ROTATIONAL KINETIC ENERGY
(CONSERVATION OF ENERGY)

GOAL: To investigate conservation of energy in a system that is rotating, moving linearly, and changing height. To dynamically measure the moment of inertia of several cylinders.

INTRODUCTION: If one rolls two cylinders with the same mass and the same diameter down an incline plane, should they reach the bottom at the same time? If they do not, what is happening?

To get a wheel (say a bicycle wheel) spinning quickly requires some effort. In the language of physics it requires work, and this work changes the amount of energy associated with the wheel. But the wheel is not changing its position so both its potential and linear kinetic energies are unchanged. The energy that is expended in spinning the wheel is stored in the form of *rotational kinetic energy.*

If one analyzes the motion of sufficiently small bits of a spinning wheel over sufficiently short time periods, then the rotational kinetic energy of the wheel is the sum of the linear kinetic energies of each small bit. *But using the concept of rotational kinetic energy greatly simplifies calculations.*

In mechanics, most problems can be solved using forces and torques, but *many of these problems can be solved more simply using the principle of conservation of energy:*

$$E_{\text{initial}} = E_{\text{final}} . \qquad (1)$$

The details of how an object got from point A to point B often are not important when energy conservation is invoked.

In this experiment, cylinders will be rolled down an incline plane (board.) One can calculate its change in potential energy, **PE**, from its mass, **m,** and change in height, Δ**h**. Linear kinetic energy, **KE** $_{LIN}$, can be calculated from its mass, **m**, and its linear velocity, **v**. And its rotational kinetic energy, **KE** $_{ROT}$ can be calculated from its moment of inertia, **I**, and its rotational velocity, ω.

$$\begin{aligned} PE &= mgh , \\ KE_{LIN} &= \tfrac{1}{2} m v^2 , \\ KE_{ROT} &= \tfrac{1}{2} I \omega^2 . \end{aligned} \qquad (2)$$

One can calculate the initial and final energy with relative ease compared to the problems of resolving force vectors and calculating linear and angular accelerations.

PROCEDURE: The procedure seems straight-forward, measure the mass, **m**, and the moment of inertia, **I**. Roll the cylinder down the incline and measure its initial and final linear velocities, $\mathbf{v}_I$ and $\mathbf{v}_f$, and initial and final angular velocities, ω_I and ω_f. Measure the change in vertical height, Δ**h**. Then calculate the initial and final total energies using eq.2 and compare the two.

Investigate how the following change the moment of inertia: solid vs. hollow (annular) cylinders; varying diameter; and varying composition.

Mass, m, can be measured with a balance. The change in height, Δ**h,** can be measured with a ruler by measuring the initial and final heights, $\mathbf{h}_I$ and $\mathbf{h}_f$.

The initial velocity, $\mathbf{v}_I$*, can be forced to be zero,* and the final velocity, $\mathbf{v}_f$, can be measured with the photogate-timer system. Because cylinders are not square, measuring their effective blocking length in the photogate may be difficult. It is suggested that one measure the final velocity using:

$$\begin{aligned} \frac{v_i + v_f}{2} &= \frac{s}{t} , \\ v_f &= \frac{2\,s}{t} . \end{aligned} \qquad (3)$$

Where **s** is the distance along the plane that the cylinder has traveled and **t** is the time required for this motion.

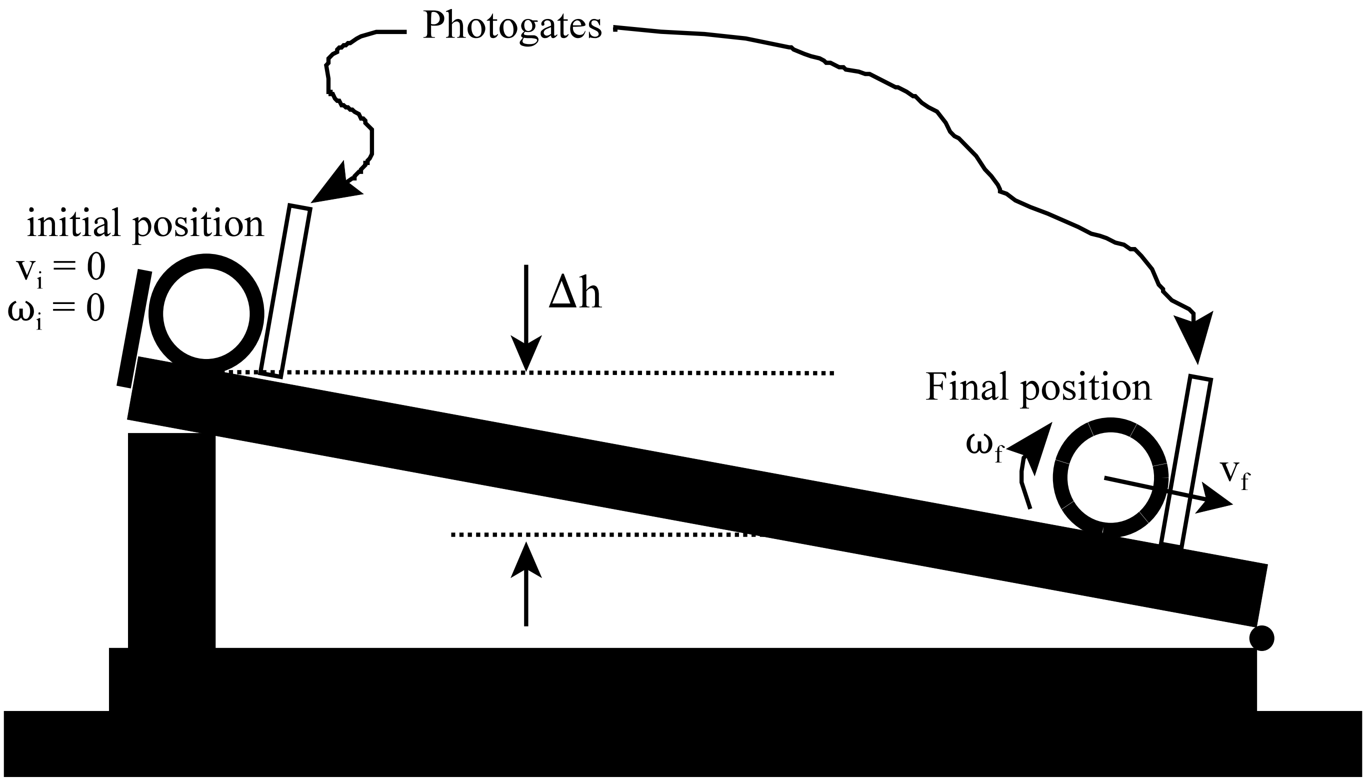

Figure 1 An inclined plane is used to study rotational kinetic energy. The linear kinetic energy, the rotational kinetic energy and the potential energy can be determined for both the initial and fianl positions.

Q i. What must be true about the acceleration for eq.3 to hold?

Use the timer in **PULSE** mode. Other methods of measuring $\mathbf{v}_f$ may be possible.

How can the angular velocities be measured? If the cylinder starts at rest, as assumed above, then the *initial angular velocity,* ω_I, *is zero.* If the cylinder rolls without slipping, then its linear and angular velocities are proportional:

$$v = r\omega , \qquad (4)$$

where **r** is the radius of the cylinder. Why? Then:

A method of measuring everything necessary has been found except for the moment of inertia, **I.** There are two approaches to take at this point:

I. The first is to assume that the law of conservation of energy is correct and experimentally determine the moments of inertia.

OR

II. Alternatively one can assume that one can calculate the moments of inertia and test the law of conservation of energy.

Of course, in either case, if the theory and experiment disagree, either the moments could be wrong or energy might not be conserved, or something else is wrong.

For all of the measurements, keep the distance, **s,** that the cylinders rolls constant. To help determine the uncertainties in your time measurements, make multiple measurement of the time that it takes to roll down the inclined plane for a given change in heights. Make measurements for several different changes in height.

There are two methods for analyzing the data. (**A i.** and **A ii**.)

A i. Assuming that the law of conservation of energy is correct and after constraining the initial conditions, one can determine moments of inertia from eq.9. Compare these results to the moments found using eq. 11 or 12.

A ii. An alternative method of analyzing the data from this experiment is to accept the formula for the moments of inertia, and test for the conservation of energy. Here one might look at the relative difference between the initial and final total energy:

$$\frac{|E_f - E_f|}{E_i} \quad \textbf{(5)}$$

Q ii. Should this value be large or small if the data is good?

If you take this approach in analyzing your data, is energy conserved within the uncertainties of your measurements?

WARNINGS:

Do not let the cylinders collide with the photogates.

EQUIPMENT:

Inclined plane & wooden blocks
Timer & photogates (2)
Meter stick, ruler & calipers
Several different cylinders

In Room

Balance (triple beam)
Tape & scissors

Equipment notes and hints:

See the appendix for information on the timer and photogates.

When measuring the distance, **s**, along the inclined plane, be careful to measure from leading edge to leading edge, or contact point to contact point.

When measuring heights, be careful to measure both heights in the same way.

At steep angles the assumption that the cylinders do not slip may not be valid.

Getting the starting conditions right and setting up the photogates will require some effort.

THEORY:

A iii. If the initial velocity is zero and there is no slipping then the equation for conservation of energy, eq.1, show that:

$$mgh_i = mgh_f + \frac{m}{2}(2s/t)^2 + \frac{I}{2}\left(\frac{2s/t}{r}\right)^2 ,$$

$$2g(h_i - h_f) = \left(1 + \frac{I}{mr^2}\right)(2s/t)^2 . \quad \textbf{(6)}$$

A iv. Put this into the form of a straight line, with the change in height being the independent variable, as it is in the experiment, gives:

$$1/t^2 = \frac{g}{2s^2}\left(1 + \frac{I}{mr^2}\right)^{-1}(h_i - h_f) . \quad \textbf{(7)}$$

Thus one can plot $(\mathbf{h_i} - \mathbf{h_f})$ vs. $\mathbf{1/t^2}$ and extract a slope, **C**, from the plot. From this slope, one can find the moment of inertia, **I.**

A v. Find this relationship:

$$I = I(C) = ????? \quad \textbf{(8)}$$

To ***calculate the moment of inertia*** about an axis, one does:

$$I = \sum m_i r_i^2 ,$$

or

$$I = \int r^2 \, dm . \quad \textbf{(9)}$$

Some results are:

$$I_{\text{sphere, diameter}} = \frac{2}{5} m r^2 . \quad \textbf{(10)}$$

$$I_{\text{Annular cylinder, own axis}} = \frac{m}{2}\left(r_{\text{inner}}^2 + r_{\text{outer}}^2\right) \cdot \quad \textbf{(12)}$$

$$I_{\text{cylinder, own axis}} = \frac{m\,r^2}{2} \,. \quad \textbf{(11)}$$

$$I_{\text{spherical shell, diameter}} = \frac{2}{3}\, m\, r^2 \,. \quad \textbf{(13)}$$

ANALYSIS:

A vi. How well do your measured moments of inertia compare to the calculated formula?

A vii. What are the uncertainties in your values? How are the uncertainties in the measurement of the time, **t,** propagated to $\mathbf{1/t}^{2}$? How does this affect the uncertainty in the slope?

A viii. How does the length, **s**, that the cylinder roll affect the measurement and the associated uncertainties? Should it be short or long?

A ix. What is the role of friction in this experiment?

A x. If there is friction and motion, should there also be energy losses to friction? Does not the friction between the cylinder and the board help to cause the cylinder to spin?

A xi. At what angle does the cylinder start to slip? Can this slipping be observed in the data before it is visible by direct observation?

A xii. Should any of your data be excluded from the evaluation of the moments of inertia because of slipping?

A xiii. What is the "y-intercept" on your $\mathbf{1/t^2}$ vs. change in height? Does this agree with what eq.7 predicts? Explain.

GOING FURTHER:

What other methods of measuring the moments of inertia can be devised?

Analyze the motion of the cylinder in terms of forces and torques. Compare the results to eq.7.

- ♦ One method of doing this analysis is to consider the rotation of the cylinder about its instantaneous point of contact with the plane. Use the parallel axis theorem to find the moment of inertia about this point. Find the torque about the contact point and solve for the angular acceleration. Relate this angular motion to the linear motion and solve the distance, **s**, in terms of **t**.

- ♦ A second method is to consider the frictional force between the cylinder and the plane. This frictional force reduces the total force down the plane, but causes a torque about the center of mass of the cylinder. The coupling between the linear and angular motion then allows the resulting equations to be solved.