Fluid Mechanics CEE 3311

LECTURE 6

Fluids in motion

L. Handia

Fluid kinematics

- Kinematics deals with velocities and streamlines without considering forces or energy
- Kinematics, developed in classical mechanics, describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without considering the forces that cause them to move.

Lagrangian and Eulerian descriptions of motion

Lagrangian – in the study of particle mechanics, where attention is focused on an individual particle, the particle is observed as a function of time.

- Its *position*, *velocity and acceleration* are listed and quantities of interest can be *calculated*.
- In the Lagrangian description *many particles* can be followed and their *influence on one another* noted.
- This becomes, however, a tremendous task as the number of particles becomes extremely large, as in fluid flow

Description of fluid motion

Eulerian – an alternative to following each fluid particle separately is to *identify a point in space* and then observe the velocity of *particles passing the point*; we can observe the rate of change of velocity as particles pass the point and we can observe if the velocity is changing with time at that particular point.

- Flow properties such as velocity are a function of both space and time. The region of flow that is being considered is called a *flow field.*

Description of fluid motion

Example.

An example may clarify these two ways of describing motion. An engineering firm is hired to make recommendations that would improve the traffic flow in a large city. The engineering firm has two alternatives: Hire college students to travel in automobiles throughout the city recording the appropriate observations (the Lagrangian approach), or hire college students to stand at the intersections and record the required information (the Eulerian approach). A correct interpretation of each set of data would lead to the same set of recommendations, that is, the same solution. In this example it may not be obvious which approach would be preferred; in fluids, however, the Eulerian description is used exclusively since the physical laws using the Eulerian description are easier to apply to actual situations.

In fluids the Eulerian description is used exclusively since the physical laws using the Eulerian description are easier to apply to actual situations.

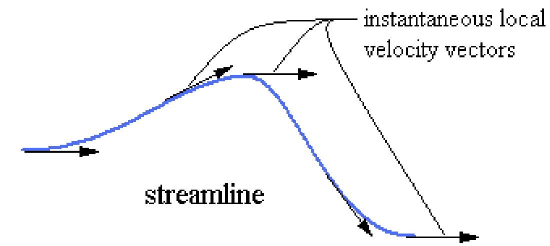
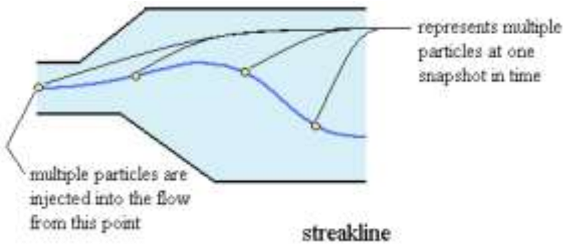
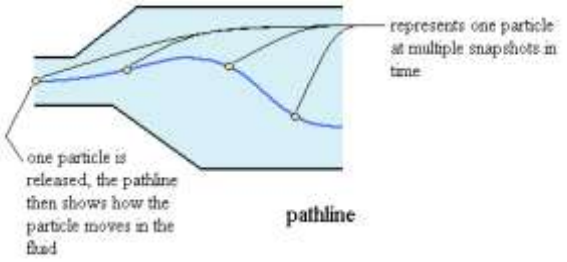
Pathlines, streaklines and streamlines

Three different lines help us in describing a flow field

1. Pathlines
2. Streaklines
3. Streamlines

Description of fluid motion

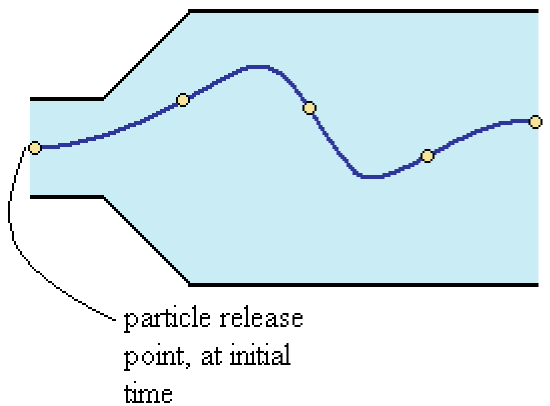
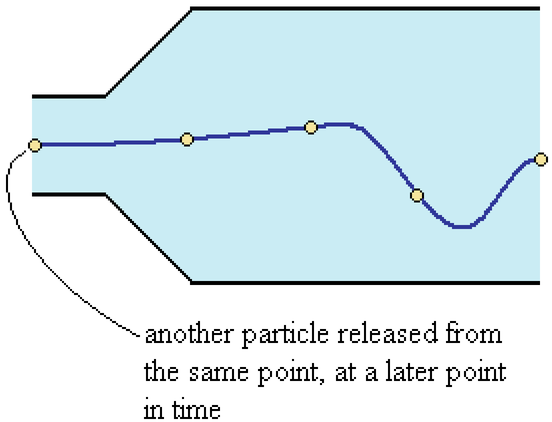
Pathlines, streaklines and streamlines

Streamlines	A line that is everywhere tangent to the instantaneous local velocity vector.	
Streaklines	Particles are continuously introduced at a certain point, the streakline shows the line formed due to the release of these particles from the specific point. It is the locus of particles that have passed sequentially through a prescribed path in the flow.	
Pathlines	Represents the path that is followed by one particle released into the fluid; the actual path traversed by a given fluid particle.	

Description of fluid motion

Pathlines, streaklines and streamlines

The images above look identical for streaklines and pathlines, but for turbulent and transitional flow, they may differ. The path one particle travels may be a different path for another particle that is released at the same point, but at a later time. See the following:

	
Pathline for a particle released at an initial point in time.	Pathline shows path for the second particle, released from the same point but at a later time, the pathline is different because the flow has changed.

Pathlines, streaklines and streamlines

- In a steady flow the streamline, pathline and streakline all coincide.
- In an unsteady flow they can be different.
- Streamlines are easily generated mathematically while pathline and streaklines are obtained through experiments.
- Streamlines are useful as indicators of the instantaneous direction of fluid motion throughout a flow field. Regions of recirculating flow and separation of a fluid off of a solid wall are easily identified with streamlines.

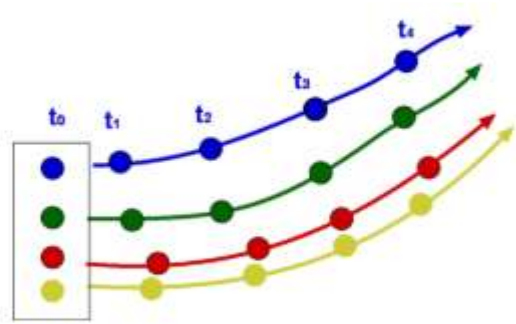
Pathlines, streaklines and streamlines

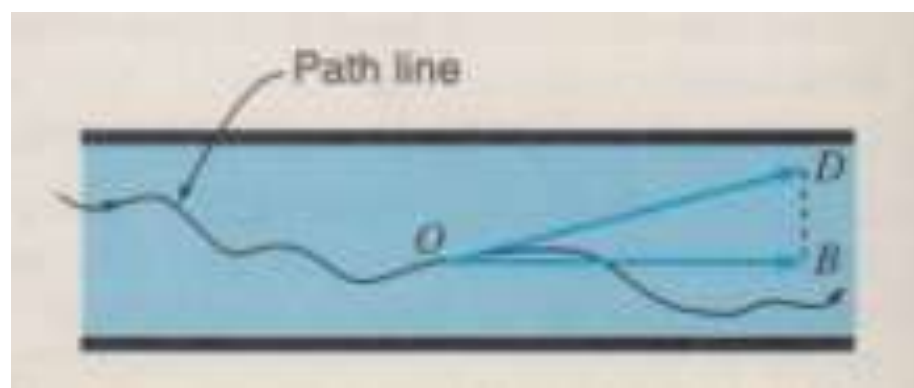
Pathline:

Definition 1: is the trace made by a single particle over a period of time. The pathline shows the direction of the velocity of the particle at successive instants of time.

Definition 2: is the locus of points traversed by a given particle as it travels in a field of flow; the pathline provides us with a "history" of the particle's locations.

Definition 3: Pathline is the line traced by a given particle. This is generated by injecting a dye into the fluid and following its path by photography or other means



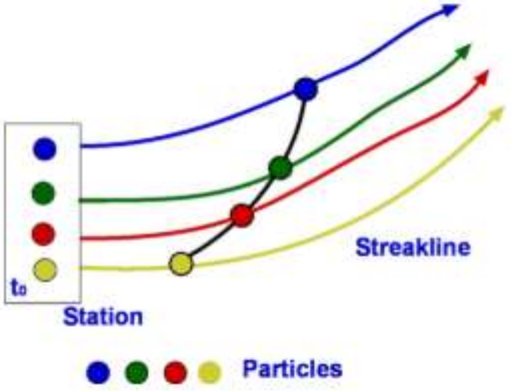


Pathlines, streaklines and streamlines

Streakline: is defined as an instantaneous line whose points are occupied by all particles originating from specified point in the flow field. Streaklines tell us where the particles are "right now".

Streakline concentrates on fluid particles that have gone through a fixed station or point. At some instant of time the position of all these particles are marked and a line is drawn through them. Such a line is called a streakline

Figure 3.2 Streaklines in the unsteady flow around a cylinder. (Photograph by Sadatashi Taneda.)



Pathlines, streaklines and streamlines

1. **Streamline**: is a line in the flow possessing the following property: the velocity vector of each particle occupying a point on the streamline is tangent to the streamline.
2. A streamline is one that drawn is tangential to the velocity vector at every point in the flow at a given instant

Streamlines show the mean direction of a number of particles at the same instant of time

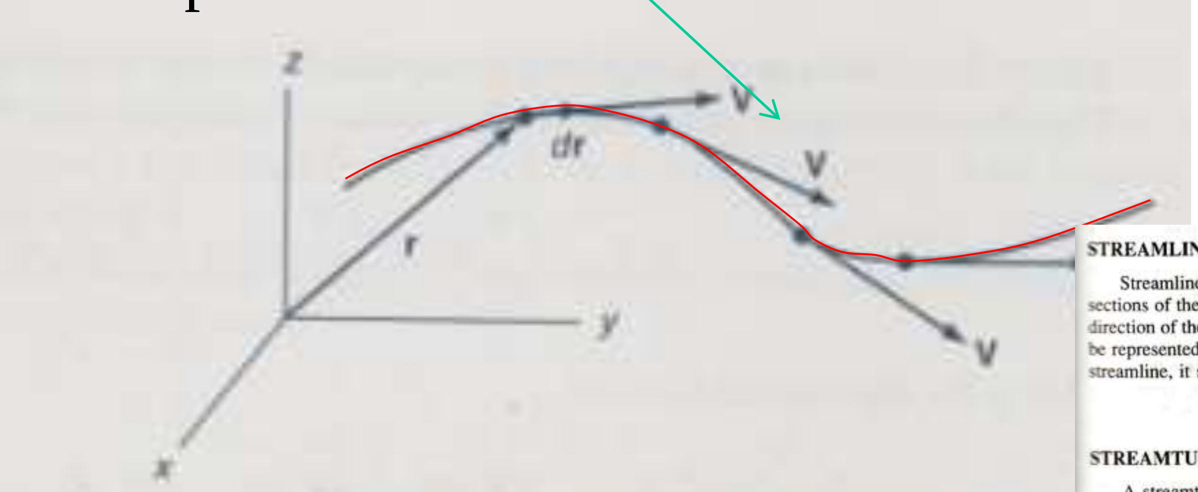


Figure 3.3 Streamline in a flow field.

STREAMLINES

Streamlines are imaginary curves drawn through a fluid to indicate the direction of motion in various sections of the flow of the fluid system. A tangent at any point on the curve represents the instantaneous direction of the velocity of the fluid particles at that point. The average direction of velocity may likewise be represented by tangents to streamlines. Since the velocity vector has a zero component normal to the streamline, it should be apparent that there can be no flow across a streamline at any point.

STREAMTUBES

A streamtube represents elementary portions of a flowing fluid bounded by a group of streamlines that confine the flow. If the streamtube's cross-sectional area is sufficiently small, the velocity of the midpoint of any cross section may be taken as the average velocity for the section as a whole. The streamtube will be used to derive the equation of continuity for steady one-dimensional incompressible flow (Problem 7.1).

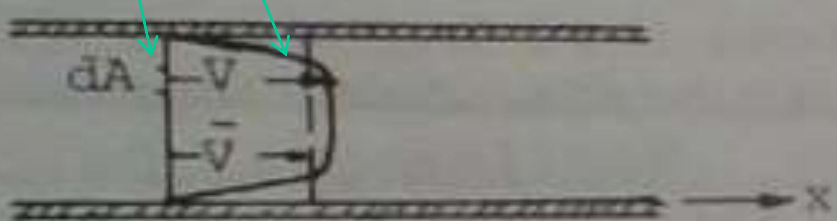
Description of fluid motion

Discharge

- Lecture handbook pp28

The discharge Q refers to the total quantity of fluid passing a given section in unit time. The velocity, in general, is variable across the section through which it flows, such as in Fig. 4.5. Then the rate of flow, discharge, past a differential area dA of the section is V.dA, and the total rate of flow Q is obtained by integration over the entire flow section.

$$Q = \int_A V.dA \qquad (4.1)$$



actual velocity V

mean velocity $\bar{V} = \frac{1}{A} \cdot \int_A V.dA$

Fig. 4.5 Flow between parallel plates

Discharge

In this example the cross-sectional area was oriented normal to the velocity vector. If other orientations are considered, such as in Fig. 4.6, where flow occurs past section A-A, it can be seen that only the normal component of the velocity, the x-component in this case, contributes to flow through the section.

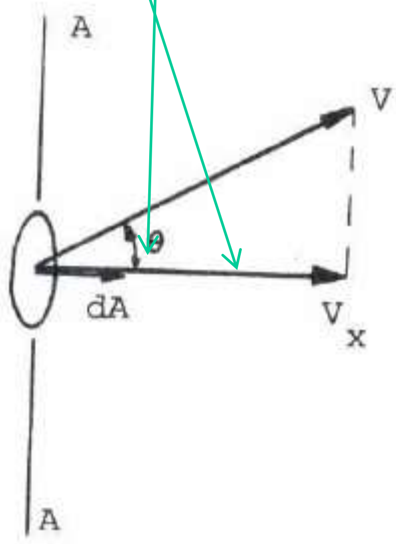


Fig. 4.6 Velocity not normal to the section

Description of fluid motion

Discharge

In formula

$$Q = \int_A V_x . dA = \int_A V . \cos \theta . dA \qquad (4.2)$$

Hence, always consider the area of a section which is normal to the total velocity or consider a velocity component which is normal to the given area.

The **mean** or **average velocity** is defined as the discharge divided by the total cross-sectional area

$$\bar{V} = Q/A$$

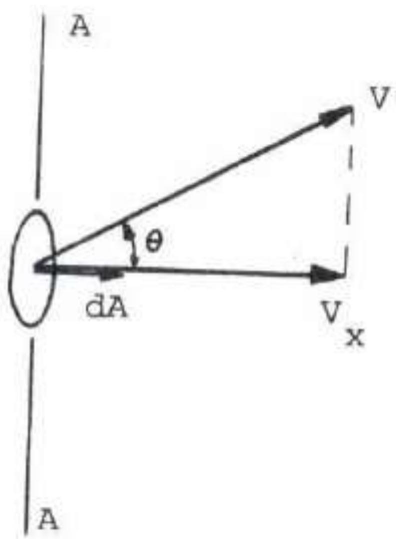


Fig. 4.6 Velocity not normal to the section

Classification of fluid motion

A. One-, Two- and Three Dimensional Flows

- In the Eulerian description of motion the velocity, in general, depends on three *space variables* and time. Such a flow is a three dimensional flow.
- A two dimensional flow is a flow in which the velocity vector depends on only two spatial variables.
- A one dimensional flow is a flow in which the velocity vector depends on only one space variable.

B. Ideal and real fluids

1. ***Ideal fluids*** $\tau = \mu \frac{du}{dy}$ $\mu = 0$

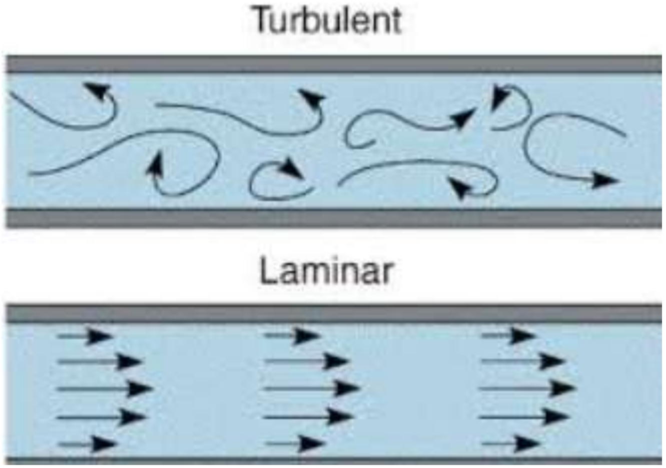
- An ideal fluid is assumed to have *no viscosity* and there are *no shear stresses*.
- There are no energy losses in the flow of an ideal fluid

2. ***Real fluids*** $\mu \neq 0$, therefore *friction* exists. These are real fluids with *energy losses* resulting from flows. Real fluids are also *compressible* although in some fluids the compressibility may be negligible for particular purpose.

C. Laminar and turbulent flows

Laminar ⇒ the fluid flows with *no significant mixing* of neighbouring fluid particles. Laminar is derived from laminate; the fluid appears to move by the sliding of laminations of *infinitesimal* thickness over adjacent layers, with relative motion of fluid particles occurring at a molecular scale. Viscous shear stresses *always* influence a laminar flow.

Turbulent flow ⇒ fluid motion varies *irregularly* so that quantities such as velocity and pressure show a *random variation* with time and space coordinates.



Classification of fluid motion

Classification between laminar and turbulent fow is defined by the Reynold number.

$$\mathrm{Re} = \frac{VL}{v} = \frac{inertial\ \ force}{viscous\ \ force}$$

where V is velocity, L characteristics length and v is the kinematic viscosity.

The Reynolds number is one of the most important parameters in hydro-mechanics.

Very small Reynolds numbers characterise by definition flows in which the viscous forces dominate and the inertial reactions are negligible.

Very high Reynolds numbers characterise flows in which finally the viscous forces become negligibly small in comparison to the inertial reactions, as for instance in fully turbulent pipe or channel flows.

Classification of fluid motion

If the Reynolds number is relatively small, the flow is laminar; if it is large the flow is turbulent.

This is more precisely stated by defining a critical Reynolds number, Re_{crit}, so that the flow is laminar if $Re < Re_{crit}$.

For example, in a flow inside a rough-walled pipe it is found that $Re_{crit} \approx 2000$. This is the minimum critical Reynolds number and is used in most engineering applications.

Classifying the fluid flow correctly is important because formulas for energy losses depend on whether the flow is laminar or turbulent. Correct choice of the formula will produce correct results leading to right designs e.g. selecting a pump based on the correct energy losses will have the right size of pump delivering the required discharge and head of fluid.

Classification of fluid motion

- D. Steady flow

Steady flow means steady with respect to time. Thus, all properties of flow at every point remain constant with respect to time

$$\frac{\partial p}{\partial t} = 0 \qquad \textit{where } p \textit{ is any property}$$

- E. Unsteady flow

In unsteady flow, the flow properties at a point change with time

$$\frac{\partial p}{\partial t} \neq 0$$

Classification of fluid motion

- F. Uniform flow

In uniform flow the cross section (shape and area) through which the flow occurs remains constant

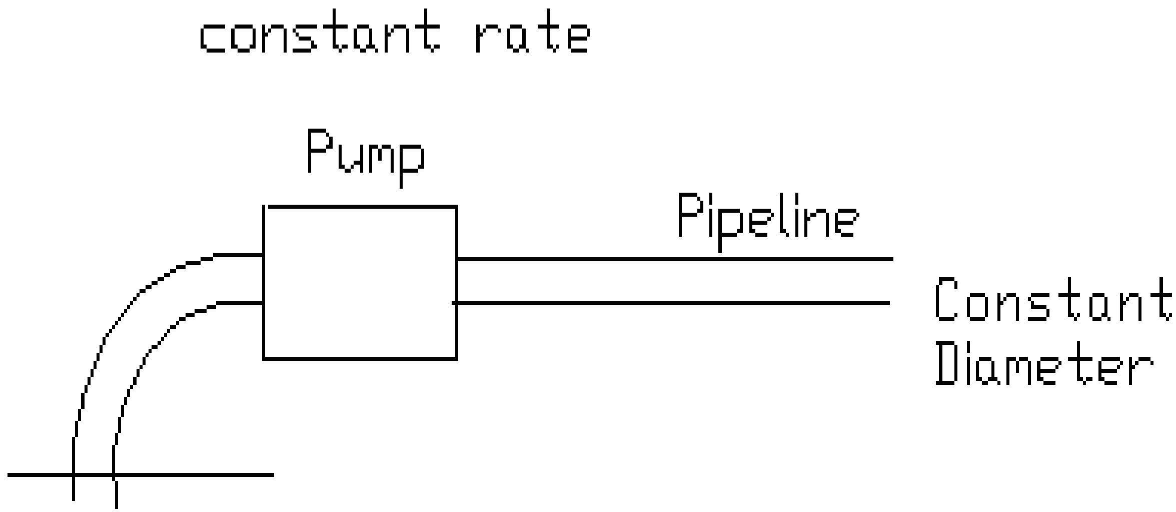
$$\frac{\partial V}{\partial s}=0$$

Where V is velocity and s is space or position

- G. Non uniform flow

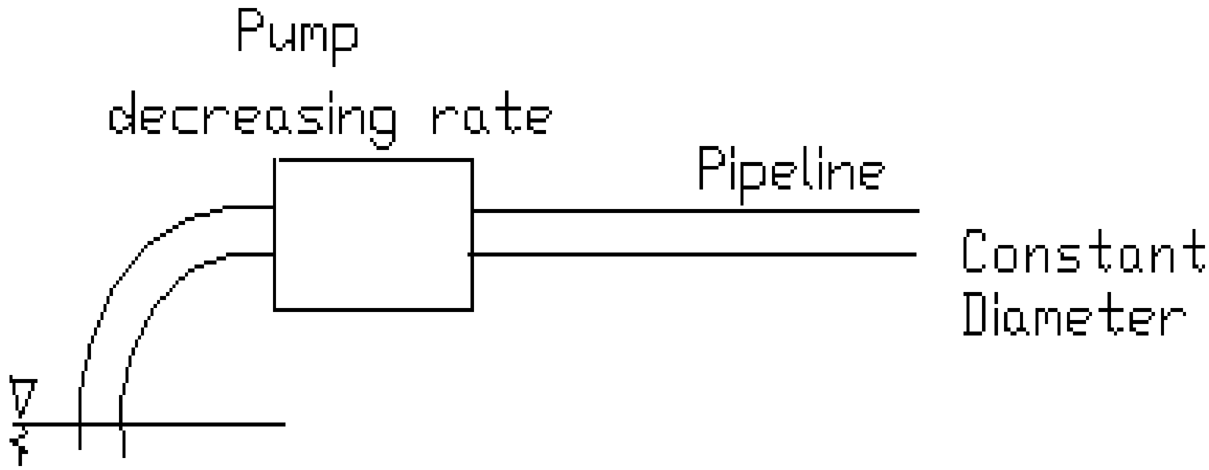
$$\frac{\partial V}{\partial s}\neq 0$$

Example



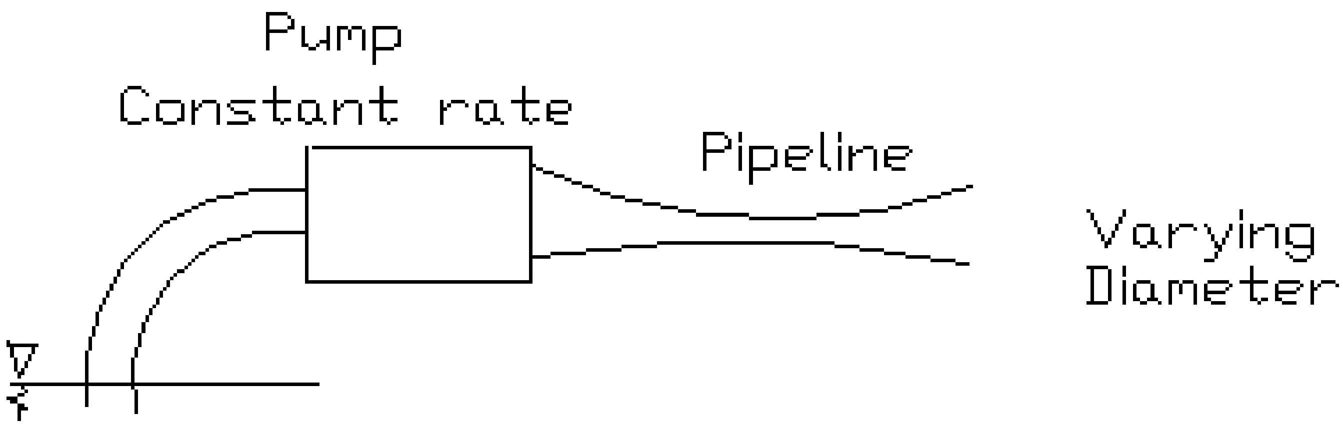
(a) Steady, Unlform

Example



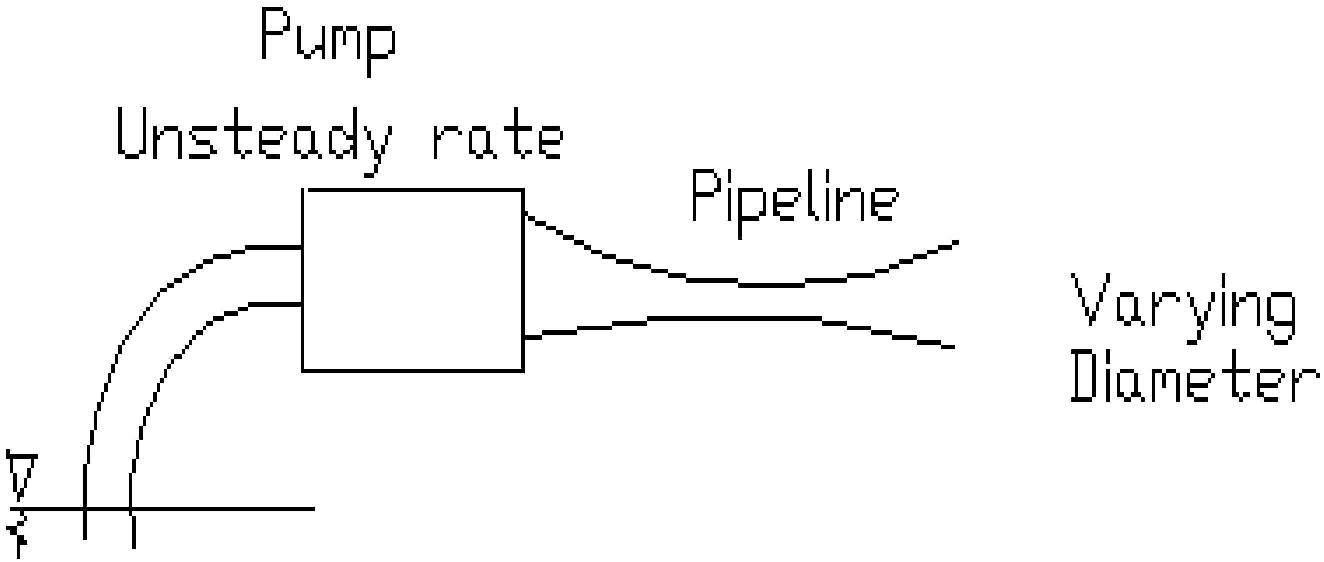
(b) Unsteady, Uniform

Example



(c) Steady, Non-unlform

Example



(d) Unsteady, Non-unlform

Acceleration

In general, the velocity of a fluid is a function both of position and time:

$$V_s = f(s, t)$$

in which V_s is the velocity along a streamline

s is the position along a streamline

Acceleration is a vector quantity that is defined as the rate at which an object changes its velocity. An object is accelerating if it is changing its velocity.

Over a small distance ds along a streamline the total increase of velocity dV_s is the sum of the increase due to its change of position and the increase due to passing an interval dt.

Mathematically

$$dV_s = \frac{\partial V_s}{\partial s} ds + \frac{\partial V_s}{\partial t} dt$$

Hence the acceleration a_s along the streamline is given by

$$a_s = \frac{dV_s}{dt} = \frac{\partial V_s}{\partial s}\frac{ds}{dt} + \frac{\partial V_s}{\partial t}\frac{dt}{dt} = V_s \frac{\partial V_s}{\partial s} + \frac{\partial V_s}{\partial t}$$

Since $V_s = ds/dt$

in which

a_s is total acceleration

$V_s \frac{\partial V_s}{\partial s}$ convective acceleration

Convective acceleration is basically defined as the rate of change of velocity due to change of position of fluid particle in fluid flow field.

$\frac{\partial V_s}{\partial t}$ local acceleration

Local acceleration is basically defined as the rate of increase of velocity with respect to time at a given point in the flow field.

$$a_s = \frac{dV_s}{dt} = \frac{\partial V_s}{\partial s}\frac{ds}{dt} + \frac{\partial V_s}{\partial t}\frac{dt}{dt} = V_s\frac{\partial V_s}{\partial s} + \frac{\partial V_s}{\partial t}$$

in which

a_s is total acceleration

$V_s \frac{\partial V_s}{\partial s}$ convective acceleration

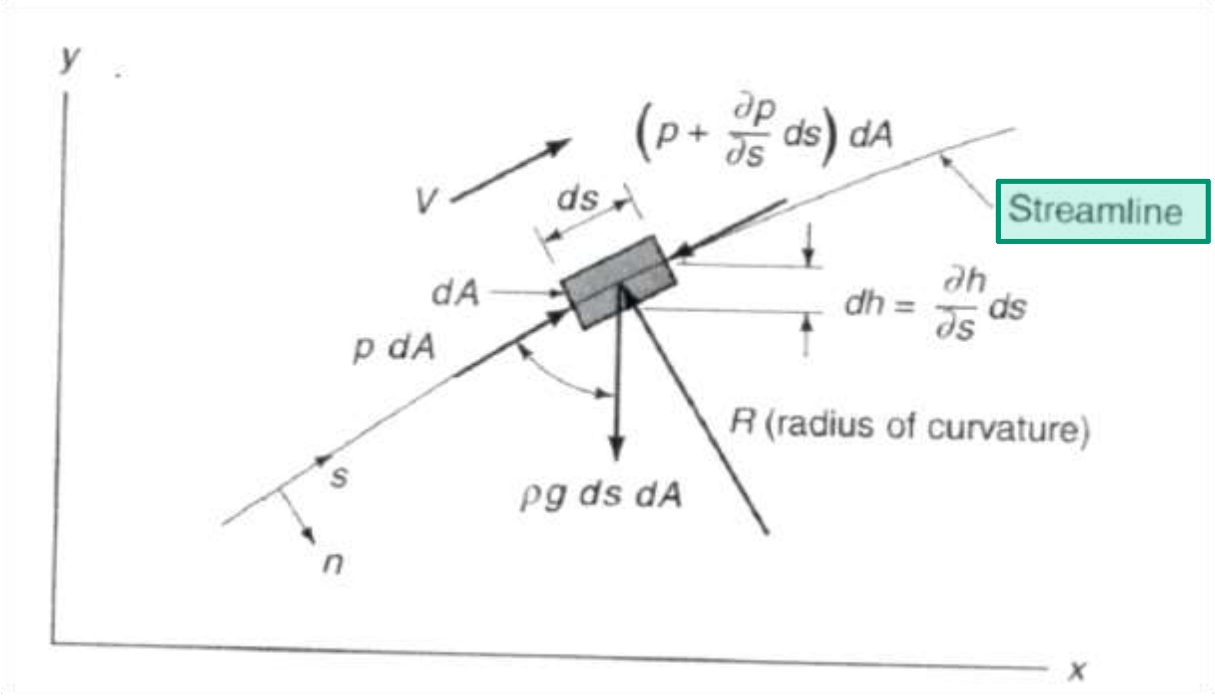
$\frac{\partial V_s}{\partial t}$ local acceleration

- In a pipe, local acceleration results if, for example, a valve is being *opened or closed*; and convective acceleration occurs in the vicinity of a *change in the pipe geometry*, such as a pipe contraction or an elbow.
- In both cases fluid particles change speed but *for very different reasons*.
- With *incompressible* fluid flow, there is convective acceleration wherever the effective flow *area changes* along the flow path.
- This is also true for *compressible* fluid flow, but , in addition, convective acceleration of a compressible fluid occurs wherever the *density varies* along the flow path *irrespective* of any changes in the effective flow area

Bernoulli Equation

The Bernoulli equation is probably used more often in fluid flow applications than any other equation.
The derivation starts with the application of Newton's second law to a fluid particle. Let us use a particle positioned as shown in Fig 6.1, with length *ds* and cross sectional area *dA*.

Second law:
The vector sum of the forces F on an object is equal to the mass m of that object multiplied by the acceleration vector a of the object: F = ma.



Bernoulli Equation

Assumption is inviscid flow (no shear stresses- no friction)

The forces acting on the particle are pressure forces and the weight, as shown. Summing forces in the direction of motion, the s-direction, there results

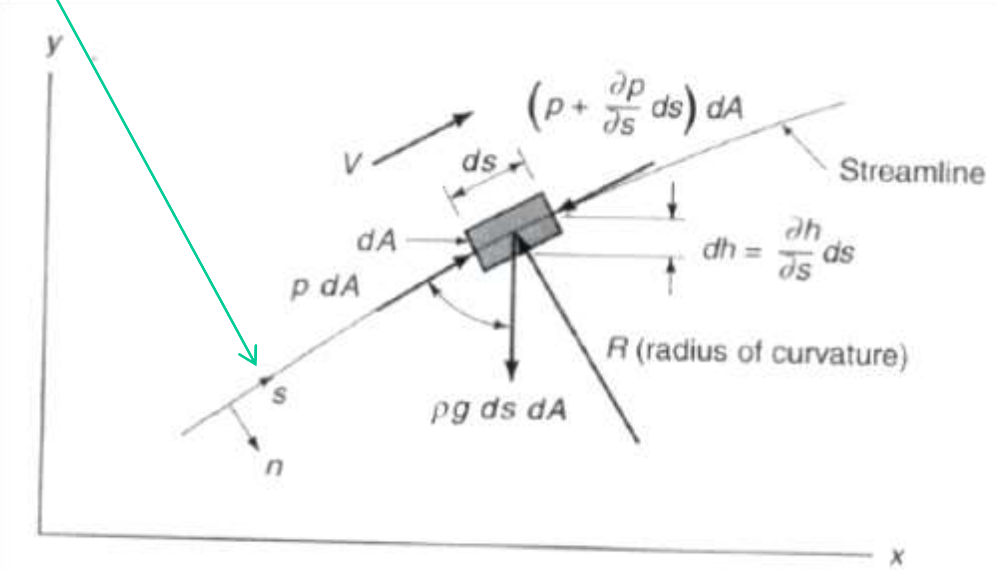
Second law:
The vector sum of the forces F on an object is equal to the mass m of that object multiplied by the acceleration vector a of the object: F = ma.

$$pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g\, ds\, dA \cos\theta = \rho\, ds\, dA\, a_s$$

where a_s is the acceleration of the particle in the s-direction. It is given by

$$a_s = V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}$$

where $\frac{\partial V}{\partial t} = 0$ since we will assume steady flow.



$$pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g \; ds \; dA \; cos\theta = \rho \; ds \; dA \; a_s$$

$$\frac{pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g \; ds \; dA \; cos\theta = \rho \; ds \; dA \; a_s}{ds \; dA}$$

$$-\frac{\partial p}{\partial s} - \rho g \; cos\theta = \rho \;\; a_s = \rho V \;\; \frac{\partial V}{\partial s} + \rho \frac{\partial V}{\partial t}$$

$$-\frac{\partial p}{\partial s} - \rho g \; cos\theta = \rho V \;\; \frac{\partial V}{\partial s} + 0 = \rho V \;\; \frac{\partial V}{\partial s}$$

$$-\frac{\partial p}{\partial s} - \rho g\cos\theta = \rho V \frac{\partial V}{\partial s} + 0 = \rho V \frac{\partial V}{\partial s}$$

Now, we assume constant density and note that $V\frac{\partial V}{\partial s} = \frac{\partial V^2/_2}{\partial s}$

i.e., ρ is the same along the streamline and through out the whole liquid

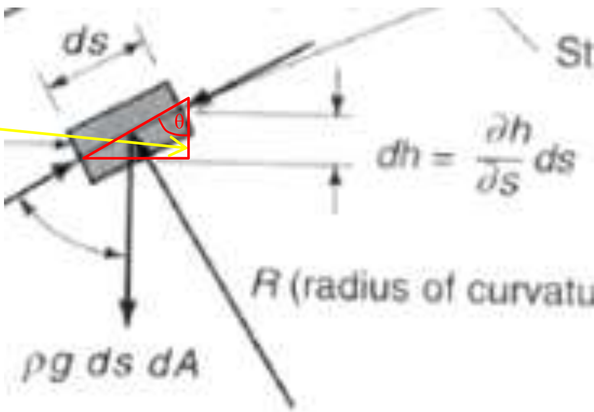
$$-\frac{\partial p}{\partial s} - \rho g\cos\theta = \rho V \frac{\partial V}{\partial s} + 0 = \rho V \frac{\partial V}{\partial s}$$

$$\frac{\partial \left(V^2/_2\right)}{\partial s} = \frac{2V}{2}\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial s}$$

$$-\frac{\partial p}{\partial s} - \rho g\cos\theta = \rho \frac{\partial V^2/_2}{\partial s}$$

Also, we see that

$$dh = ds\cos\theta = \frac{\partial h}{\partial s}ds$$



so that (or since)

$$\cos\theta = \frac{\partial h}{\partial s}$$

$$-\frac{\partial P}{\partial s} - \rho g\frac{\partial h}{\partial s} = \rho\frac{dV^2/_2}{\partial s}$$

$$-\frac{\partial P}{\partial s}-\rho g\frac{\partial h}{\partial s}=\rho\frac{d\,V^2/2}{\partial s}$$

$$\frac{\partial P}{\partial s}+\rho g\frac{\partial h}{\partial s}+\rho\frac{d\,V^2/2}{\partial s}=0$$

$$\frac{\partial}{\partial s}\left(P+\rho g h+\rho V^2/2\right)=0$$

$$\frac{\partial}{\partial s}\left(P + \rho g h + \rho V^2 \big/ 2\right) = 0$$

$$\frac{\partial}{\partial s}\left(\frac{P + \rho g h + \rho V^2 \big/ 2}{\rho}\right) = 0$$

$$\frac{\partial}{\partial s}\left(\frac{V^2}{2} + \frac{P}{\rho} + gh\right) = 0$$

This is satisfied, if along a streamline

$$\frac{\partial}{\partial s}\left(\frac{V^2}{2} + \frac{P}{\rho} + gh\right) = \text{constant}$$

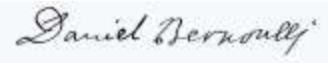
$$\frac{V^2}{2} + \frac{P}{\rho} + gh = \text{constant}$$

or, between two points on the same streamline

$$\frac{V_1^2}{2} + \frac{P_1}{\rho} + gh_1 = \frac{V_2^2}{2} + \frac{P_2}{\rho} + gh_2$$

This is the well known **Bernoulli equation** in honour of Daniel Bernoulli (1700-1782), the Swiss physicist who presented this theorem in 1738. Note the assumptions:

Born	8 February 1700 Groningen, Dutch Republic
Died	17 March 1782 (aged 82) Basel, Republic of the Swiss
Nationality	Swiss
Alma mater	University of Basel (M.D., 1721) Heidelberg University University of Strasbourg
Known for	Bernoulli's principle Early kinetic theory of gases Thermodynamics
Scientific career	
Fields	Mathematics, physics, medicine
Thesis	*"Dissertatio physico-medica de respiratione" (Dissertation on the medical physics of respiration)* (1721)
Signature	



Daniel Bernoulli

Bernoulli equation

- Assumptions of the Bernoulli equation:
  - Slide 31 – Inviscid fluid (no shear stresses-no friction). An inviscid flow is the flow of an ideal fluid
  - Slide 31 – Steady flow ($^{\partial V}/_{dt} = 0$)
  - Slide 35 – Along a streamline ($a_s = V \frac{\partial \mathrm{V}}{\partial s}$)
  - Slide 33 – Constant density $\left({}^{\partial \rho}\!/_{\partial s} = 0 \right)$

Bernoulli equation

- If the equation is divided by g, Bernoulli equation becomes

$$\frac{V_1^2}{2} + \frac{P_1}{\rho} + gh_1 = \frac{V_2^2}{2} + \frac{P_2}{\rho} + gh_2$$

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + h_1 = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + h_2$$

- ❑ $\frac{p}{\rho g}$ is the static pressure head
- ❑ $\frac{V^2}{2g}$ is the dynamic pressure
- ❑ $\frac{p}{\rho g} + h$ is the piezometric head
- ❑ $\frac{V^2}{2g} + \frac{p}{\rho g}$ is the total pressure head or stagnation pressure head
- ❑ The sum of all 3 terms is the total head or energy head

from Lecture 2, $P = \gamma h$

therefore, $h = \frac{P}{\rho g}$

Bernoulli equation

Application of the Bernoulli equation

- Care must be taken never to use the equation in an unsteady flow or if viscous effects are significant. The equation is used:
    1. to determine how high the water from a fireman's hose will reach

1,000,000 USD

ROSENBAUER AIRPORT FIREFIGHTING TRUCK
PANTHER 8X8 FLF 14000/250

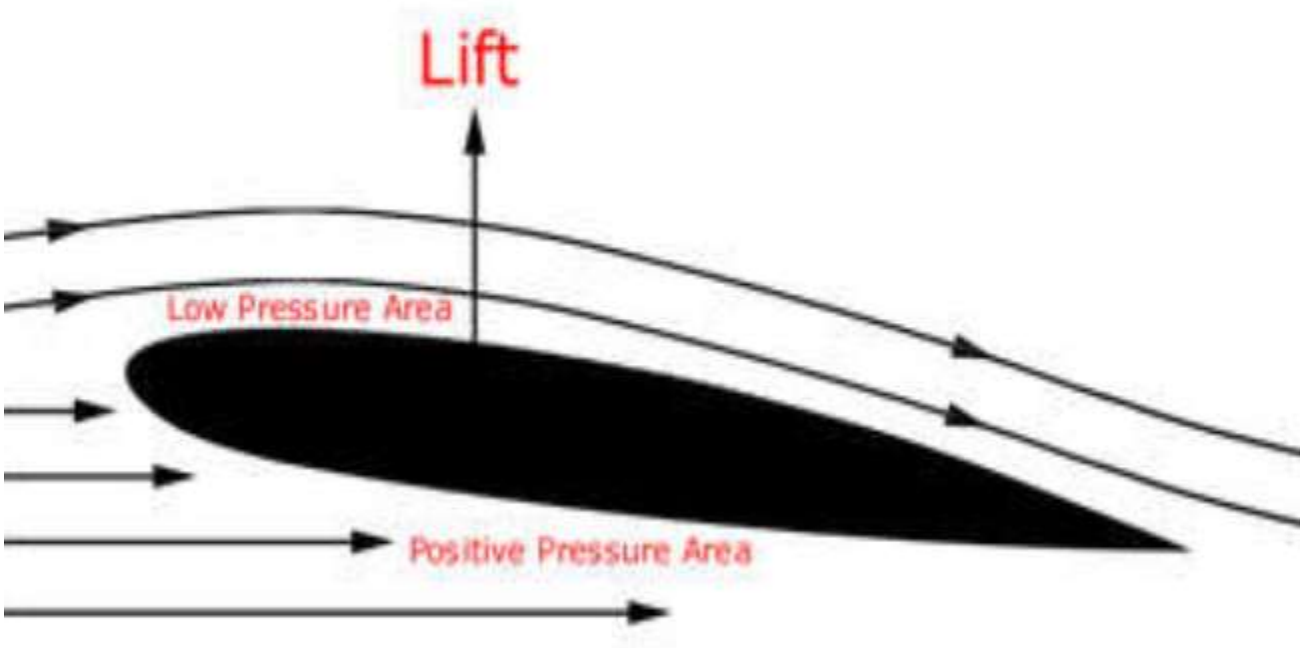
42,000 USD

PHOTOS: US$1million Fire Tender Truck Fails To Quench Fire In Kitwe, Goods Get Burnt

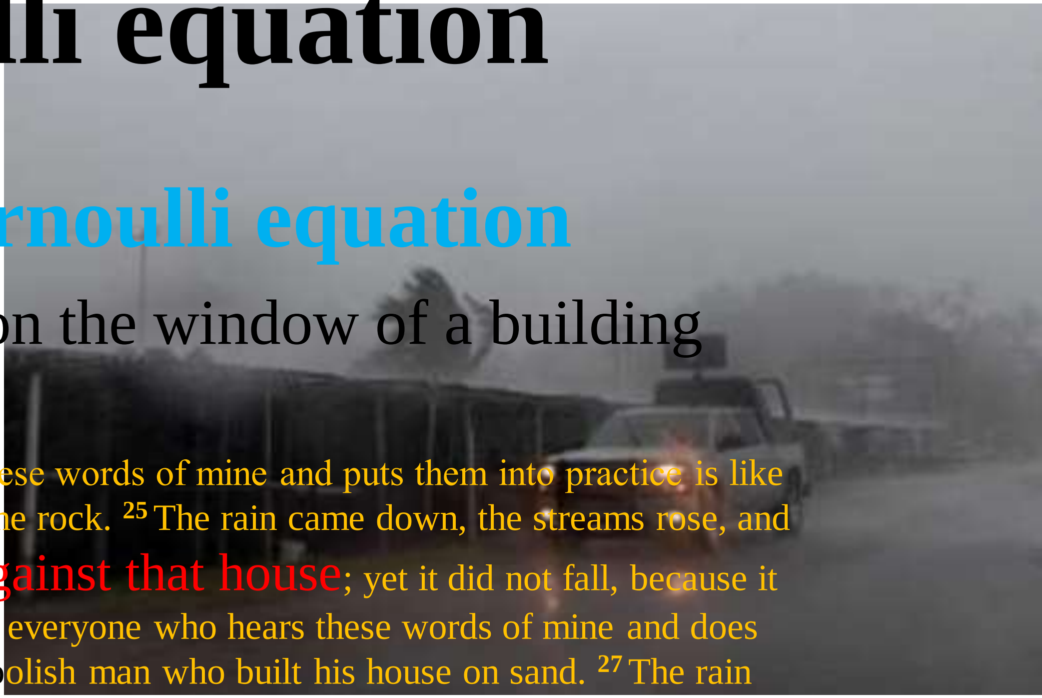
Bernoulli equation

Application of the Bernoulli equation

2. to find the pressure on the surface of a low-speed airfoil (airfoil is a shape of a wing, blade or sail)



Bernoulli equation



Application of the Bernoulli equation

3. to find the wind force on the window of a building
(Matthew 7:24-27)

24 “Therefore everyone who hears these words of mine and puts them into practice is like
a wise man who built his house on the rock. 25 The rain came down, the streams rose, and
the winds blew and beat against that house; yet it did not fall, because it
had its foundation on the rock. 26 But everyone who hears these words of mine and does
not put them into practice is like a foolish man who built his house on sand. 27 The rain
came down, the streams rose, and the winds blew and beat against that
house, and it fell with a great crash.”

(One of the primary ways windows fail in hurricanes is broken glass from wind pressure, according to Graham Architectural Products. Hurricane wind speeds range from 119km/h to above 249km/h)

Bernoulli equation

Application of the Bernoulli equation

4. In internal flows over relatively short distances, e.g., flow through a contraction or flow from a plenum (air filled space that receives air from a blower for distribution)

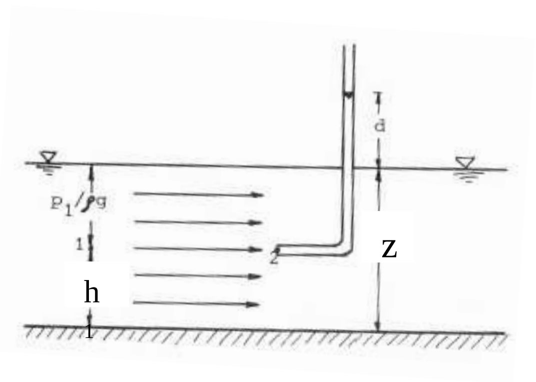
Bernoulli equation

Application of the Bernoulli equation

5. Pitot tubes

Beginning his career as a mathematician and astronomer, Pitot won election to the Academy of Sciences in 1724. He became interested in the problem of flow of water in rivers and canals and discovered that much contemporary theory was erroneous—for example, the idea that the velocity of flowing water increased with depth. He devised a tube, with an opening facing the flow, that provided a convenient and reasonably accurate measurement of flow velocity and that has found wide application ever since (e.g., in anemometers for measuring wind speed).

Henri Pitot	
Born	3 May 1695 Aramon, Gard, France
Died	27 December 1771 (aged 76) Aramon, Gard, France
Nationality	French
Known for	Pitot tube
Scientific career	
Fields	Hydraulics



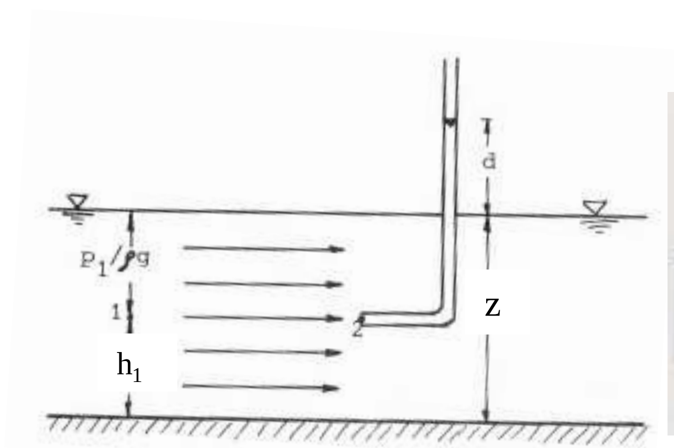
Bernoulli equation

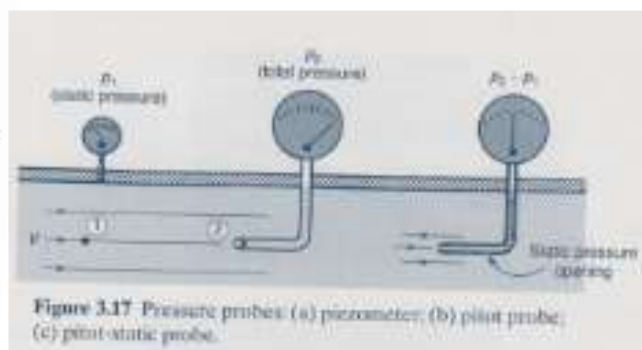
Application of the Bernoulli equation

5. Pitot tubes

A. Simple pitot tube

- The velocity can be measured simply by installing a simple pitot tube.





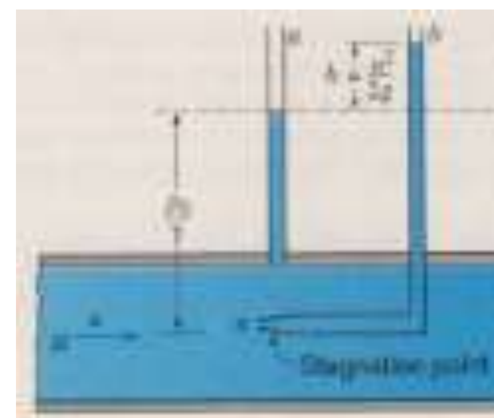


Fig. A: simple pitot tube

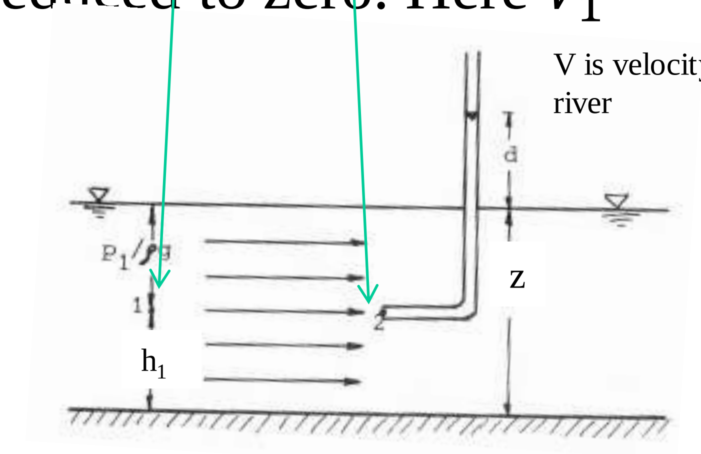
Bernoulli equation

Application of the Bernoulli equation

Applying Bernoulli equation between 1 and 2.

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + h_1 = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + h_2$$

- $h_2 = h_1$ and $V_2 = 0$. Point 2 is the co-called stagnation point, where the velocity is reduced to zero. Here $V_1 = \text{V}$.



V is velocity of water in the river

- $\frac{V^2}{2g} + \frac{p_1}{\rho g} + h_1 = 0 + \frac{p_2}{\rho g} + h_2$

since $\frac{P_1}{\rho g} + h_1 = z \; and \; \frac{P_2}{\rho g} + h_2 = z + d$

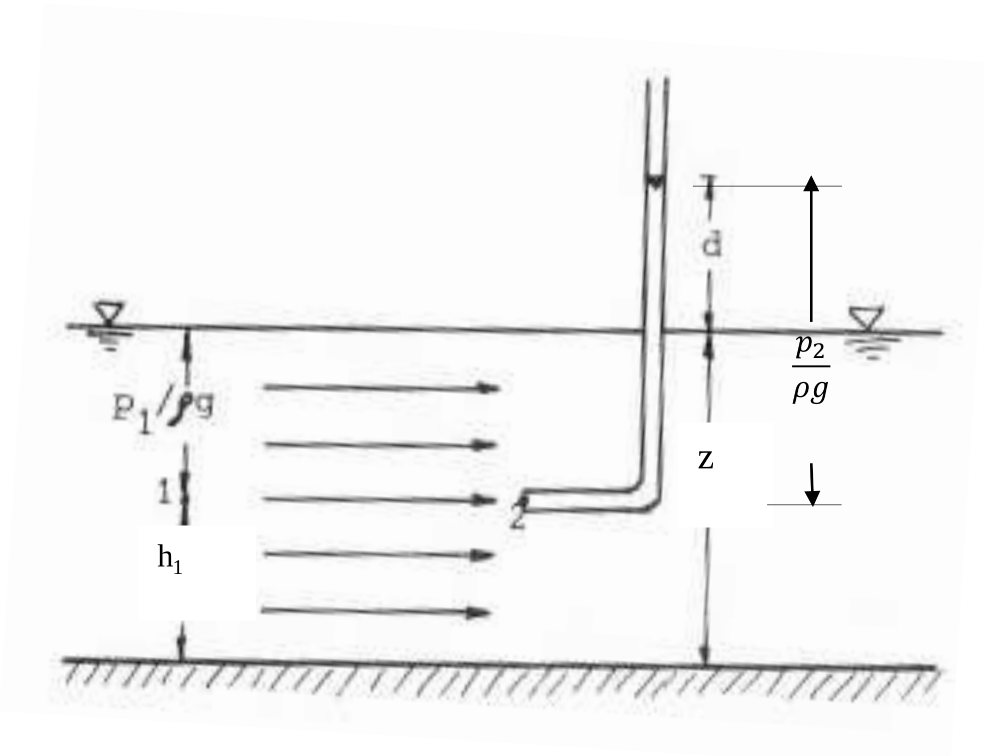
- $\frac{V^2}{2g} + z = z + d$

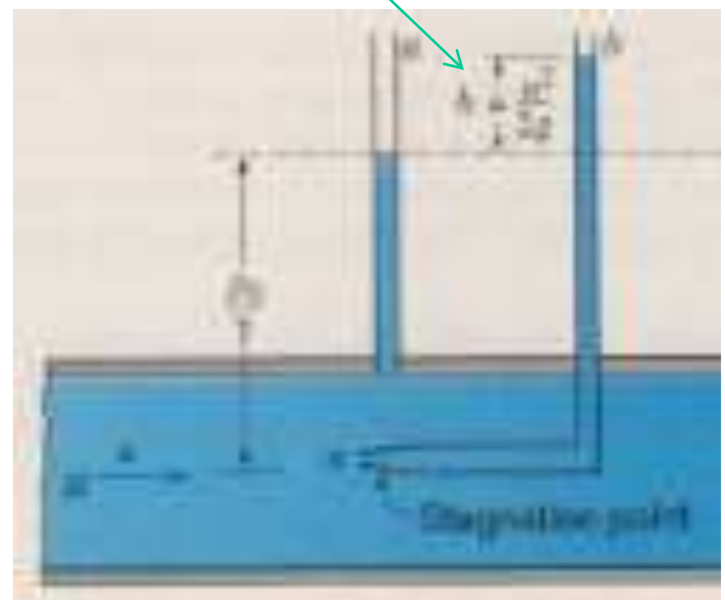
- $V = \sqrt{2gd}$

By measuring d, which is easy, one can compute the velocity of flow

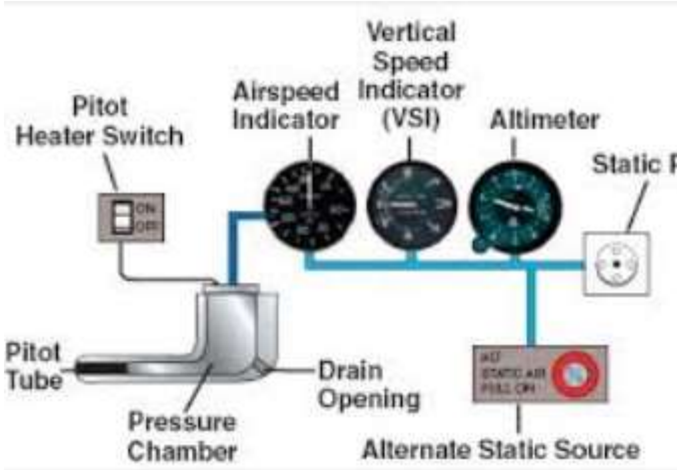
Simply stated, the rise d in the tube at point 2 is due to the velocity head being converted to pressure head as elevation head is the same at points 1 and 2. See next slide

Simply stated, the rise d in the tube at point 2 is due to the velocity head being converted to pressure head as elevation head is the same at points 1 and 2





Uses of pitot tubes



As the airplane moves through the air, air enters the pitot tube and puts pressure (dynamic) on a diaphragm inside the airspeed indicator.

Weather instruments at Mount Washington Observatory. Pitot tube static anemometer is on the right.

It is widely used to determine the airspeed of an aircraft, water speed of a boat, and to measure liquid, air and gas flow velocities in industrial applications.

Pitot tubes on aircraft commonly have heating elements called pitot heat to prevent the tube from becoming clogged with ice. The failure of these systems can have *catastrophic consequences*, as in the case of Austral Líneas Aéreas Flight 2553, Birgenair Flight 301 (investigators suspected that some kind of insect could have created a nest inside the pitot tube: the prime suspect is the black and yellow mud dauber wasp), Northwest Airlines Flight 6231, Aeroperú Flight 603 (blocked static port), and of one X-31. The French air safety authority BEA said that pitot tube icing was a contributing factor in the crash of Air France Flight 447 into the Atlantic Ocean. In 2008 Air Caraïbes reported two incidents of pitot tube icing malfunctions on its A330s.

Did a Small Metal Tube Bring Down an Indonesian Airliner?

- As teams search for Lion Air Flight 610’s black boxes, experts speculate that a faulty pitot tube, used for measuring airspeed, may have contributed to the crash.
- Rescue workers collecting wreckage Tuesday from Lion Air Flight 610, which crashed into the sea northeast of Jakarta, Indonesia, on Monday.CreditMast Irham/EPA, via Shutterstock
- Image
- Rescue workers collecting wreckage Tuesday from Lion Air Flight 610, which crashed into the sea northeast of Jakarta, Indonesia, on Monday.CreditCreditMast Irham/EPA, via Shutterstock
- **By Russell Goldman**
- Oct. 30, 2018
- Divers scoured the Java Sea on Tuesday looking for clues that could explain why a brand new airliner fell out of the sky just moments after takeoff, killing all 189 people on board.
- Before Lion Air Flight 610 lost contact on Monday, the plane displayed erratic changes in its speed, altitude and direction, causing experts to speculate that a problem with the aircraft's instruments used to calculate airspeed and altitude may have contributed to the crash.
- Those indicators, or pitot tubes, have been implicated in previous aviation disasters, but experts said that determining the cause of the crash would ultimately require the recovery of the plane’s flight data recorders, the so-called black boxes.

Rescue workers collecting wreckage Tuesday from Lion Air Flight 610, which crashed into the sea northeast of Jakarta, Indonesia, on Monday. Mast Irham/EPA, via Shutterstock

Bernoulli equation

Application of the Bernoulli equation

5. Pitot tubes

   B. Pitot static tube

   - This is used to measure the difference between total and static pressure with one probe.

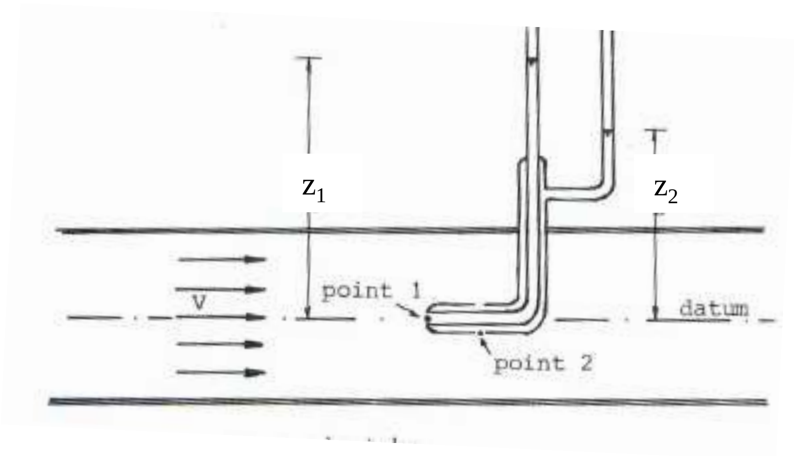


Fig. B: pitot static tube

Bernoulli equation

Application of the Bernoulli equation

Applying Bernoulli equation between 1 and 2.

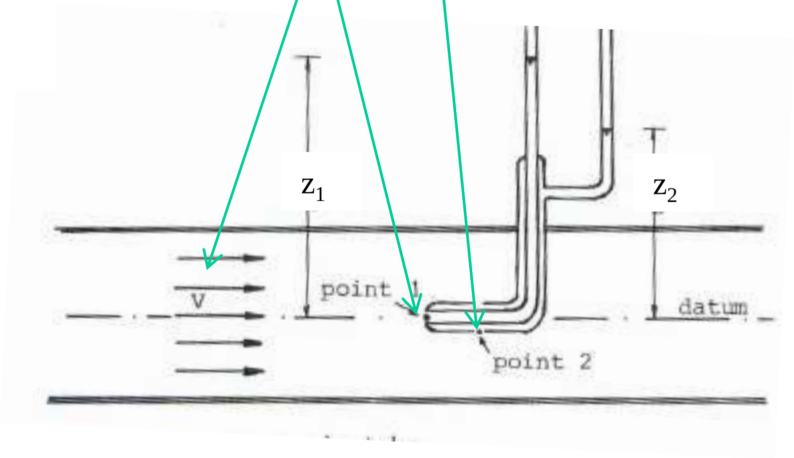
$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + h_1 = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + h_2$$

- $V_1 = 0$. Point 1 is the co-called stagnation point, where the velocity is reduced to zero. Here $V_2 = V$. And $h_1 = h_2 = 0$
- $0 + \frac{p_1}{\rho g} \qquad = \frac{V^2}{2g} + \frac{p_2}{\rho g}$

Static pressure head

Total pressure head

- $z_1 = \frac{V^2}{2g} + z_2$
- $V = \sqrt{2g(z_1 - z_2)}$



6. Flow through a sharp edged orifice

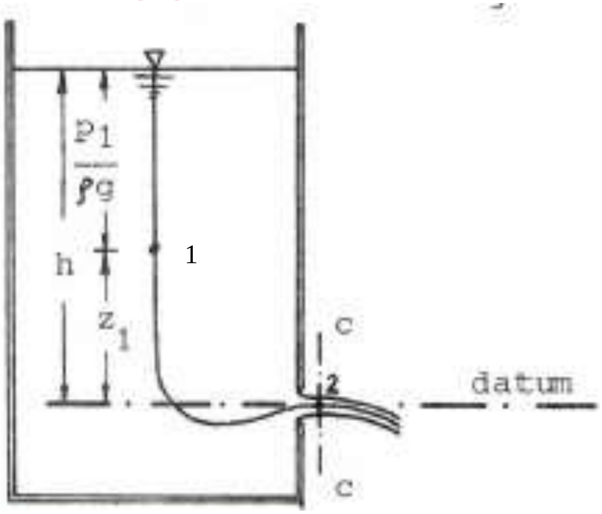
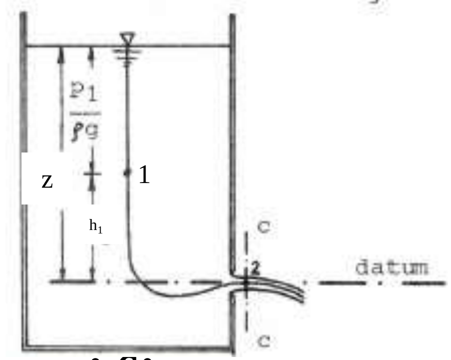


Fig Flow through a sharp-edged orifice

- At section c-c the contraction of the jet is maximal. This section is called the vena contracta and at this point the flow is uniform (streamlines are straight and parallel) & therefore the pressure is also uniform i.e., the pressure at the vena contracta equals that of the atmospheric pressure.

Application of the Bernoulli equation



- $\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + h_1 = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + h_2$
- The tank is large compared to the orifice and point 1 sufficiently far from the orifice. Therefore, V_1 may be assumed negligible compared to V_2
- $0 + z = \frac{V_2^2}{2g} + 0 + 0$
- $V_2 = \sqrt{2gz}$

By measuring z, one can compute the velocity of flow

As water level reduces in a tank due to being drawn, one notices that the velocity of water reduces

Application of the Bernoulli equation

7. Flow through a sharp crested notch

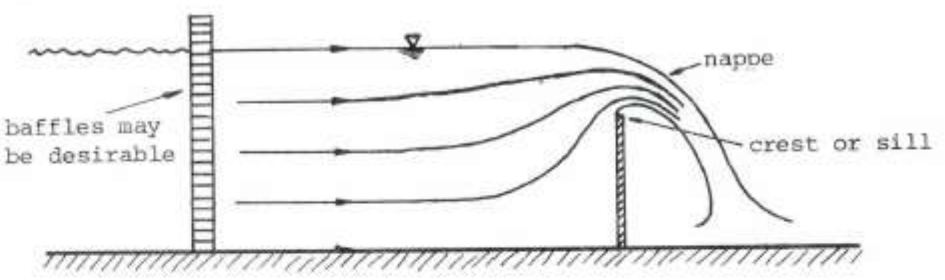


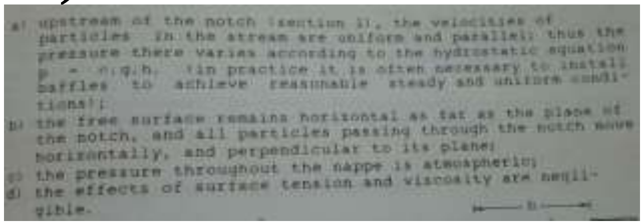
Fig Flow through a sharp-edged notch

Baffle: a device used to restrain the flow of a fluid, gas, etc. or to prevent the spreading of sound or light in a particular direction.

In hydraulic engineering, a nappe is **a sheet or curtain of water that flows over a weir or dam**. The upper and lower water surface have well-defined characteristics that are created by the crest of a dam or weir.

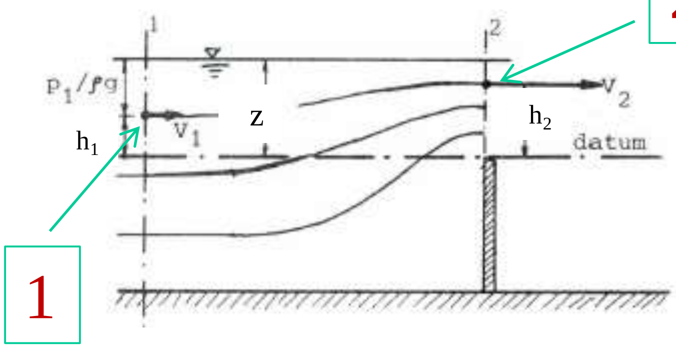
Application of the Bernoulli equation

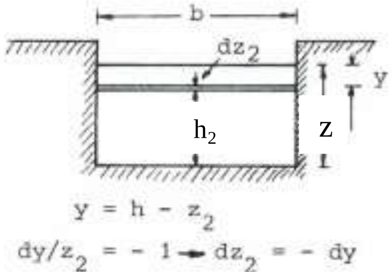
- Assumptions (see lecture handbook)



a) upstream of the notch (section 1), the velocities of particles in the stream are uniform and parallel; thus the pressure there varies according to the hydrostatic equation p = ρgh. (in practice it is often necessary to install baffles to achieve reasonable steady and uniform conditions);
b) the free surface remains horizontal as far as the plane of the notch, and all particles passing through the notch move horizontally, and perpendicular to its plane;
c) the pressure throughout the nappe is atmospheric;
d) the effects of surface tension and viscosity are negligible.

- $\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + h_1 = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + h_2$
- V_1 is negligible compared to V_2 and p_2=p_a= 0 Assumption c
- $0 + z \quad = \frac{V_2^2}{2g} + h_2$
- $V_2 = \sqrt{2g(z - h_2)}$

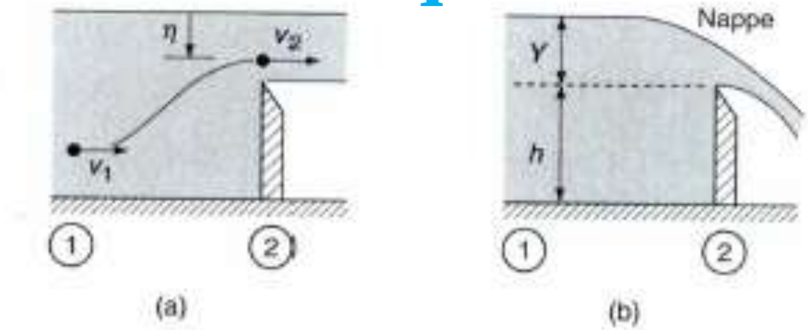




Note: points 1 & 2 are along a streamline

Application of the Bernoulli equation

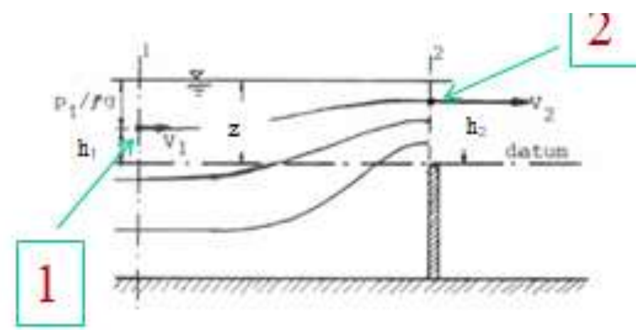
$V_2 = \sqrt{2g(z - h_2)}$



- If η replaces $(z\text{-}h_2)$
- $V_2 = \sqrt{2g\eta}$
- If b is the width of the crest to the normal flow, the ideal discharge is given as
- $Q = b\int_0^Y V_2 d\,\eta = b\int_0^Y \sqrt{2g\eta}\,d\eta = b\frac{2}{3}\sqrt{2g}\,Y^{3/2}$

The only variable is the depth Y and so it is easy to calculate the discharge

predict the real flow shown in F



Application of the Bernoulli equation

- Experiments have shown that the magnitude of the exponent is nearly correct but that a discharge coefficient C_d must be applied to accurately predict the real flow shown in Fig (b) C_d

For rectangular weir

$$C_d = 0.61 + 0.08\frac{Y}{h}$$

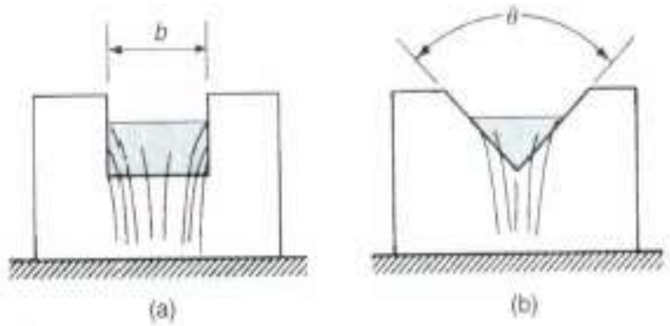


Fig Contracted rectangular and V-notch weirs

- $Q = C_d \frac{2}{3}\sqrt{2g}\ \mathrm{b}\, Y^{3/2}$

Example 1

Determine the discharge of water over a rectangular sharp-crested weir, $b = 1.25$ m, $Y = 0.35$ m, $h = 1.47$ m, with side walls and with end contractions. If a 90° V-notch weir were to replace the rectangular weir, what would be the required Y for a similar discharge?

Solution: For the rectangular weir, using Eq. 10.4.26, the discharge coefficient is

$$C_d = 0.61 + 0.08\frac{Y}{h}$$

$$= 0.61 + 0.08 \times \frac{0.35}{1.47}$$

$$= 0.63$$

Substitute into Eq. 10.4.25 and calculate

$$Q = C_d \tfrac{2}{3}\sqrt{2g}\, bY^{3/2}$$
$$= 0.63 \times \tfrac{2}{3} \times \sqrt{2 \times 9.81} \times 1.25 \times 0.35^{3/2}$$
$$= 0.48\ \text{m}^3/\text{s}$$

With end contractions the effective width of the weir is reduced by $0.2Y$, resulting in

$$Q = C_d \tfrac{2}{3}\sqrt{2g}\,(b - 0.2Y)Y^{3/2}$$
$$= 0.63 \times \tfrac{2}{3} \times \sqrt{2 \times 9.81} \times (1.25 - 0.2 \times 0.35) \times 0.35^{3/2}$$
$$= 0.45\ \text{m}^3/\text{s}$$

With a discharge of $Q = 0.48$ m^3/s, use Eq. 10.4.27 to find Y for the 90° V-notch weir:

$$Y = \left[\frac{Q}{C_d \times \frac{8}{15} \times \sqrt{2g}\tan(\theta/2)}\right]^{2/5}$$
$$= \left[\frac{0.482}{0.58 \times \frac{8}{15} \times \sqrt{2 \times 9.81} \times \tan 45^\circ}\right]^{2/5} = 0.66\ \text{m}$$

Example 2

The wind reaches a speed of 100 km/h in a storm. Calculate the force acting on a 1 m × 2 m window facing the storm. The window is in a high-rise building, so the wind speed is not reduced due to ground effects. Use $\rho = 1.2\ \text{kg/m}^3$.

Solution: The window facing the storm will be in a stagnation region where the wind speed is brought to zero. Working with gage pressures, the pressure p upstream in the wind is zero. The velocity V must have units of m/s. It is

$$V = 100 \frac{\text{km}}{\text{h}} \times \frac{1\ \text{h}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \text{km}} = 27.8\ \text{m/s}$$

Bernoulli's equation then allows us to calculate the pressure on the window as follows:

$$\frac{V_2^2}{2g} + \frac{p_2}{\gamma} + h_2 = \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + h_1$$

(with $V_2 = 0$ and $p_1 = 0$ struck out)

$$\therefore \quad p_2 = \frac{\rho V_1^2}{2}$$

$$= \frac{1.2 \times (27.8)^2}{2} = 464\ \text{N/m}^2$$

where we have used $h_2 = h_1, p_1 = 0, V_2 = 0$, and $\gamma = \rho g$. Multiply by the area and find the force to be

$$F = pA$$

$$= 464 \times 1 \times 2 = 928\ \text{N}$$