{7}{10}mv^2 = \frac{7}{10}mgr$

Find h.

$\Delta K = \frac{7}{10}mgr = -\Delta U = mgh - mg(2r),$ so $h = \boxed{\frac{27}{10}r}.$

66. Strategy Use conservation of energy and the relationship between speed and radial acceleration.

Solution At the top of the loop, the cylinder's speed must be at least the speed that results in a radial acceleration of g.

$\frac{v^2}{r} = g,$ so $v^2 = gr.$

The cylinder's kinetic energy is $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\left(\frac{v}{r}\right)^2 = mv^2 = mgr,$ and it must equal the potential energy difference $mgh - mg(2r).$

Thus, $mgr = mgh - 2mgr = mg(h - 2r),$ so $h = \boxed{3r}.$

67. Strategy Consider the rotational inertia of each object. Use conservation of energy and the relationship between speed and radial acceleration.

Solution Since $I_{\text{sphere}} = \frac{2}{5}mr^2 < mr^2 = I_{\text{hollow cylinder}},$

h will decrease. The smaller the rotational inertia, the less gravitational energy will go into rotational energy, and the more will go into translational energy.

Redo the calculation with the solid sphere. At the top of the loop, the sphere's speed must be at least the speed that results in a radial acceleration of g.

$\frac{v^2}{r} = g,$ so $v^2 = gr.$ Thus, its kinetic energy is $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{5}mr^2\left(\frac{v}{r}\right)^2 = \frac{7}{10}mv^2 = \frac{7}{10}mgr,$ and the kinetic energy must equal the potential energy difference $mgh - mg(2r).$ Find h.

$\frac{7}{10}mgr = mgh - 2mgr,$ so $\frac{7}{10}r = h - 2r,$ or $h = 2.7r.$

Problem 67 had a minimum of $h = 3r$. With a solid sphere, the minimum is $h = 2.7r,$ which is a little less than $3r$.

68. (a) Strategy Let $r_1 = 0.00500$ m and $r_2 = 0.0200$ m. The tangential speed of the axle and the speed of the yo-yo are the same. Use conservation of energy.

Solution Find the speed of the yo-yo.

$$\Delta K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr_2^2\right)\left(\frac{v^2}{r_1^2}\right) = \frac{1}{2}mv^2 + \frac{1}{4}m\left(\frac{r_2}{r_1}\right)^2 v^2 = -\Delta U = mgh, \text{ so}$$

$$v = \sqrt{\frac{4gh}{2+(r_2/r_1)^2}} = \sqrt{\frac{4(9.80\ \text{m/s}^2)(1.00\ \text{m})}{2+\left(\frac{0.0200\ \text{m}}{0.00500\ \text{m}}\right)^2}} = \boxed{1.5\ \text{m/s}}.$$

(b) Strategy Assume constant acceleration.

Solution Find the time is takes the yo-yo to fall.

$$\Delta y = v_{\text{av}}\Delta t = \frac{1}{2}(v_{\text{fy}} + v_{\text{iy}})\Delta t = \frac{v}{2}\Delta t, \text{ so } \Delta t = \frac{2\Delta y}{v} = \frac{2(1.00\ \text{m})}{1.476\ \text{m/s}} = \boxed{1.36\ \text{s}}.$$

69. Strategy The rotational inertia of a uniform disk is $I = \frac{1}{2}MR^2$. Use Eq. (8-14).

Solution Find the magnitude of the angular momentum of the turntable.

$$L = I\omega = \frac{1}{2}MR^2\omega = \frac{1}{2}(5.00\ \text{kg})(0.100\ \text{m})^2(0.550\ \text{rev/s})(2\pi\ \text{rad/rev}) = \boxed{0.0864\ \text{kg}\cdot\text{m}^2/\text{s}}$$

70. Strategy The rotational inertia of a uniform solid sphere is $I = \frac{2}{5}MR^2$. Use Eq. (8-14).

Solution Find the magnitude of the angular momentum of the Earth.

$$L = I\omega = \frac{2}{5}MR^2\omega = \frac{2}{5}(5.97\times10^{24}\ \text{kg})(6.37\times10^6\ \text{m})^2\left(\frac{1\ \text{rev}}{24\ \text{h}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = \boxed{7.0\times10^{33}\ \text{kg}\cdot\text{m}^2/\text{s}}$$

71. Strategy The rotational inertia of a hoop is $I = MR^2$. Use Eq. (8-14).

Solution Find the magnitude of the angular momentum of the flywheel.

$$L = I\omega = MR^2\omega = (5.6\times10^4\ \text{kg})(2.6\ \text{m})^2(350\ \text{rpm})\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{1.4\times10^7\ \text{kg}\cdot\text{m}^2/\text{s}}$$

72. Strategy Since the torque is constant, it is equal to the change in angular momentum divided by the time interval.

Solution Find the applied torque.

$$\tau = \frac{\Delta L}{\Delta t} = \frac{115\ \text{kg}\cdot\text{m}^2/\text{s} - 240\ \text{kg}\cdot\text{m}^2/\text{s}}{2.5\ \text{s}} = -50\ \text{N}\cdot\text{m}.$$

The torque applied is $\boxed{50\ \text{N}\cdot\text{m opposite the rotation of the wheel}}$.

73. Strategy Since the torque is constant, it is equal to the change in angular momentum divided by the time interval.

Solution Find the time to stop the spinning wheel

$$\tau = \frac{\Delta L}{\Delta t}, \text{ so } \Delta t = \frac{\Delta L}{\tau} = \frac{-6.40\ \text{kg}\cdot\text{m}^2/\text{s}}{-4.00\ \text{N}\cdot\text{m}} = \boxed{1.60\ \text{s}}.$$

74. Strategy Use conservation of angular momentum and Eq. (8-14).

Solution Find the skater's new rate of rotation.

$L_i = I_i\omega_i = L_f = I_f\omega_f$, so $\omega_f = \dfrac{I_i}{I_f}\omega_i = \dfrac{1}{0.67}(1.0 \text{ rev/s}) = \boxed{1.5 \text{ rev/s}}$.

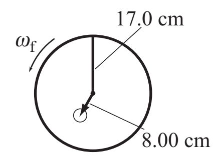
75. Strategy Use conservation of angular momentum and Eq. (8-14).

Solution Find the skater's final angular velocity.

$L_i = I_i\omega_i = L_f = I_f\omega_f$, so $\omega_f = \dfrac{I_i}{I_f}\omega_i = \dfrac{2.50}{1.60}(10.0 \text{ rad/s}) = \boxed{15.6 \text{ rad/s}}$.

76. Strategy The initial rotational inertia is $I_i = \frac{1}{2}MR^2$, and the final rotational inertia is $I_f = \frac{1}{2}MR^2 + mr^2$, where M is the mass of the disk, m is the mass of the clod of clay, R is the radius of the disk, and r is the distance from the center of the disk (axis of rotation) to the center of the clod. Use conservation of angular momentum.

Solution Solve for the final angular speed.



$L_i = I_i\omega_i = \dfrac{1}{2}MR^2\omega_i = L_f = I_f\omega_f = \left(\dfrac{1}{2}MR^2 + mr^2\right)\omega_f$, so

$$\omega_f = \frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + mr^2}\omega_i = \left(1 + \frac{2mr^2}{MR^2}\right)^{-1}\omega_i = \left[1 + \frac{2(0.12 \text{ g})(0.0800 \text{ m})^2}{(0.80 \text{ kg})(0.170 \text{ m})^2}\right]^{-1}(18.0 \text{ Hz})$$

$= \boxed{16.9 \text{ Hz}}$.

77. Strategy The rotational inertias of the wheel and guinea pig are $I_w = MR^2$ and $I_g = mR^2$, respectively, where M is the mass of the wheel, m is the mass of the guinea pig, and R is the radius of the wheel. Use conservation of angular momentum and $v = r\omega$.

Solution Find the angular velocity of the wheel.

$L_w = L_g$, so $I_w\omega_w = MR^2\omega_w = I_g\omega_g = mR^2\omega_g = mRv_g$.

Thus, $\omega_w = \dfrac{mv_g}{MR} = \dfrac{(0.500 \text{ kg})(0.200 \text{ m/s})}{(2.00 \text{ kg})(0.400 \text{ m})} = \boxed{0.125 \text{ rad/s}}$.

78. Strategy Use conservation of angular momentum and Eq. (8-14).

Solution Find the diver's initial angular velocity.

$L_i = I_i\omega_i = L_f = I_f\omega_f$, so $\omega_i = \dfrac{I_f}{I_i}\omega_f = \dfrac{1}{3.00}\left(\dfrac{2.00 \text{ rev}}{1.33 \text{ s}}\right)\left(\dfrac{2\pi \text{ rad}}{\text{rev}}\right) = \boxed{3.15 \text{ rad/s}}$.

79. Strategy Use Eqs. (5-2), and (8-14).

Solution

(a) Find the time elapsed during the dive in the tuck position.

$$\Delta y = -h = v_{iy}\Delta t - \frac{1}{2}g(\Delta t)^2 = 0 - \frac{1}{2}g(\Delta t)^2, \text{ so } \Delta t = \sqrt{\frac{2h}{g}}.$$

Find the number of turns (revolutions).

$$\Delta\theta = \omega\Delta t = \frac{L}{I}\sqrt{\frac{2h}{g}}, \text{ so } \left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\Delta\theta = \left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\frac{106\ \text{kg}\cdot\text{m}^2/\text{s}}{8.0\ \text{kg}\cdot\text{m}^2}\sqrt{\frac{2(10.0\text{ m})}{9.80\ \text{m/s}^2}} = \boxed{3.0}.$$

(b) Find the number of turns during the dive in the pike position.

$$\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\frac{106\ \text{kg}\cdot\text{m}^2/\text{s}}{15.5\ \text{kg}\cdot\text{m}^2}\sqrt{\frac{2(10.0\text{ m})}{9.80\ \text{m/s}^2}} = \boxed{1.6}$$

80. Strategy The initial rotational inertia is $I_i = \frac{1}{2}MR^2$, and the final rotational inertia is $I_f = \frac{1}{2}MR^2 + (F_g/g)R^2$, where M is the mass of the merry-go-round, F_g is the weight of the child, R is the radius of the merry-go-round and the distance from the center of the merry-go-round (axis of rotation) to the child. Use conservation of angular momentum.

Solution Solve for the final angular speed.

$$L_i = I_i\omega_i = \frac{1}{2}MR^2\omega_i = L_f = I_f\omega_f = \left(\frac{1}{2}MR^2 + \frac{F_g}{g}R^2\right)\omega_f, \text{ so}$$

$$\omega_f = \frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + \frac{F_g}{g}R^2}\omega_i = \left(1 + \frac{2F_g}{gM}\right)^{-1}\omega_i = \left[1 + \frac{2(180\text{ N})}{(9.80\ \text{m/s}^2)(160\text{ kg})}\right]^{-1}(0.75\ \text{rev/s}) = \boxed{0.61\ \text{rev/s}}.$$

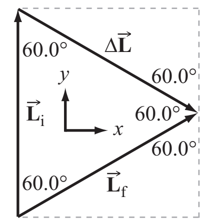
Compute the change in rotational kinetic energy.

$$\Delta K_r = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}\left(\frac{1}{2}MR^2 + \frac{F_g}{g}R^2\right)\omega_f^2 - \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2$$

$$= \frac{1}{2}\left\{\left[\frac{1}{2}(160\text{ kg})(2.0\text{ m})^2 + \frac{(180\text{ N})(2.0\text{ m})^2}{9.80\ \text{m/s}^2}\right]\left(0.61\ \frac{\text{rev}}{\text{s}}\right)^2 - \left[\frac{1}{2}(160\text{ kg})(2.0\text{ m})^2\right]\left(0.75\ \frac{\text{rev}}{\text{s}}\right)^2\right\}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)^2$$

$$= \boxed{-660\text{ J}}$$

81. Strategy The average torque is equal to the magnitude of the change in angular momentum divided by the time interval.



Solution Let $\vec{\mathbf{L}}_i = L$ in the $+y$-direction. Then $\Delta\vec{\mathbf{L}}$ has components $\Delta L_x = L\sin\theta$ and $\Delta L_y = L\cos\theta - L = L(\cos\theta - 1).$ So,

$$\left|\Delta\vec{\mathbf{L}}\right| = \sqrt{(L\sin\theta)^2 + [L(\cos\theta - 1)]^2} = L\sqrt{\sin^2 60.0° + (\cos 60.0° - 1)^2} = 1.00L.$$

Compute the magnitude of the required torque.

$$\tau = \left|\frac{\Delta\vec{\mathbf{L}}}{\Delta t}\right| = \frac{1.00L}{\Delta t} = \frac{1.00I\omega}{\Delta t} = \frac{\frac{1}{2}mr^2\omega}{\Delta t}$$

$$= \frac{(1.00\times 10^5\text{ kg})(2.00\text{ m})^2(300.0\text{ rpm})}{2(3.00\text{ s})}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{2.10\times 10^6\text{ N}\cdot\text{m}}$$

82. Strategy Consider how the angular momentum of the rotating disk affects the motion of the ship.

Solution

The disk should rotate in a horizontal plane so that the angular momentum vector is vertical. This does not make it difficult to steer; the ship can change direction without affecting the direction of the angular momentum.

83. Strategy The rotational inertia of the Moon is $I = mr^2$. Use conservation of angular momentum and $v = \omega r$.

Solution Find the ratio of the Moon's orbital speed at perigee to that at apogee.

$$I_a\omega_a = mr_a^2\frac{v_a}{r_a} = mr_a v_a = I_p\omega_p = mr_p^2\frac{v_p}{r_p} = mr_p v_p,\text{ so }\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{4.07\times 10^5\text{ km}}{3.56\times 10^5\text{ km}} = \boxed{1.14}.$$

84. Strategy The rotational inertia of each blade (uniform rod) is $I = \frac{1}{3}ML^2$, where L is the length of each blade. Find the angular acceleration of the fan using the definition; and use Eq. (8-9) to find the torque applied to the fan by the motor.

Solution The angular acceleration is $\alpha = \Delta\omega/\Delta t$. Find the torque.

$$\Sigma\tau = I\alpha = 4\left(\frac{1}{3}ML^2\right)\frac{\Delta\omega}{\Delta t} = \frac{4ML^2\Delta\omega}{3\Delta t} = \frac{4(0.35\text{ kg})(0.60\text{ m})^2(1.8\text{ rev/s})}{3(4.35\text{ s})}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) = \boxed{0.44\text{ N}\cdot\text{m}}$$

85. Strategy Use Eq. (8-3). The force due to the weight is mg.

Solution Find the torque.

$$\tau = F_\perp r = mgr = (10.0\text{ kg})(9.80\text{ N/kg})(1.0\text{ m}) = \boxed{98\text{ N}\cdot\text{m}}$$

86. Strategy The rotational inertia of the rod is $I = \frac{1}{3}mL^2$. Use conservation of energy.

Solution Find the speed of the lower end of the uniform rod when moving at its lowest point.

$$\Delta K = K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\left(\frac{v}{L}\right)^2 = \frac{1}{6}mv^2 = -\Delta U = mgh = mg\frac{L}{2},\text{ so }v = \boxed{\sqrt{3gL}}.$$

87. Strategy The rotational inertia of the gymnast is $\frac{1}{3}m(2r)^2$, where $r = 1.0$ m. Use conservation of energy.

Solution Find the angular speed at the bottom of the swing.

$$\Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[\frac{1}{3}m(2r)^2\right]\omega^2 = \frac{2}{3}mr^2\omega^2 = -\Delta U = mg(2r) = 2rmg, \text{ so}$$

$$\omega = \sqrt{\frac{3g}{r}} = \sqrt{\frac{3(9.80\ \text{m/s}^2)}{1.0\ \text{m}}} = \boxed{5.4\ \text{rad/s}}.$$

88. Strategy Use Eqs. (8-8). Choose the axis of rotation at the hinge attaching the crane to the cab (the pivot).

Solution Find T_1.

$$\Sigma\tau = 0 = T_2(12.2\ \text{m})\sin 10.0° + T_1(12.2\ \text{m})\sin 5.0° - (18\ \text{kN})(6.1\ \text{m})\sin 40.0° - (67\ \text{kN})(12.2\ \text{m})\sin 40.0° \text{ and}$$

$$\sum F_y = T_1 - 67\ \text{kN} = 0, \text{ so } \boxed{T_1 = 67\ \text{kN}}.$$

Find T_2.

$$T_2 = \frac{[(18\ \text{kN})(6.1\ \text{m}) + (67\ \text{kN})(12.2\ \text{m})]\sin 40.0° - (67\ \text{kN})(12.2\ \text{m})\sin 5.0°}{(12.2\ \text{m})\sin 10.0°}, \text{ so } \boxed{T_2 = 250\ \text{kN}}.$$

At the pivot:

$$\sum F_y = F_{py} - 18\ \text{kN} - 67\ \text{kN} - T_1\cos 45.0° - T_2\cos 50.0° = 0, \text{ so}$$

$$F_{py} = 18\ \text{kN} + 67\ \text{kN} + (247.7\ \text{kN})\cos 50.0° + (67\ \text{kN})\cos 45.0° = 291.6\ \text{kN}.$$

$$\sum F_x = F_{px} - T_1\sin 45.0° - T_2\sin 50.0° = 0, \text{ so}$$

$$F_{px} = (247.7\ \text{kN})\sin 50.0° + (67\ \text{kN})\sin 45.0° = 237.1\ \text{kN}.$$

Find the magnitude.

$$F_p = \sqrt{(237.1\ \text{kN})^2 + (291.6\ \text{kN})^2} = 380\ \text{kN}$$

Find the direction.

$$\theta = \tan^{-1}\frac{291.6}{237.1} = 51°$$

So, $\boxed{\vec{\mathbf{F}}_p = 380\ \text{kN at } 51° \text{ with the horizontal}}$.

89. Strategy Use conservation of energy.

Solution Find the final speeds of each object.
Solid sphere:

$$\Delta K = K_{\text{tr}} + K_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2 = -\Delta U = U_{\text{i}} - U_{\text{f}} = mgh,$$

so $v = \sqrt{\frac{10gh}{7}}$.

Hollow sphere:

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{3}mv^2 = \frac{5}{6}mv^2 = mgh, \text{ so } v = \sqrt{\frac{6gh}{5}}.$$

Solid cylinder:

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2 = mgh, \text{ so } v = \sqrt{\frac{4gh}{3}}.$$

Hollow cylinder:

$$\frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2 = mgh, \text{ so } v = \sqrt{gh}.$$

Cube:

$$\frac{1}{2}mv^2 = mgh, \text{ so } v = \sqrt{2gh}.$$

So, $v_{\text{cube}} > v_{\text{solid sphere}} > v_{\text{solid cylinder}} > v_{\text{hollow sphere}} > v_{\text{hollow cylinder}}$.

The objects reach the bottom in the following order from first to last: cube, solid sphere, solid cylinder, hollow sphere, and hollow cylinder.

90. (a) Strategy The rotational inertia of a uniform solid cylinder is $I = \frac{1}{2}mr^2$. Use conservation of energy.

Solution Find the speed v of the cylinder after it has fallen a height h.

$$\Delta K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - 0 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{3}{4}mv^2 = -\Delta U = mgh, \text{ so}$$

$$v^2 = \frac{4}{3}gh = 2a_y\Delta y = 2a_y(0-h), \text{ or } a_y = -\frac{2}{3}g = -\frac{2}{3}(9.80 \text{ m/s}^2) = -6.53 \text{ m/s}^2.$$

The acceleration of the cylinder is $\boxed{6.53 \text{ m/s}^2 \text{ down}}$.

(b) Strategy Use Newton's second law.

Solution The cords each pull upward on the cylinder with tension T.

$$\Sigma F_y = 2T - mg = ma_y, \text{ so } T = \frac{1}{2}m(a_y + g) = \frac{1}{2}(2.6 \text{ kg})(-6.533 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = \boxed{4.2 \text{ N}}.$$

91. Strategy Compute the net torque on the piece of uniform metal.

Solution Let the length of the piece of metal be L.

$$\Sigma\tau = 0 = kx\sin\theta L - mg\cos\theta\frac{L}{2},\ \text{so}\ x = \frac{mg}{2k\tan\theta} = \frac{(53.0\text{ kg})(9.80\text{ m/s}^2)}{2(275\text{ N/m})\tan 50.0°} = \boxed{0.792\text{ m}}.$$

92. Strategy Let the subscripts be 1 for the painter, 2 for the can, and 3 for the plank.

Solution

(a) Choose the axis of rotation at the point of contact between the plank and the right sawhorse.

$$\Sigma\tau = 0 = m_3gd_3 - m_1gd_1 - m_2gd_2,\ \text{so}\ d_1 = \frac{m_3d_3 - m_2d_2}{m_1}.$$

The distance from the right-hand edge is $1.40\text{ m} - d_1 = d$.

$$d = 1.40\text{ m} - \frac{m_3d_3 - m_2d_2}{m_1} = 1.40\text{ m} - \frac{(20.0\text{ kg})(3.00\text{ m} - 1.40\text{ m}) - (4.0\text{ kg})(1.40\text{ m} - 0.14\text{ m})}{61\text{ kg}}$$

$$= \boxed{0.96\text{ m from the RH edge}}$$

(b) Choose the axis of rotation at the point of contact between the plank and the left sawhorse.

$$\Sigma\tau = 0 = m_1gd_1 - m_2gd_2 - m_3gd_3,\ \text{so}\ d_1 = \frac{m_2d_2 + m_3d_3}{m_1}.$$

The distance from the left-hand edge is $1.40\text{ m} - d_1 = d$.

$$d = 1.40\text{ m} - \frac{m_2d_2 + m_3d_3}{m_1} = 1.40\text{ m} - \frac{(4.0\text{ kg})(6.00\text{ m} - 1.40\text{ m} - 0.14\text{ m}) + (20.0\text{ kg})(1.60\text{ m})}{61\text{ kg}}$$

$$= \boxed{0.58\text{ m from the LH edge}}$$

93. (a) Strategy The rotational inertia of a uniform solid disk is $I = \frac{1}{2}MR^2$.

Solution Compute the rotational inertia.

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(200.0\text{ kg})(0.40\text{ m})^2 = \boxed{16\text{ kg}\cdot\text{m}^2}$$

(b) Strategy Use Eq. (8-1).

Solution Compute the initial rotational kinetic energy.

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(16\text{ kg}\cdot\text{m}^2)(3160\text{ rad/s})^2 = \boxed{8.0\times10^7\text{ J}}$$

(c) Strategy and Solution The ratio of the rotational to the translational kinetic energies is

$$\frac{K_{\text{rot}}}{K_{\text{tr}}} = \frac{K_{\text{rot}}}{\frac{1}{2}mv^2} = \frac{2(8.0\times10^7\text{ J})}{(1000.0\text{ kg})(22.4\text{ m/s})^2} = \boxed{320}.$$

(d) Strategy Set the work done by air resistance equal to the stored energy in the flywheel.

Solution Find the distance d the car can travel.

$$Fd = K_{\text{rot}},\ \text{so}\ d = \frac{K_{\text{rot}}}{F} = \frac{8.0\times10^7\text{ J}}{670.0\text{ N}} = \boxed{120\text{ km}}.$$

94. (a) Strategy The rotational inertia of a uniform solid sphere is $I = \frac{2}{5}MR^2$. Use Eq. (8-1).

Solution Find the kinetic energy of the Earth.

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{1}{5}(5.974\times10^{24}\text{ kg})(6.371\times10^6\text{ m})^2\left(\frac{2\pi\text{ rad}}{24\text{ h}}\right)^2\left(\frac{1\text{ h}}{3600\text{ s}}\right)^2 = \boxed{2.6\times10^{29}\text{ J}}$$

(b) Strategy and Solution $T = \dfrac{2\pi}{\omega}$ and $K_{\text{rot}} \propto \omega^2$, so $\omega \propto \sqrt{K_{\text{rot}}}$ and $\dfrac{T_\text{f}}{T_\text{i}} = \dfrac{\omega_\text{i}}{\omega_\text{f}} = \sqrt{\dfrac{K_\text{i}}{K_\text{f}}}$. The change in the period is $T_\text{f} - T_\text{i} = \left(\sqrt{\dfrac{K_\text{i}}{K_\text{f}}} - 1\right)T_\text{i} = \left(\sqrt{\dfrac{1}{0.990}} - 1\right)(24\text{ h})\left(\dfrac{60\text{ min}}{1\text{ h}}\right) = 7\text{ min.}$

The length of the day would increase by 7 minutes.

(c) Strategy Divide 1.0% of the Earth's rotational kinetic energy by the world's energy usage.

Solution One percent of the Earth's rotational kinetic energy would supply the world's energy needs (at today's usage) for $\dfrac{0.010(2.6\times10^{29}\text{ J})}{1.0\times10^{21}\text{ J/yr}} = \boxed{2.6\text{ million years}}$.

95. Strategy Refer to the figure. Use Eqs. (7-9) and (8-4).

Solution

(a) $|\tau_\text{i}| = |F_\text{i}r_{\perp\text{i}}| = |m_\text{i}gr_\text{i}\cos\theta_\text{i}| = |x_\text{i}m_\text{i}g|$

If $\sum x_\text{i}m_\text{i}g > 0$, then the system rotates CW ($\tau < 0$), and if $\sum x_\text{i}m_\text{i}g < 0$, then the system rotates CCW ($\tau > 0$). Therefore $\tau_\text{i} = -x_\text{i}m_\text{i}g$.

(b) Since $\tau_\text{i} = -x_\text{i}m_\text{i}g$ and the center of gravity is at (x_CG, y_CG), $\sum\tau_\text{i} = -\sum x_\text{i}m_\text{i}g = -g\left(\dfrac{\Sigma x_\text{i}m_\text{i}}{M}\right)M = -x_\text{CG}Mg$.

(c)
$$-\sum x_\text{i}m_\text{i}g = -x_\text{CG}Mg$$
$$\frac{\sum x_\text{i}m_\text{i}g}{M} = x_\text{CG}g$$
$$x_\text{CM} = x_\text{CG}$$

96. (a) Strategy The rotational inertia of a uniform disk is $I = \frac{1}{2}MR^2$.

Solution Find the radius.

$$I = \frac{1}{2}MR^2, \text{ so } R = \sqrt{\frac{2I}{M}} = \sqrt{\frac{2(4.55\times10^6\ \text{kg}\cdot\text{m}^2)}{7.27\times10^5\ \text{kg}}} = \boxed{3.54\ \text{m}}.$$

(b) Strategy The rotational inertia of a hollow cylinder is $I = MR^2$.

Solution Find the radius.

$$I = MR^2, \text{ so } R = \sqrt{\frac{I}{M}} = \sqrt{\frac{4.55\times10^6\ \text{kg}\cdot\text{m}^2}{7.27\times10^5\ \text{kg}}} = \boxed{2.50\ \text{m}}.$$

(c) Strategy Use the definition of average power, the work-kinetic energy theorem, and Eq. (8-1).

Solution The rate at which the energy of the flywheel is decreased is

$$P_{\text{av}} = \frac{W}{\Delta t} = \frac{\Delta K}{\Delta t} = \frac{\frac{1}{2}I\omega_\text{f}^2 - \frac{1}{2}I\omega_\text{i}^2}{\Delta t} = \frac{I}{2\Delta t}(\omega_\text{f}^2 - \omega_\text{i}^2).$$

The average power supplied is $-P_{\text{av}}$.

$$\frac{I}{2\Delta t}(\omega_\text{i}^2 - \omega_\text{f}^2) = \frac{4.55\times10^6\ \text{kg}\cdot\text{m}^2}{2(5.00\ \text{s})}[(386\ \text{rpm})^2 - (252\ \text{rpm})^2]\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)^2\left(\frac{1\ \text{min}}{60\ \text{s}}\right)^2 = \boxed{4.3\times10^8\ \text{W}}$$

97. Strategy The system is in equilibrium. Use Eqs. (8-8).

Solution Find h.

$$\frac{h}{(1.26\ \text{m})/2} = \tan 75°, \text{ so } h = [(1.26\ \text{m})/2]\tan 75° = (0.630\ \text{m})\tan 75°.$$

At the top of the ladder, each leg exerts a horizontal force on the other. These forces are equal in magnitude and opposite in direction, since the system is in equilibrium. Let the magnitude of this force be F. The tension T in the rope is directed to the left at the connection point on the right leg, so for the right leg, we have

$\Sigma F_x = F - T = 0$ or $T = F$.

Calculate the torque about the contact point of the right leg of the ladder and the ground.

$$\Sigma\tau = (0.630\ \text{m})mg - Fh = 0, \text{ so } T = F = \frac{(0.630\ \text{m})mg}{h} = \frac{(0.630\ \text{m})mg}{(0.630\ \text{m})\tan 75°} = \frac{mg}{\tan 75°}.$$

The tension in the rope is the same along its length, so

$$T_{\text{rope}} = \frac{mg}{\tan 75°} = \frac{(42\ \text{kg})(9.80\ \text{N/kg})}{\tan 75°} = \boxed{110\ \text{N}}.$$

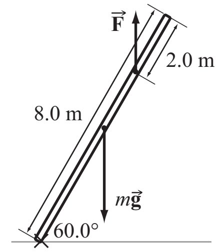
98. Strategy The system is in equilibrium. Choose the axis of rotation at the point of contact between the ladder and the floor.

Solution

(a) Find the vertical force.

$\Sigma\tau = 0 = F(6.0\text{ m})\cos 60.0° - mg(4.0\text{ m})\cos 60.0°,$ so

$F = \frac{4.0}{6.0}mg = \frac{4.0}{6.0}(15\text{ kg})(9.80\text{ N/kg}) = \boxed{98\text{ N}}.$



(b) This does not help the person trying to lift the ladder, since the torque problem is not alleviated by exerting a force at the point of rotation.

99. (a) Strategy The rotational inertia of a uniform thin rod is $I = \frac{1}{3}ML^2.$

Solution Compute the rotational inertia of the limb.

$I = \frac{1}{3}ML^2 = \frac{1}{3}(0.0280\text{ kg})(0.0380\text{ m})^2 = \boxed{1.35\times10^{-5}\text{ kg}\cdot\text{m}^2}$

(b) Strategy Use Eq. (8-9).

Solution Compute the muscular force required to achieve the blow.

$\Sigma\tau = Fr = I\alpha = \frac{1}{3}ML^2\frac{\Delta\omega}{\Delta t},$ so $F = \frac{(0.0280\text{ kg})(0.0380\text{ m})^2(175\text{ rad/s})}{3(1.50\times10^{-3}\text{ s})(3.00\times10^{-3}\text{ m})} = \boxed{524\text{ N}}.$

100. Strategy Use Eqs. (8-8) and (8-9). Let $a_x = a = -a_y.$

Solution For the two blocks, we have $\sum F_x = T_1 = m_1a_x = m_1a$ and $\sum F_y = 0;$

$\sum F_x = 0$ and $\sum F_y = T_2 - m_2g = m_2a_y = -m_2a,$ so $T_2 = m_2g - m_2a.$

For the pulley, we have $\sum\tau = -T_1R + T_2R = I\alpha = I\frac{a}{R},$ so $T_1 - T_2 = -\frac{Ia}{R^2}.$

Find the acceleration of the blocks.

$T_1 - T_2 = m_1a + m_2a - m_2g = -\frac{Ia}{R^2},$ so $\left(m_1 + m_2 + \frac{I}{R^2}\right)a = m_2g$ or $a = \boxed{\frac{m_2g}{m_1 + m_2 + I/R^2}}.$

101. Strategy The rotational inertial of a uniform disk is $I = \frac{1}{2}MR^2.$ Use Eq. (8-14).

Solution Find the magnitude of the angular momentum of the disk.

$L = I\omega = \frac{1}{2}MR^2\omega = \frac{1}{2}(2.0\text{ kg})(0.100\text{ m})^2(3.0\text{ rev/s})(2\pi\text{ rad/rev}) = \boxed{0.19\text{ kg}\cdot\text{m}^2/\text{s}}$

102. (a) Strategy Since the hoop started at rest, the final angular velocity is twice the average angular velocity.

Solution Find the angular velocity of the hoop when it arrives at the bottom of the inclined plane.

$$\omega_f = 2\omega_{av} = 2\frac{v_{av}}{r} = 2\left(\frac{2\pi}{C}\right)\frac{\Delta x}{\Delta t} = \frac{4\pi(10.0\text{ m})}{(2.00\text{ m})(10.0\text{ s})} = \boxed{6.28\text{ rad/s}}$$

(b) Strategy The rotational inertia of a hoop is $I = MR^2$. Use Eq. (8-14).

Solution Find the angular momentum of the hoop when it reaches the bottom of the incline.

$$L = I\omega = MR^2\omega = M\left(\frac{C}{2\pi}\right)^2\omega = (1.50\text{ kg})\left(\frac{2.00\text{ m}}{2\pi}\right)^2(2\pi\text{ rad/s}) = \boxed{0.955\text{ kg}\cdot\text{m}^2/\text{s}}$$

(c) Strategy Consider the forces acting on the hoop.

Solution The gravitational force acts on the hoop in the direction parallel to the line between the axis of the hoop and the point of contact between the rim of the hoop and the inclined plane, so it supplies no torque. The force of friction acts at the rim of the hoop, perpendicularly to the line between the axis of the hoop and the point of contact between the rim of the hoop and the inclined plane; therefore, it is the force of $\boxed{\text{friction}}$ that supplied the net torque.

(d) Strategy The average torque on the hoop is equal to the change in angular momentum of the hoop divided by the time interval of the change.

Solution Find the force of friction.

$$\frac{\Delta L}{\Delta t} = \tau_{av} = fr = f\frac{C}{2\pi}, \text{ so } f = \frac{2\pi}{C}\frac{\Delta L}{\Delta t} = \frac{2\pi(0.955\text{ kg}\cdot\text{m}^2/\text{s})}{(2.00\text{ m})(10.0\text{ s})} = \boxed{0.300\text{ N}}.$$

103. Strategy Since the mass is concentrated at the tip, $I = MR^2$. Use Eq. (8-14).

Solution Compute the angular momenta of the second and hour hands of the clock.

(a) $$L = I\omega = MR^2\omega = (0.10\text{ kg})(0.300\text{ m})^2\left(\frac{1\text{ rev}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) = \boxed{9.4\times10^{-4}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) $$L = (0.20\text{ kg})(0.200\text{ m})^2\left(\frac{1\text{ rev}}{12\text{ h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) = \boxed{1.2\times10^{-6}\text{ kg}\cdot\text{m}^2/\text{s}}$$

104. (a) Strategy and Solution $\tau = F_{\perp}r = (0)r = 0,$ since the force due to gravity is parallel to the radial distance between the planet and the Sun.

(b) Strategy The rotational inertial of a planet is $I = mr^2$. Use Eq. (8-14).

Solution

$$L = I\omega = \boxed{mr^2\omega}$$

(c) Strategy and Solution If Δt is small, the area swept out is approximately

$$A = \frac{1}{2}rv\Delta t = \frac{1}{2}r(r\omega)\Delta t = \boxed{\frac{1}{2}r^2\omega\Delta t}.$$

(Since Δt is small, the area is approximately a triangle with height equal to r and base equal to $v\Delta t$.)

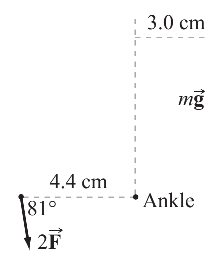
(d) Strategy and Solution $\frac{A}{\Delta t} = \frac{1}{2}r^2\omega,$ and $L = mr^2\omega$ is constant, so $r^2\omega$ is constant. Thus, $\frac{A}{\Delta t},$ the area swept out per unit time, is constant.

105. Strategy The system is in equilibrium. Choose the axis of rotation at the ankle.

Solution Find the force that each calf muscle needs to exert while the woman is standing.

$$\Sigma\tau = 0 = 2F(4.4\text{ cm})\sin 81° - mg(3.0\text{ cm}),\text{ so}$$

$$F = \frac{mg(3.0\text{ cm})}{2(4.4\text{ cm})\sin 81°} = \frac{(68\text{ kg})(9.80\text{ N/kg})(3.0\text{ cm})}{2(4.4\text{ cm})\sin 81°} = \boxed{230\text{ N}}.$$



106. (a) Strategy Use conservation of angular momentum and Eq. (8-14).

Solution Calculate the angular velocity after the child moves out to the rim of the merry-go-round.

$$L_i = I_i\omega_i = L_f = I_f\omega_f = (I_i + I_{child})\omega_f = (I_i + mR^2)\omega_f,\text{ so } \omega_f = \boxed{\frac{I_i\omega_i}{I_i + mR^2}}.$$

(b) Strategy Use Eqs. (8-1) and (8-14).

Solution Calculate the rotational kinetic energy and angular momentum before and after.

Before:

$$K_{rot} = \boxed{\frac{1}{2}I_i\omega_i^2}\text{ and } L = \boxed{I_i\omega_i}.$$

After:

$$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}(I_i + mR^2)\left(\frac{I_i\omega_i}{I_i + mR^2}\right)^2 = \boxed{\frac{1}{2}\frac{I_i^2\omega_i^2}{I_i + mR^2}}\text{ and } L = \boxed{I_i\omega_i}.$$

107. (a) Strategy The weight is equal to the change in the combined readings of the scales.

Solution Compute the student's weight.

$W = 394.0\text{ N} + 541.0\text{ N} - 100.0\text{ N} - 100.0\text{ N} = \boxed{735.0\text{ N}}$

(b) Strategy The system is in equilibrium. Choose the axis of rotation at the point of contact between the plank and scale B.

Solution Find x_1.

$$\Sigma\tau = 0 = m_s g x_1 - F_A L + m_p g\left(\frac{L}{2}\right),\text{ so } x_1 = \frac{F_A L - m_p g\left(\frac{L}{2}\right)}{mg} = \frac{(2.2\text{ m})\left[394.0\text{ N} - \frac{1}{2}(200.0\text{ N})\right]}{735.0\text{ N}} = \boxed{0.88\text{ m}}.$$

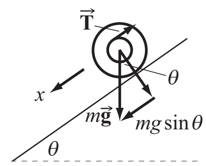
(c) Strategy The height of the student is $h = 1.60$ m.

Solution Find the height y of the student's center of gravity.

$$y = x_1\frac{h}{h} = (0.88\text{ m})\frac{h}{1.60\text{ m}} = \boxed{0.55h}$$

108. (a) Strategy Use Eq. (8-9) and Newton's second law. Let the +*x*-direction be down the plane.

Solution Find the tension in the thread.



$$\Sigma\tau = Tr = I\alpha = \frac{Ia_{CM}}{R},\text{ so } T = \frac{Ia_{CM}}{rR}.$$

Find the spool's acceleration.

$$-T + mg\sin\theta = -\frac{Ia_{CM}}{rR} + mg\sin\theta = ma_{CM},\text{ so}$$

$$g\sin\theta = a_{CM}\left(1 + \frac{I}{mrR}\right)\text{ or } a_{CM} = \frac{g\sin\theta}{1 + \frac{I}{mrR}}.$$

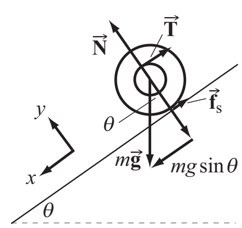
$$\boxed{\text{The spool spins and moves down the incline with } a_{CM} = \frac{g\sin\theta}{1 + \frac{I}{mrR}}.}$$

(b) Strategy Use Eqs. (8-8).

Solution Find the magnitude and direction of the frictional force.

$\sum F_x = mg\sin\theta - f_s - T = 0$ and $\sum F_y = N - mg\cos\theta = 0.$

Choose the axis of rotation at the axis of the spool.



$$\Sigma\tau = 0 = f_s R - Tr = f_s R - (mg\sin\theta - f_s)r,\text{ so } f_s = \frac{mg\sin\theta}{1 + R/r}.$$

The force of friction is $\boxed{\frac{mg\sin\theta}{1 + R/r}\text{ up the incline}}$.

(c) Strategy and Solution $\mu_{s,\,min}N = f_s$, so $\mu_{s,\,min} = \frac{f_s}{N} = \frac{mg\sin\theta}{mg\cos\theta(1 + R/r)} = \boxed{\frac{\tan\theta}{1 + R/r}}.$

109. Strategy Since the bike travels with constant velocity, the acceleration is zero and $\Sigma\tau = 0$.

Solution Find the magnitude of the force with which the chain pulls.

$$\Sigma\tau = 0 = fr_2 - F_C r_1,\text{ so } F_C = \frac{r_2}{r_1}f = 6.0(3.8\text{ N}) = \boxed{23\text{ N}}.$$

110. Strategy Use conservation of energy.

Solution

(a) Find the speed with which the roustabout reaches the ground.

$$\Delta K = \frac{1}{2}mv^2 = -\Delta U = mgL,\text{ so } v = \boxed{\sqrt{2gL}}.$$

(b) Find the speed with which the roustabout reaches the ground.

$$\Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\left(\frac{v}{L}\right)^2 = \frac{1}{6}Mv^2 = -\Delta U = Mg\frac{L}{2},\text{ so } v = \boxed{\sqrt{3gL}}.$$

(c) Since $\sqrt{2gL} < \sqrt{3gL}$, $\boxed{\text{the roustabout should jump}}$.

111. Strategy Use conservation of angular momentum.

Solution Find the new rate of rotation.

$L_f = I_f\omega_f = L_i = I_i\omega_i$, so

$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{(2.40\text{ kg}\cdot\text{m}^2)(0.50\text{ rev/s})}{2.40\text{ kg}\cdot\text{m}^2 - 2\left[\frac{1}{3}(3.00\text{ kg})(0.65\text{ m})^2\right] - 2(1.00\text{ kg})(0.65\text{ m})^2 + 2\left[\frac{1}{3}(3.00\text{ kg})(0.22\text{ m})^2\right] + 2(1.00\text{ kg})(0.22\text{ m})^2} = \boxed{1.3\text{ rev/s}}.$$

112. Strategy Choose the axis of rotation at the elbow. The scale pushes with an upward force of 96 N.

Solution Find the force exerted by the triceps muscle.

$$\Sigma\tau = 0 = (96\text{ N})(38\text{ cm}) - F_t(2.5\text{ cm}),\text{ so } F_t = \frac{38}{2.5}(96\text{ N}) = \boxed{1.5\text{ kN}}.$$

113. Strategy Choose the axis of rotation at the contact point between the horizontal surface and the tip of the left leg.

Solution Find the maximum wind speeds in which the blowfly and dog can stand.

(a) $\tau_{net} = 0 = F_{wind}r\sin\theta - mgr\cos\theta$, so

$$\frac{mg}{\tan\theta} = F_{wind} = cAv^2\text{ or } v = \sqrt{\frac{mg}{cA\tan\theta}} = \sqrt{\frac{(0.070\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{(1.3\text{ N}\cdot\text{s}^2/\text{m}^4)(0.10\times10^{-4}\text{ m}^2)\tan 30.0^\circ}} = \boxed{9.6\text{ m/s}}.$$

(b) $$v = \sqrt{\frac{(0.070\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{(1.3\text{ N}\cdot\text{s}^2/\text{m}^4)(0.10\times10^{-4}\text{ m}^2)\tan 80.0^\circ}} = \boxed{3.1\text{ m/s}}$$

(c) $$v = \sqrt{\frac{(10.0\text{ kg})(9.80\text{ m/s}^2)}{(1.3\text{ N}\cdot\text{s}^2/\text{m}^4)(0.030\text{ m}^2)\tan 80.0^\circ}} = \boxed{21\text{ m/s}}$$

114. (a) Strategy Refer to Example 8.7. The system is in equilibrium until the ladder begins to slip.

Solution Use Newton's second law.
$\Sigma F_x = N_w - f = 0$, so $f = N_w$.
At the person's highest point, the frictional force has its maximum possible magnitude, $f = \mu_s N_f$.
Thus, $N_w = \mu_s N_f$.
$\Sigma F_y = N_f - Mg - mg = 0$, so $N_w = \mu_s g(M + m)$.
Choose the axis of rotation at the contact point between the ladder and the floor.
$\Sigma\tau = 0 = -N_w L\sin\theta + mg\left(\frac{1}{2}L\cos\theta\right) + Mgd\cos\theta$, so

$$d = \frac{N_w L\sin\theta - \frac{1}{2}mgL\cos\theta}{Mg\cos\theta} = \frac{\mu_s g(M+m)L\sin\theta - \frac{1}{2}mgL\cos\theta}{Mg\cos\theta} = \frac{\left[\mu_s(M+m)\sin\theta - \frac{m}{2}\cos\theta\right]L}{M\cos\theta}$$
$$= \boxed{\left(\mu_s\frac{M+m}{M}\tan\theta - \frac{m}{2M}\right)L}$$

(b) Strategy and Solution Since $\tan\theta$ increases as θ increases on the interval $0 \le \theta < 90°$, and since d increases if $\tan\theta$ increases [which is evident from the equation found in part (a)], placing the ladder at a larger angle θ allows a person to climb farther up the ladder without having it slip.

(c) Strategy Set $d = L$.

Solution Find the minimum angle that enables the person to climb all the way to the top of the ladder.

$$L = \left(\mu_s\frac{M+m}{M}\tan\theta - \frac{m}{2M}\right)L$$
$$1 + \frac{m}{2M} = \mu_s\frac{M+m}{M}\tan\theta$$
$$\frac{2M+m}{2} = \mu_s(M+m)\tan\theta$$
$$\frac{2M+m}{2\mu_s(M+m)} = \tan\theta$$
$$\theta = \tan^{-1}\frac{2M+m}{2\mu_s(M+m)} = \tan^{-1}\frac{2(60.0\text{ kg}) + 15.0\text{ kg}}{2(0.45)(60.0\text{ kg} + 15.0\text{ kg})} = \boxed{63°}$$