Kinematic Analysis of Structures

(Strain analysis)

Deformation of rocks under stress

Topics

1. Rigid-body and non rigid-body deformations
2. Measurement of strain
3. Strain ellipse and strain equations
4. Pure shear and simple shear
5. Stress-strain relationships in rocks
6. Determination of strain in rocks

Concept of detailed structural analysis

In studying basic structures and structural systems, we base our work on a branch of structural geology known as:

detailed structural analysis

with particular emphasis on strain analysis.

Three fundamental interlocking strategies are harnessed in detailed structural analysis:

1. **descriptive analysis,**
2. **kinematic analysis and,**
3. **dynamic analysis.**

Each of these looks at geologic structures from a different point.

Descriptive analysis is concerned with recognizing and describing structures and measuring their :

- **location,**
- **geometries, and**
- **orientations.**

We describe, literally inside and out, their physical and geometric components.

Kinematic analysis focuses on interpreting the deformational movements responsible for the development of the structures.

The basic movements are :

- **translation** (changes in position),
- **rotation** (change in orientation)
- **distortion** (change in shape), and
- **dilation** (change in size)

(figure 1.25)

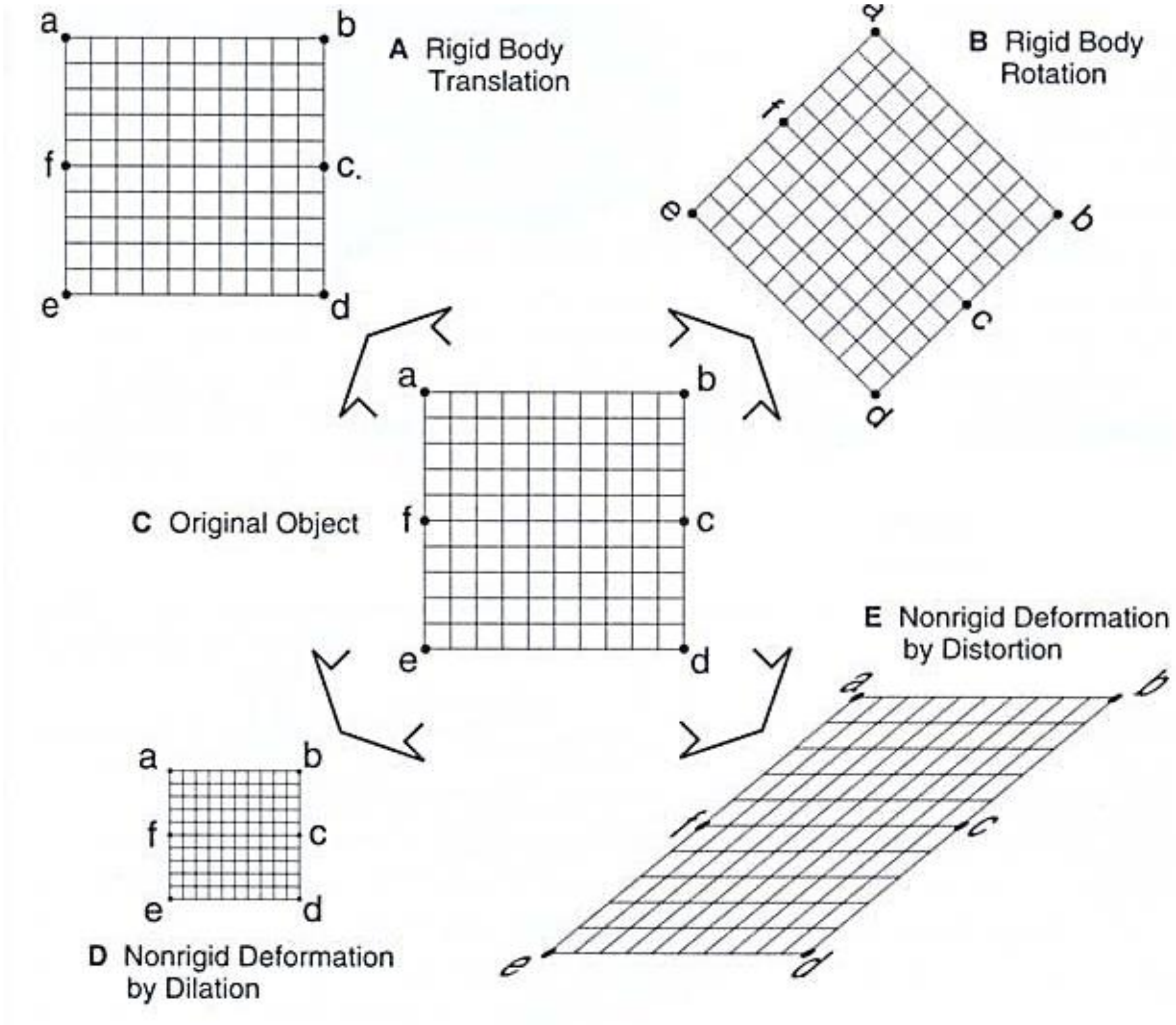


Fig. 2.2. **Originally undeformed body (C) in center of diagram (i.e., square abde) is deformed by (A) rigid body translation, (B) rigid body rotation, (D) non-rigid body dilation , and (E) non-rigid body distortion**

Dynamic analysis interprets:

deformational movements in terms of forces and stresses responsible for the formation of structures, as well as evaluating the strength of the materials during deformation.

Thus, Dynamic analysis interprets forces, stresses and mechanics that give rise to structures.

The major aim of the analysis is to describe the

- **orientation and**
- **magnitude**

of the stress, and the response of the material.

Kinematic analysis

Kinematic analysis takes off where descriptive analysis ends. It is about movements.

Kinematic analysis deals with recognizing and describing the changes that, during deformation, are brought about by movement of the body.

If some part of a body, is forced to change its location, or position, it undergoes translation (fig. 1.33A).

When forced to change its orientation, it undergoes rotation (figure 1.33B).

When forced to change shape, it undergoes distortion (fig. 1.33D).

Put figures here of Fig. 1.33a. 1/33B and 1.33C)

The overall goal of kinematic analysis is to interpret the combination of translation, rotations, distortions, and dilations that altered the location, orientation, shape and size of a body of rock.

Kinematic analysis is carried out at all scales, from submicroscopic to regional.

Evaluating changes in shape and size brought about by deformation is the focus of strain analysis.

Strain analysis is basic to modern structural analysis. It requires a quantitative evaluation of changes in original size and shape of the geologic objects.

Of main interest is the analysis of secondary deformation, like

- **the rotation of rock layers during folding,**
- **the displacements along a fault or within a fault system; or**
- **the opening up of wall rock as a dike is intruded.**

What is Rock Deformation?

It is a collective displacements of points in a body relative to an external reference frame

Thus Deformation is the transformation from a some initial to some final geometry by means of :

- **(rigidbody) translation,**
- **(rigidbody) rotation,**
- **strain (distortion), and/or**
- **volume change (dialation).**

Deformation of a rock body occurs in response to a force.

Deformation relates the positions of particles before and after the deformation history, and the positions of points before and after deformation can be connected with vectors.

These vectors are called displacement vectors, and a field of such vectors is referred to as the displacement field.

Components of Deformation

Deformation involves any one, or a combination, of the following four components:

Ways that rocks respond to stress:

1. **Translation**
2. **Rotation**
3. **Dilation**
4. **Distortion**

} **Strain**

Rigid and non-rigid body deformation

There are two different strategies for kinematic analysis of deformational structures, and which one is used depends upon whether the rocks under consideration have behaved:

- **as rigid or**
- **as non-rigid bodies**

during the deformation.

The distinction between rigid and non-rigid rests on:

- whether the rock body, at the scale of observation, moves intact without a change in shape or size (rigid body deformation),

- or with a change in shape or size (non-rigid body deformation).

The test is whether each point in the body, during the deformation, maintains the same exact location relative to neighboring points.

If so it is a rigid body deformation.

If not, it is a non-rigid body deformation.

1. Rigid body deformation

During *rigid body deformation*, rock is: translated and or rotated in such a way that original size and shape are preserved.

A schematic example of rigid body translation without rotation is portrayed in figure 2.2A. Block *abde* has shifted its position, but has changed in such a way that its original size and shape are maintained.

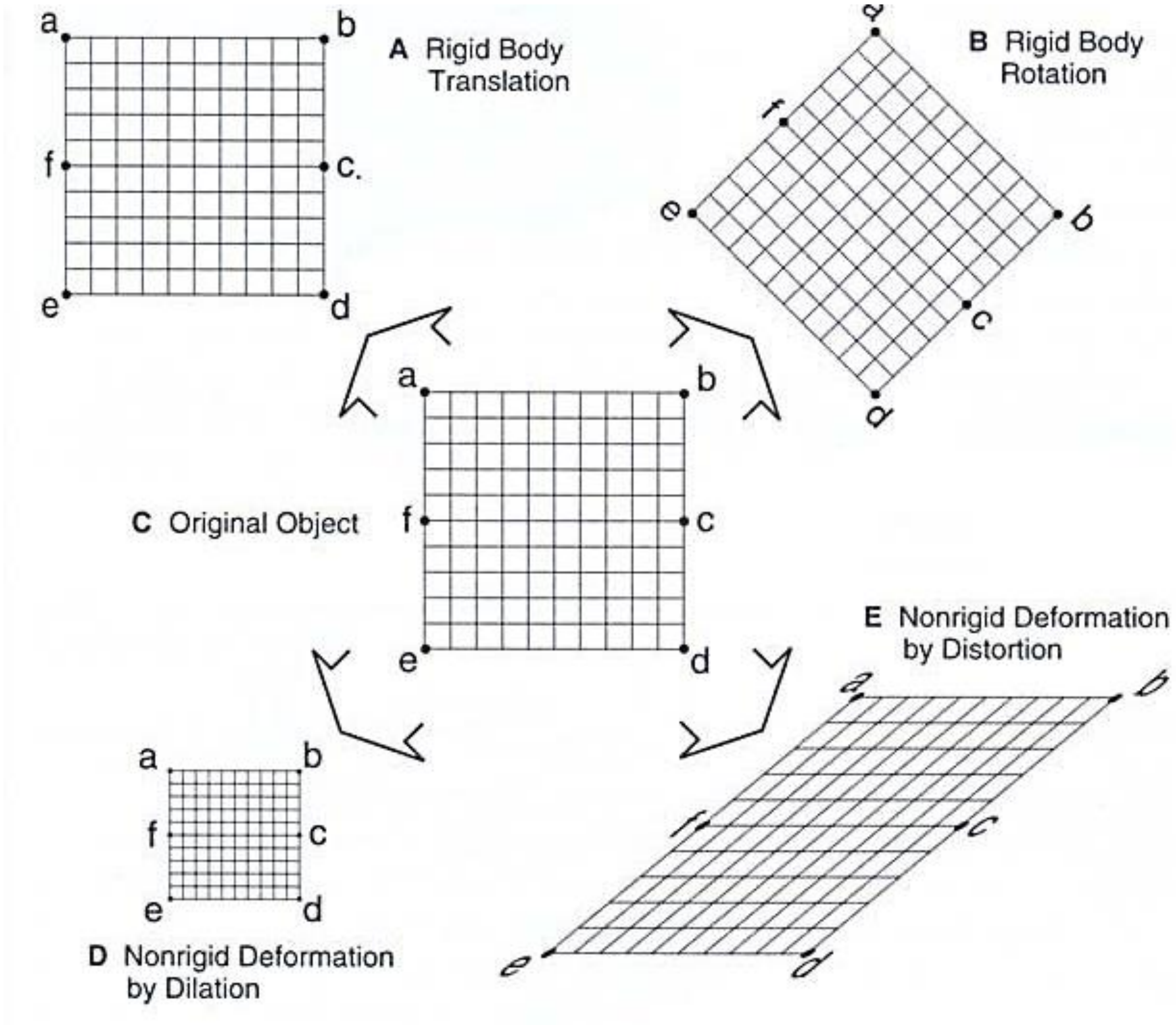
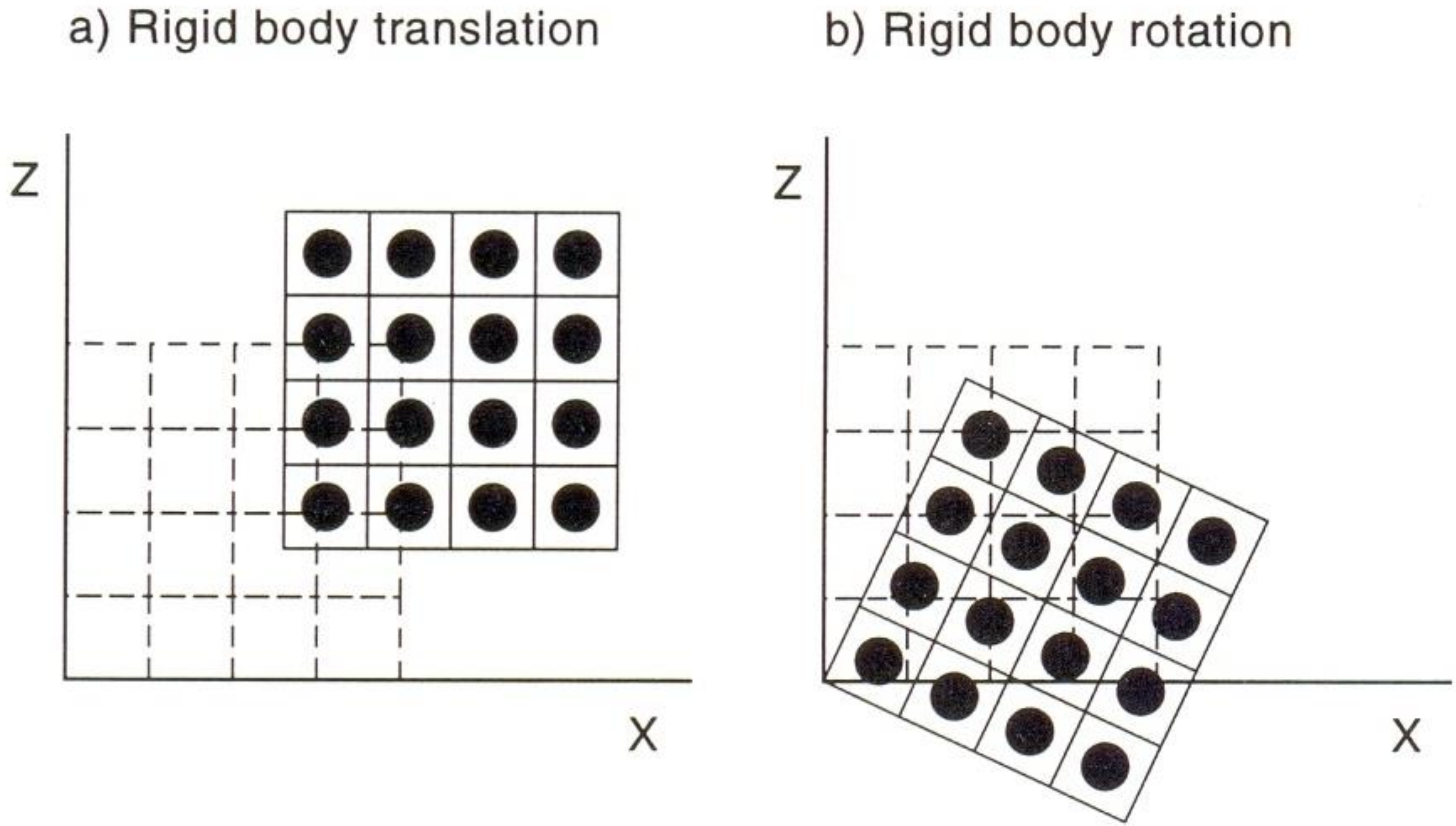


Fig. 2.2. Originally undeformed body (C) in center of diagram (i.e., square abde) is deformed by (A) rigid body translation, (B) rigid body rotation, (D) non-rigid body dilation , and (E) non-rigid body distortion

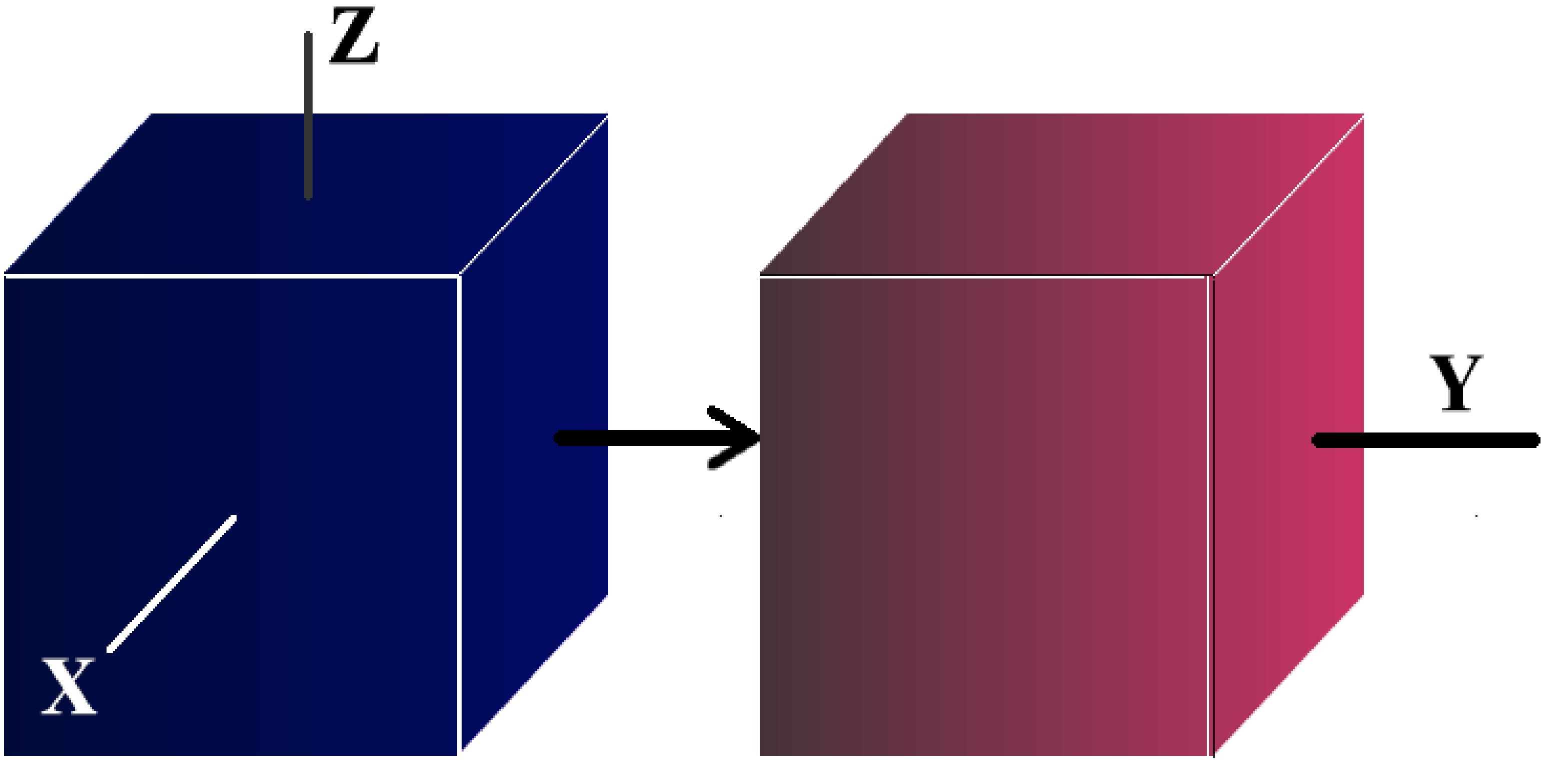
There is no change in the configuration of points within the block.

Similarly, Fig. 2.2B portraits rigid body deformation, in this case a rotation.

Again, there is no change in the configuration of points within the block.


a) Rigid body translation
Z
X
b) Rigid body rotation
Z
X

Translation Parallel to the Y axis



Rigid Body Translation

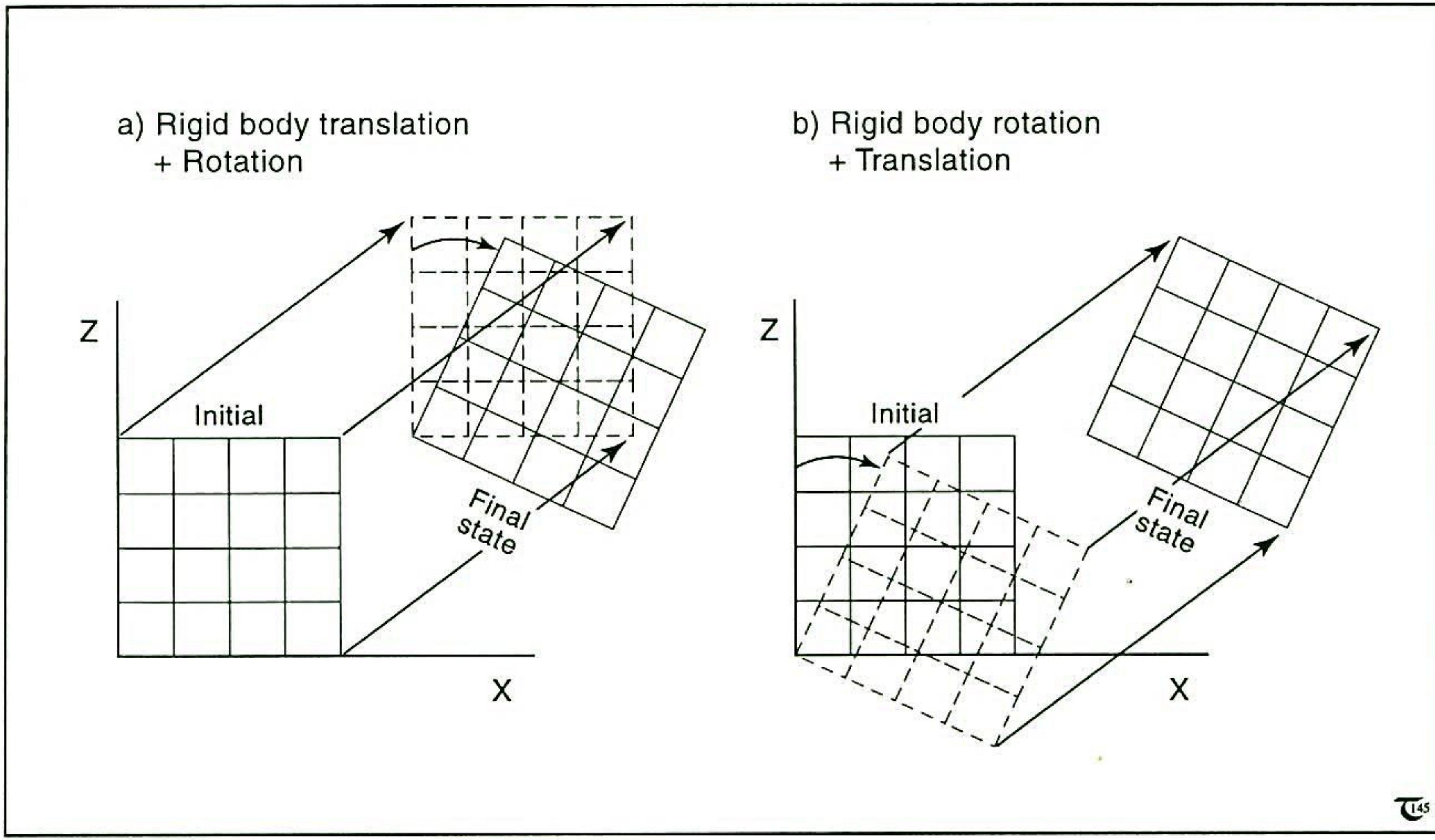


Figure 12-11: a) & b) Rigid body rotation and translation are commutative deformation processes.

A. Translation

During pure translation, a body of rock is displaced in such a way that all points within the body move along parallel path relative to some external reference frame.

Sliding down a steep snow-covered slope on a really stiff cafeteria tray is an example of pure translation, provided you do not "fishtail" or spin out (Figure 2.7).

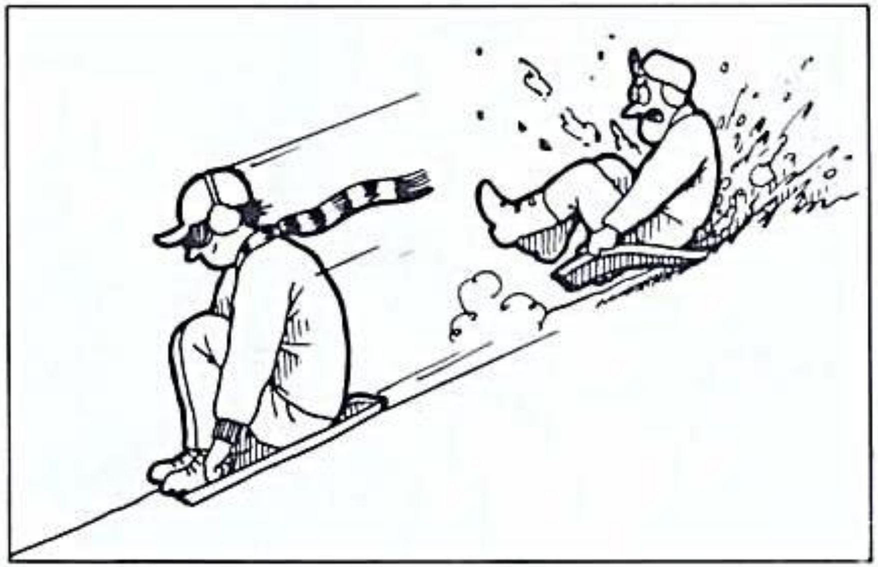


Figure 2.7 If the movement is purely rigid body translation, all the points in the body will move exactly the same distance along exactly the same path. If not, ouch! (Artwork by D. A. Fischer.)

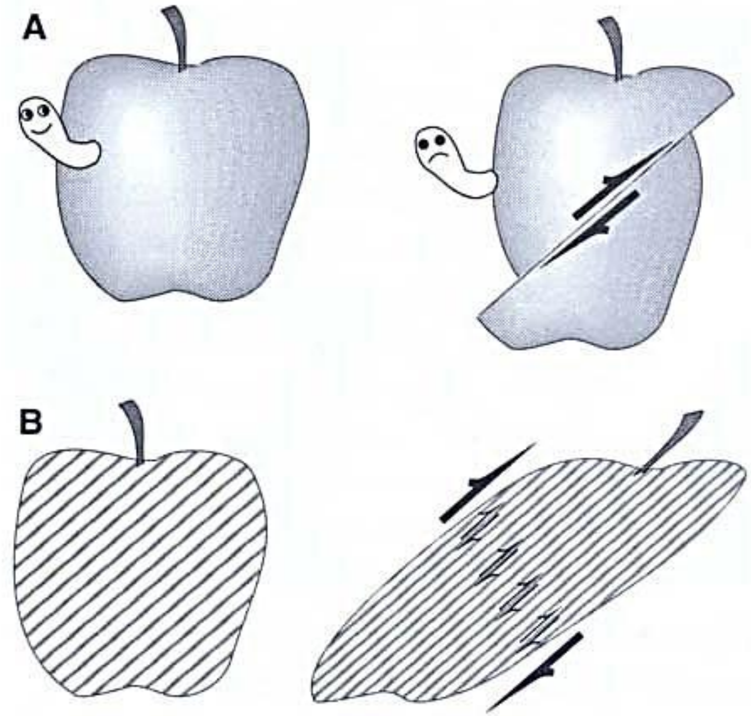


Figure 2.6 (*A*) Rigid versus (*B*) non–rigid deformation of objects by faulting is partly a matter of the closeness of spacing of the structures within the chosen field of view.

Thus, during a rock body deformation involving movement of the body from one place to another, i.e., change in position (translation)

- **Particles within the body do not change relative position**
- **No rotation or strain are involved**
- **e.g., passengers in a car, movement of a fault block**

Rigid bodies translate past one another along surfaces where the integrity of the rock body, at whatever scale is interrupted, and along which movement is possible.

Surfaces that accommodate translation in the geologic world include faults, joints, and bedding planes.

Translation can be considered at extreme scales as well;

- **on the one hand, the world;**
- **on the other hand, the crystal lattice.**

The concept of plate tectonics, is a view of the entire outer shell of the earth composed of an array of broad, thin rigid plates of crust and uppermost mantle (lithosphere).

The rigid plates translate (and rotate) with respect to one another, moving on hot, ductile, non-rigid mantle material (asthenosphere). The plate motions are described mathematically as a rigid body translations and rotations.

At the opposite end, individual tiny crystals are built in a lattice framework of atoms. We will see that suitably oriented layers of atoms within a lattice may translate with respect to other layers when forced to do so.

B. Rotation

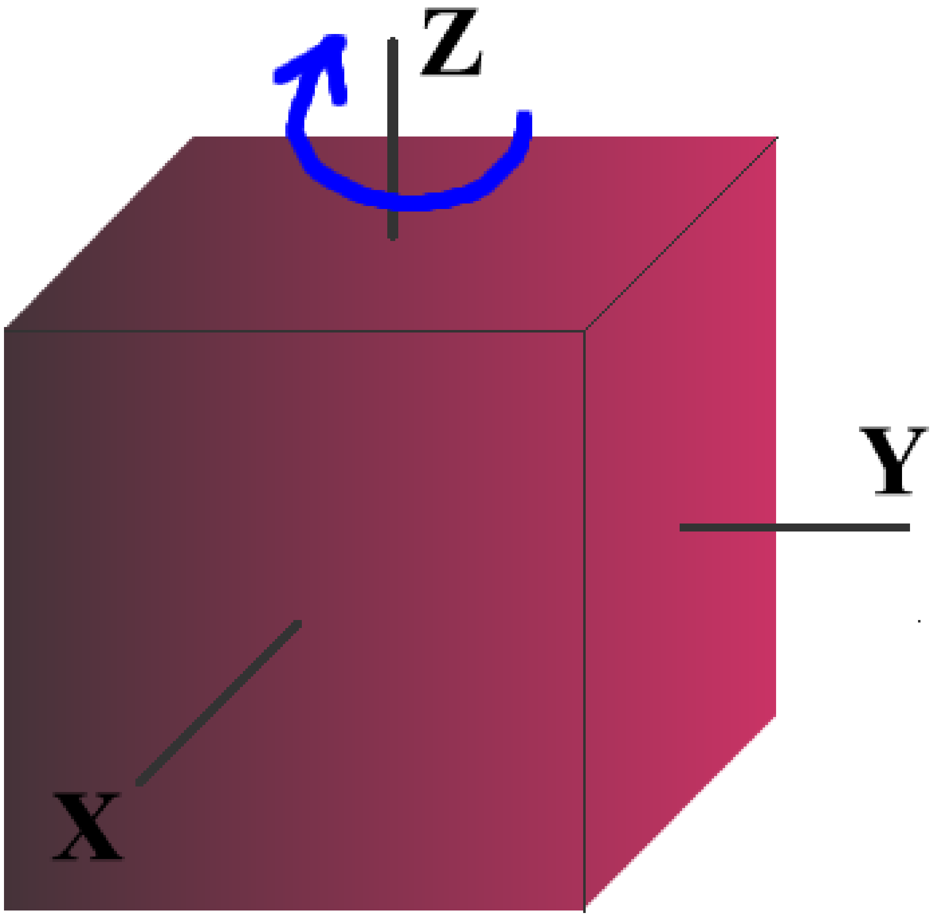
Rotation is a rigid body operation that changes the configuration of point in a way best described by rotation about some common axis.

The changes in locations of points are described by:

1. **The orientation of the axis of rotation (trend and plunge),**
2. **the sense of rotation (clockwise vs. counterclockwise), and,**
3. **the magnitude of rotation (measured in degrees).**

Based on this set of facts, the location of points before and after rotation can be calculated.

Clockwise Rotation about the z-axis



Rigid body Rotation

Spin of the body around an axis

- **Particles within the body do not change relative position**
- **No translation or strain is involved**
- **Particle lines rotate relative to an external coordinate system**
  - **Examples**
    - **Rotation of a car**
    - **Rotation of a fault block**

2. Non-rigid body deformation (Strain)

Strain **is a change in size or shape in response to stress**

Strain **results from non-rigid body deformation when rock undergoes a:**

A. Change in shape (distortion) and or
B. Change in size or volume (dilation)

Strain expressed as distortion or dilation results from a change in configuration of points within a body.

A single body, during a single deformational event, may experience both dilation and distortion.

Points within strained bodies do not retain their original spacing and configuration relative to one another. The original spacing of points within the body is changed.

A. Distortion or Strain

Distortion is an operation that involves the change in the spacing of points within a body of rock in such a way that the overall shape of the body is altered with or without a change in volume.

Changes of points in body relative to each other

- **Particle lines may rotate relative to an external coordinate system**

Example: squeezing a paste

Where pure dilation takes place without change in shape, internal points of reference spread apart or pack closer together in such a way that line lengths between points become uniformly longer or shorter.

Overall shape remains the same.

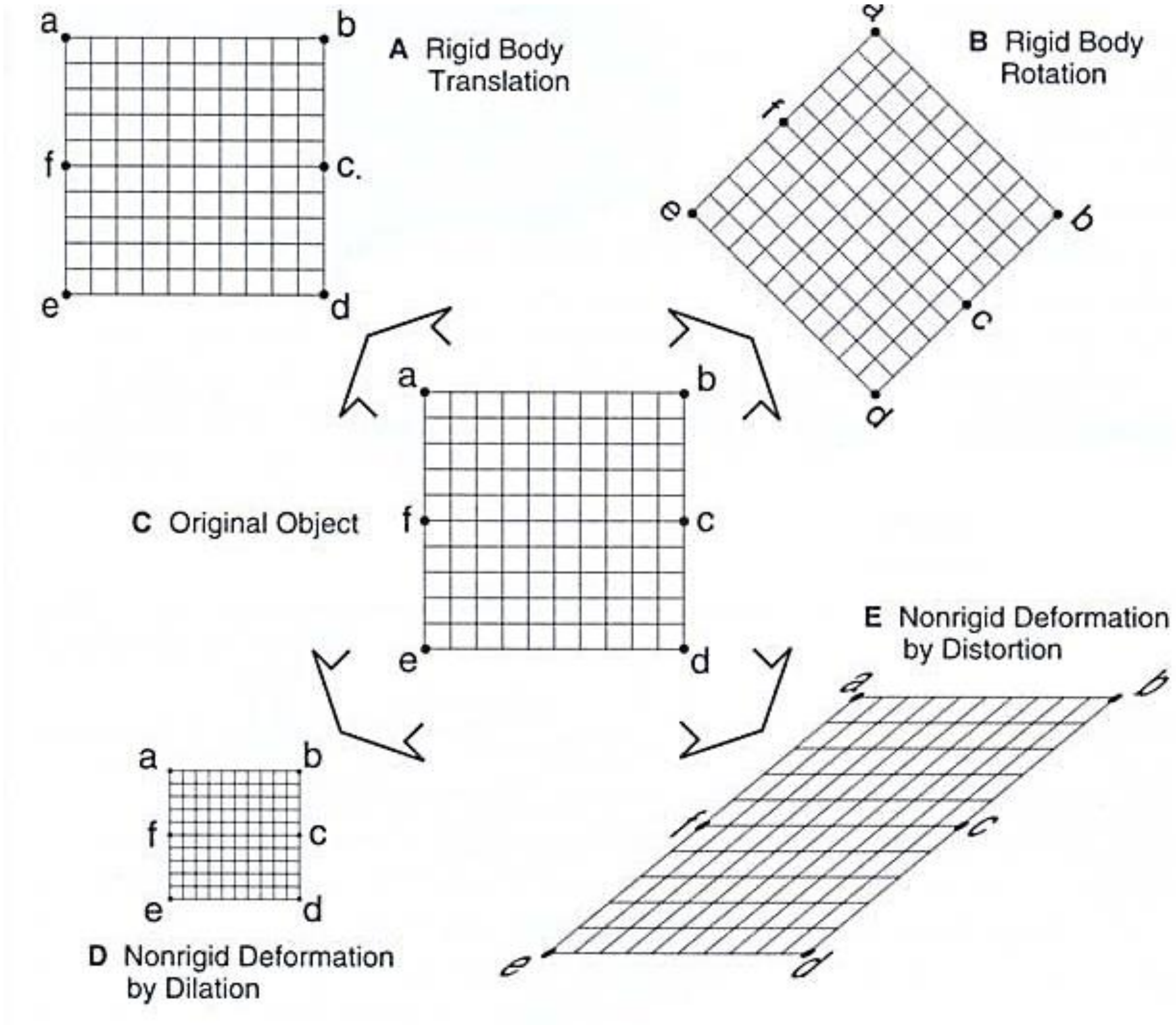


Fig. 2.2. Originally undeformed body (C) in center of diagram (i.e., square abde) is deformed by (A) rigid body translation, (B) rigid body rotation, (D) non-rigid body dilation , and (E) non-rigid body distortion

During distortion, the changes in spacing of points in a body are such that the overall shape of the body is altered, with or without a change in size/volume (figure 2.2).

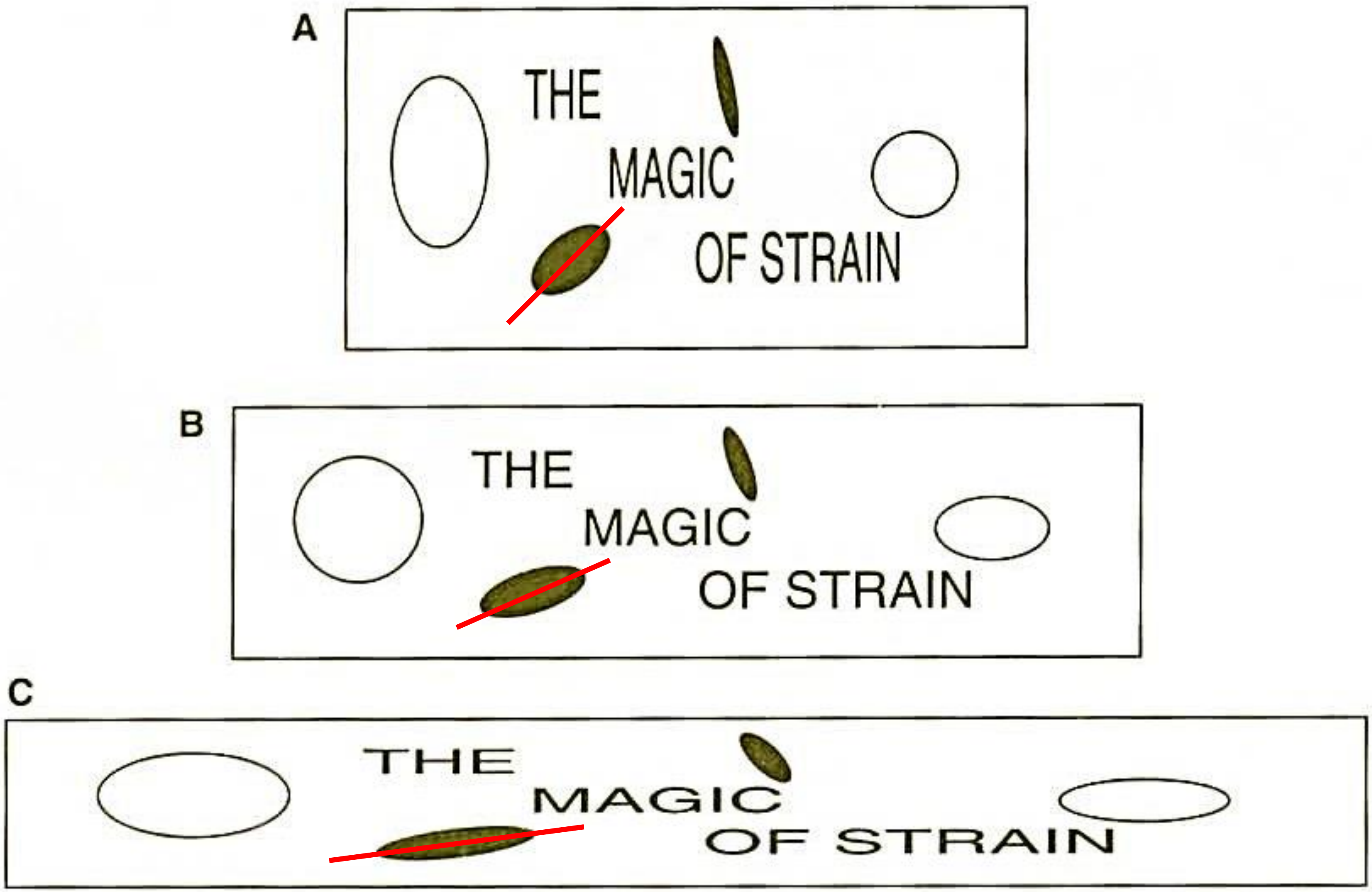
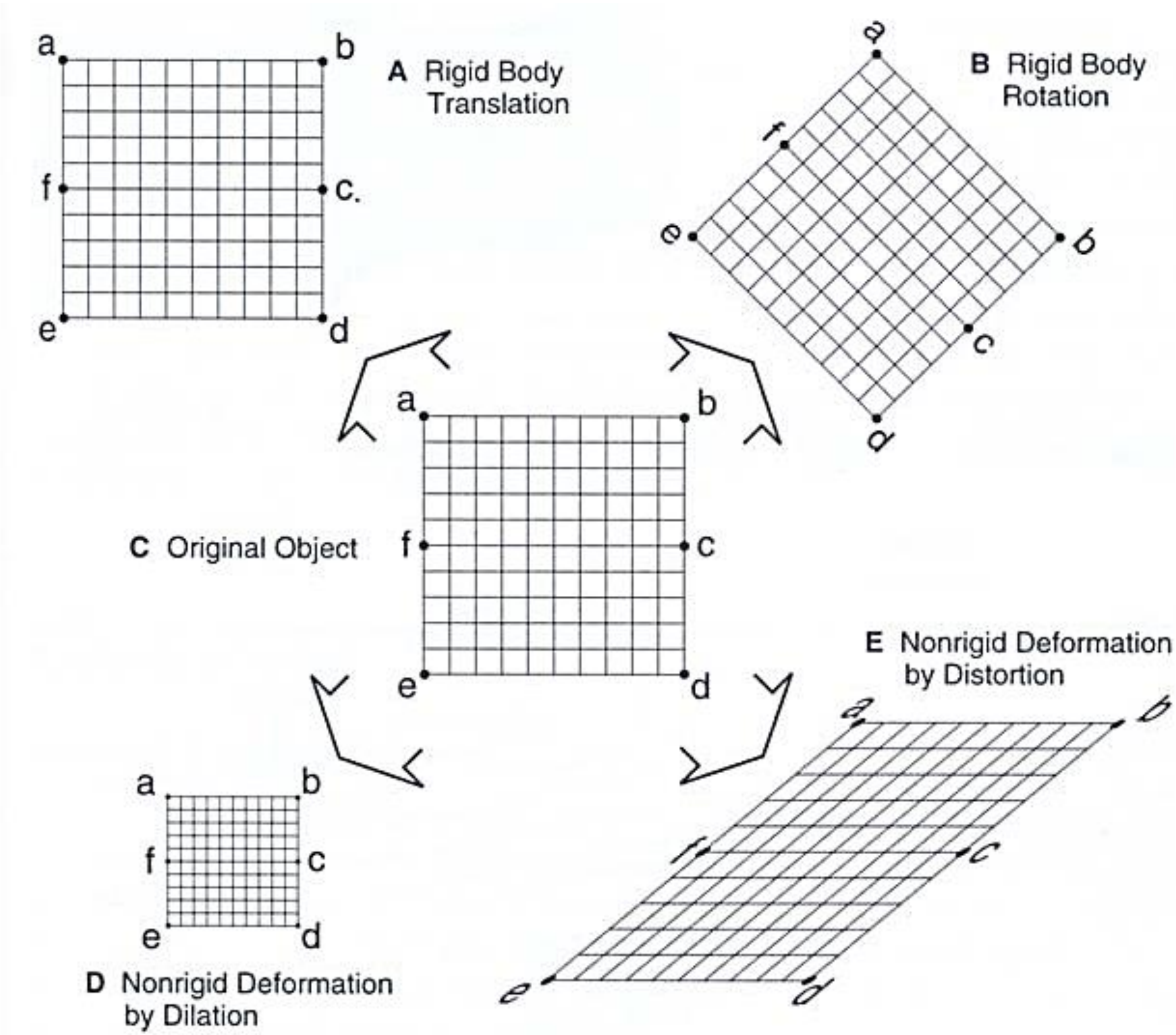


Figure 2.26. (A) A deformable block is embossed with a circle, a vertically oriented ellipse, two black ellipses, and words. (B) When flattened and extended, changes in the shapes and orientations of the reference objects on the front face of the block record the nature of the internal strain. The amount of flattening and stretching is just enough to transform the original vertical ellipse into a perfect circle. (C) With even more flattening tighter and tigher; the two black ellipses continuously rotate toward the direction of stretching; and the letters of the words continuously change font.



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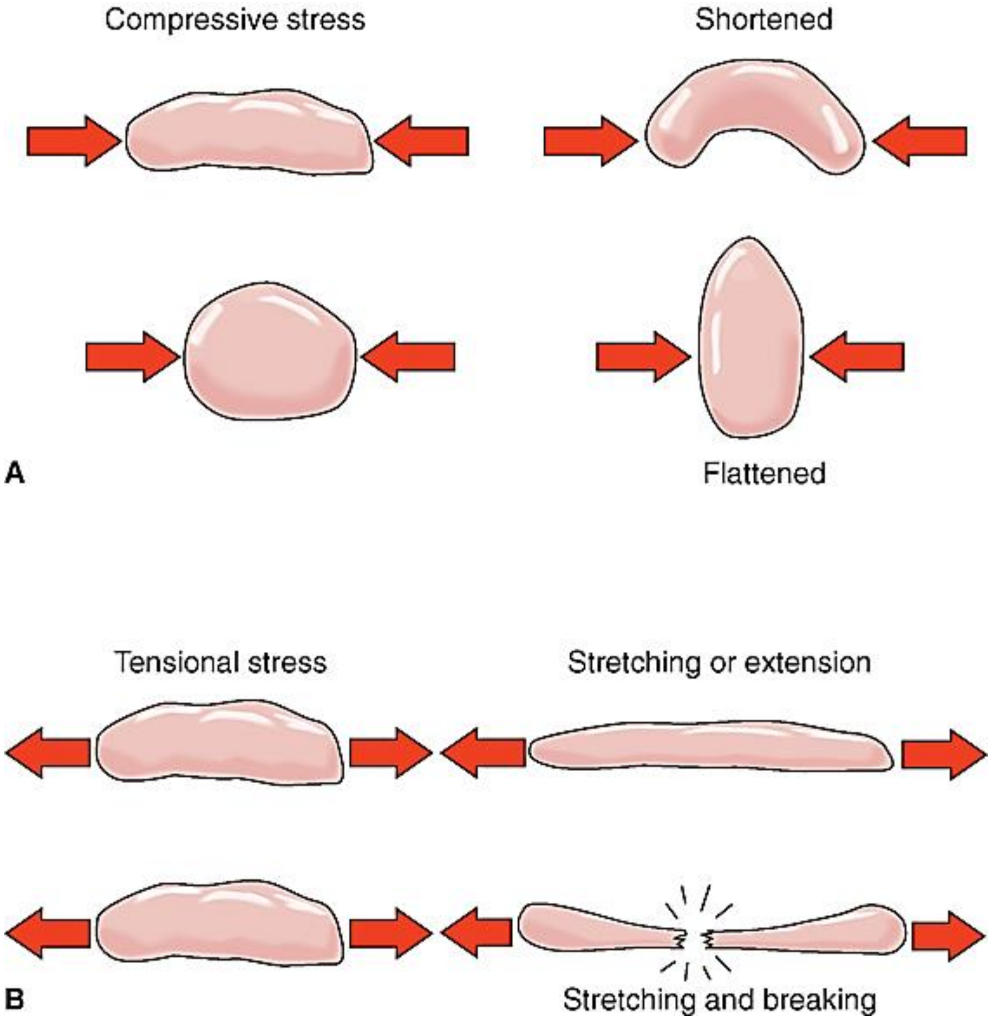
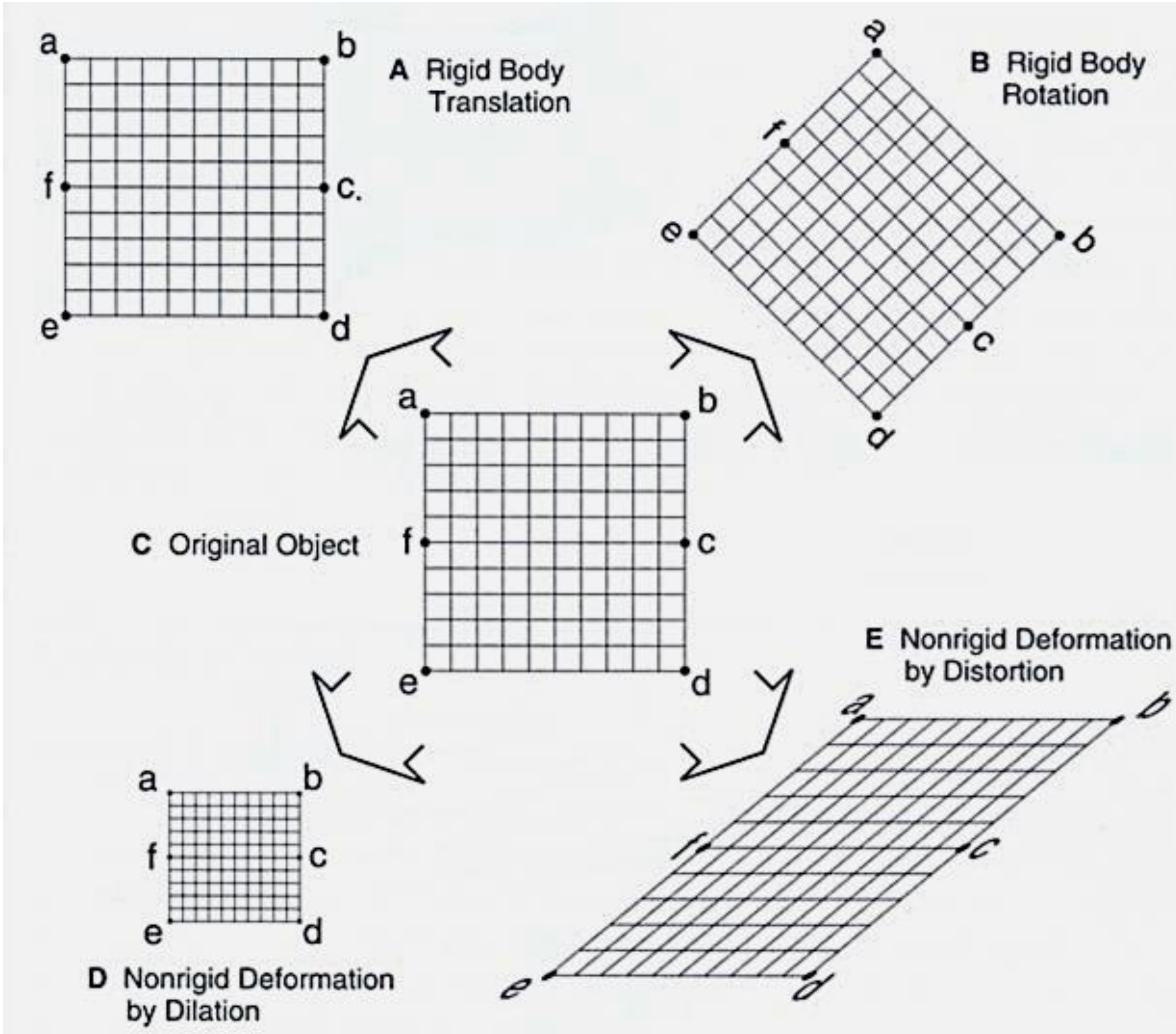
Material Strain



Pure distortion is a change in a shape without a change in size. As shown in figure **2.2E**, the change in shape of a body from a square to a rhomb is made possible by systematic changes in spacing between points in the body.

The distance between points *a* and *e* increases from 10 units to 16 units. Angular relations between alignment of points change as well. For example, angle *aed* is reduced from 90° to 40°.



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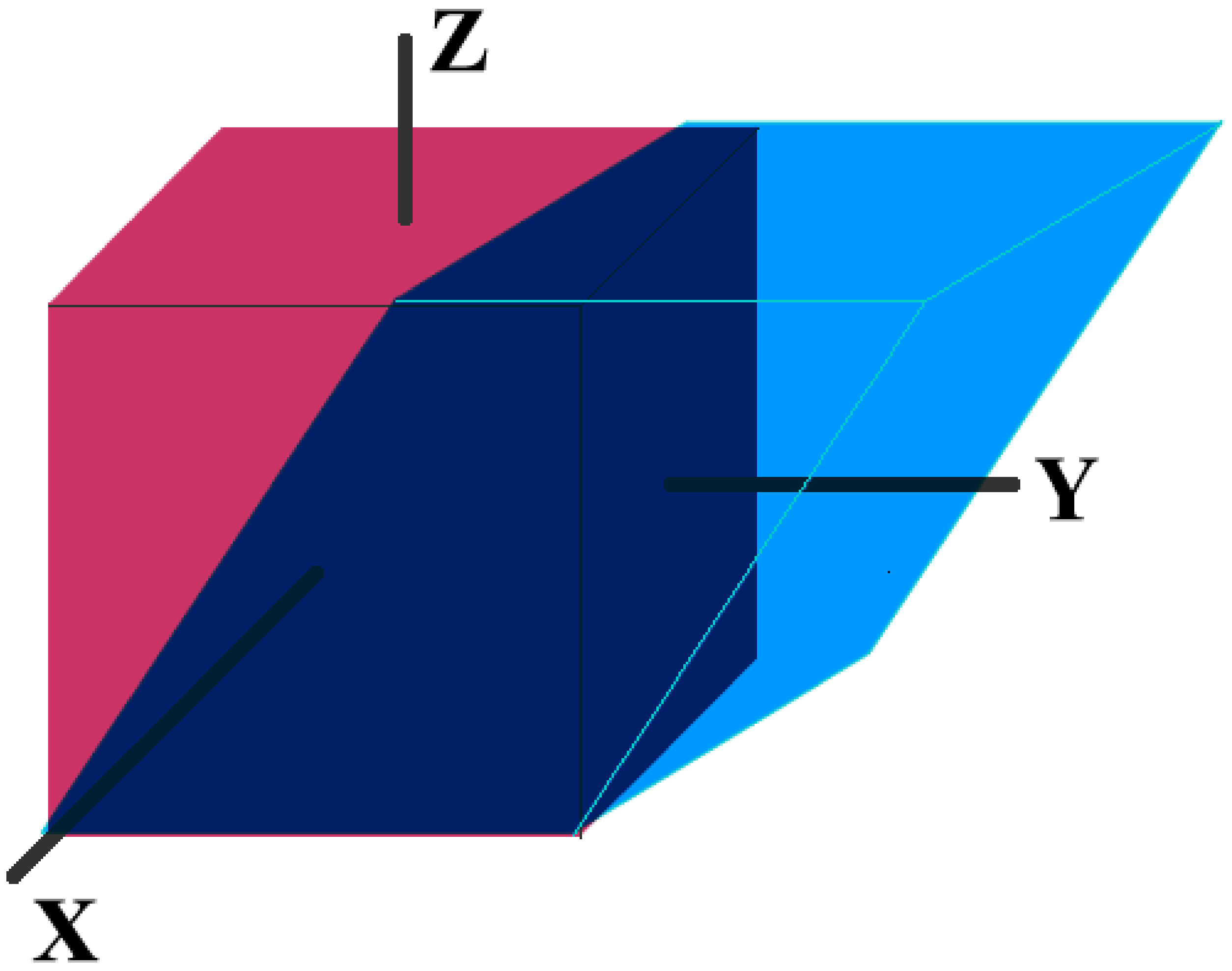
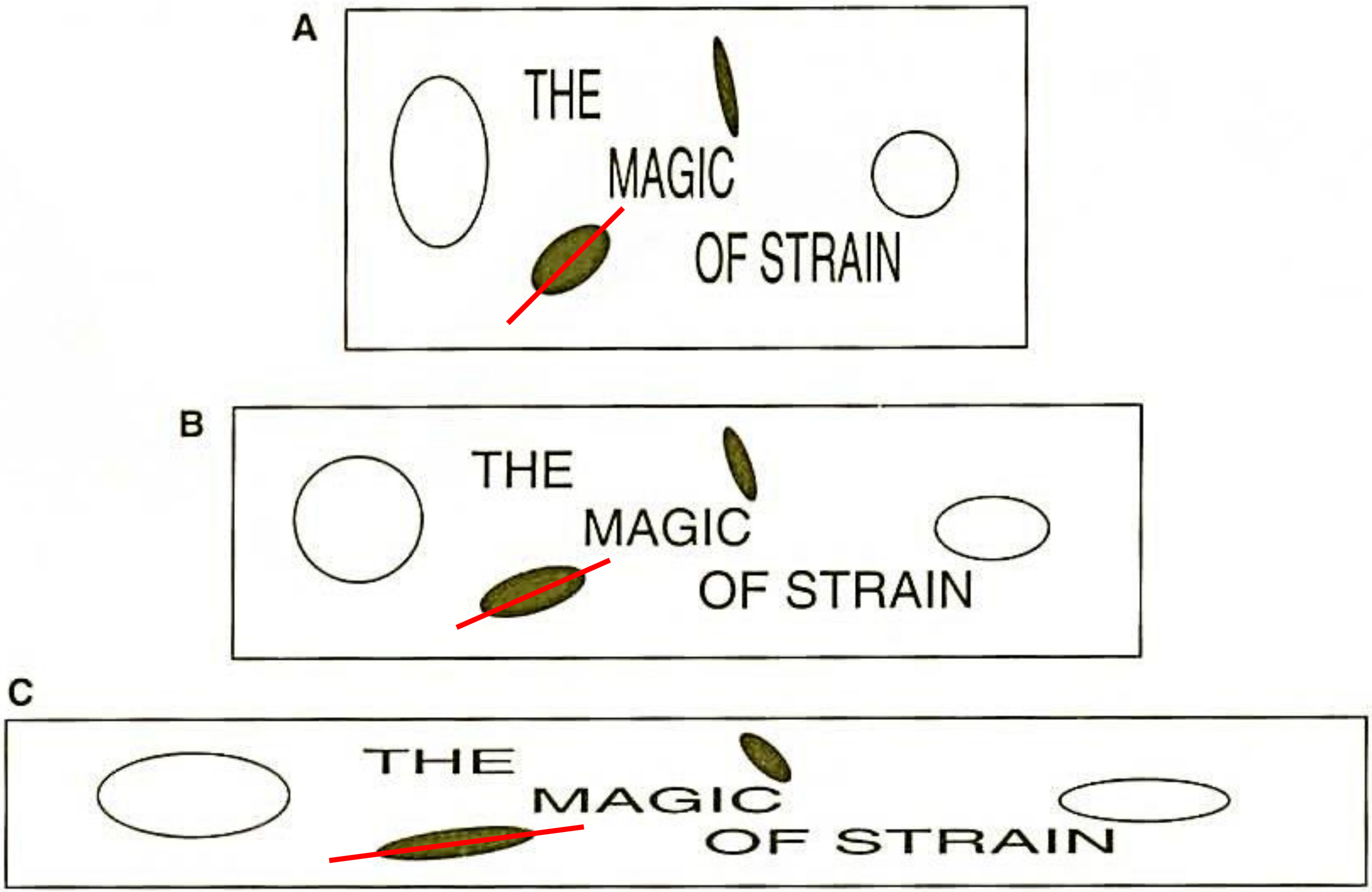
Strain or Distortion



When a non-rigid body deformation results in the systematic distortion of what we normally regard as solid rock, the results can be unusually interesting.

A circles changes into ellipse through non-rigid body distortion.

However, in rocks we deal with processes that lead to both movement and distortion.



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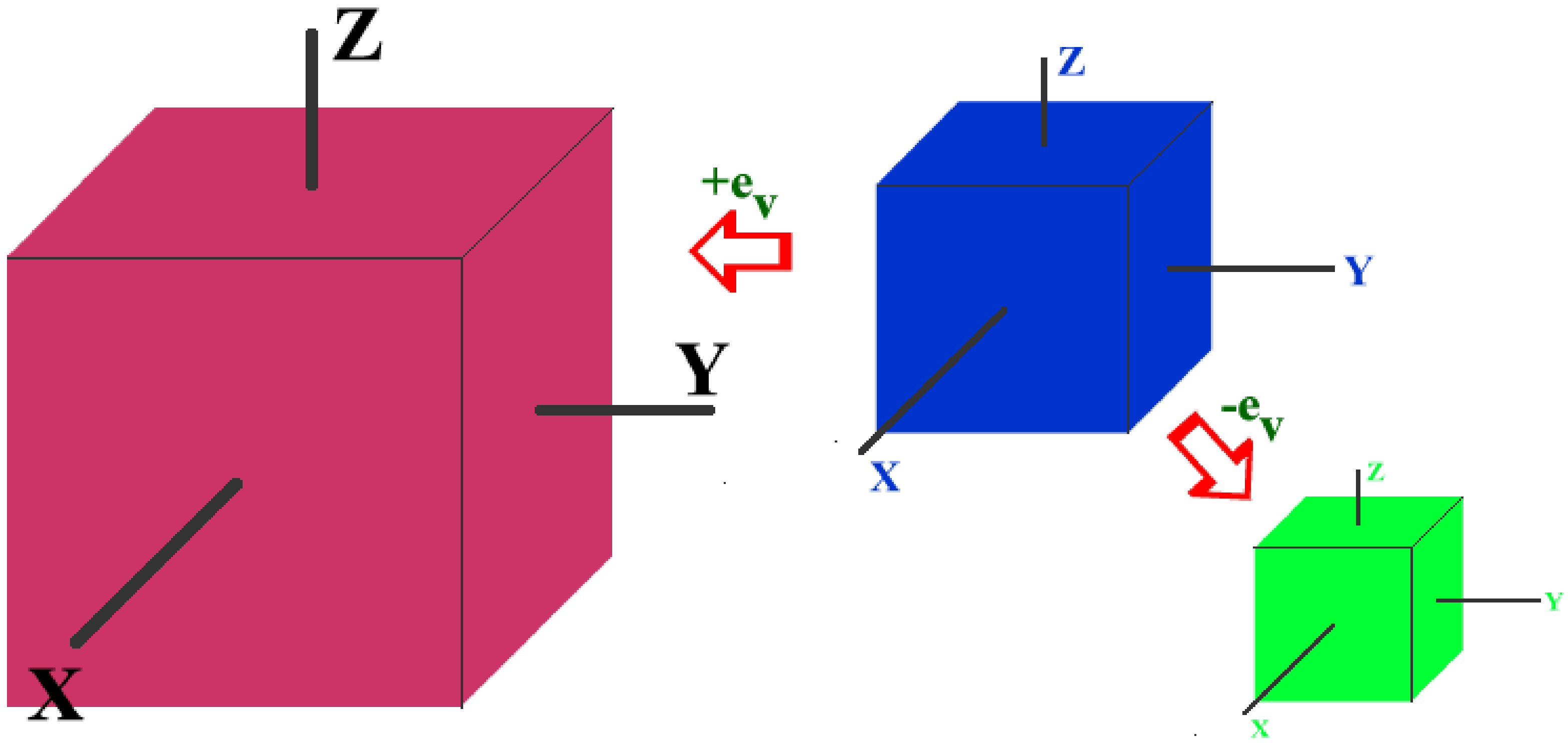
B. Dilation

It is an operation involving a change in volume or size.

Pure dilation is a change in size without a change in shape. Fig 2.2D pictures a decrease in size (negative dilation): the spacing of points within the original body is cut in half.

Pure Dilation



In pure dilation:

- The overall shape remains the same
- Internal points of reference spread apart or pack closer together
- Line lengths between points become uniformly longer or shorter

Significant dilation accompanies such non-rigid structural processes as:

- **the shrinkage of mud to produce mud cracks,**
- **the compaction of sediments during burial to produce thinning,**
- **the cooling of basalt to produce columnar joints,**

Homogeneous
and
Inhomogeneous strains

The chief constraints that the "homogeneous deformation" clause imposes are:

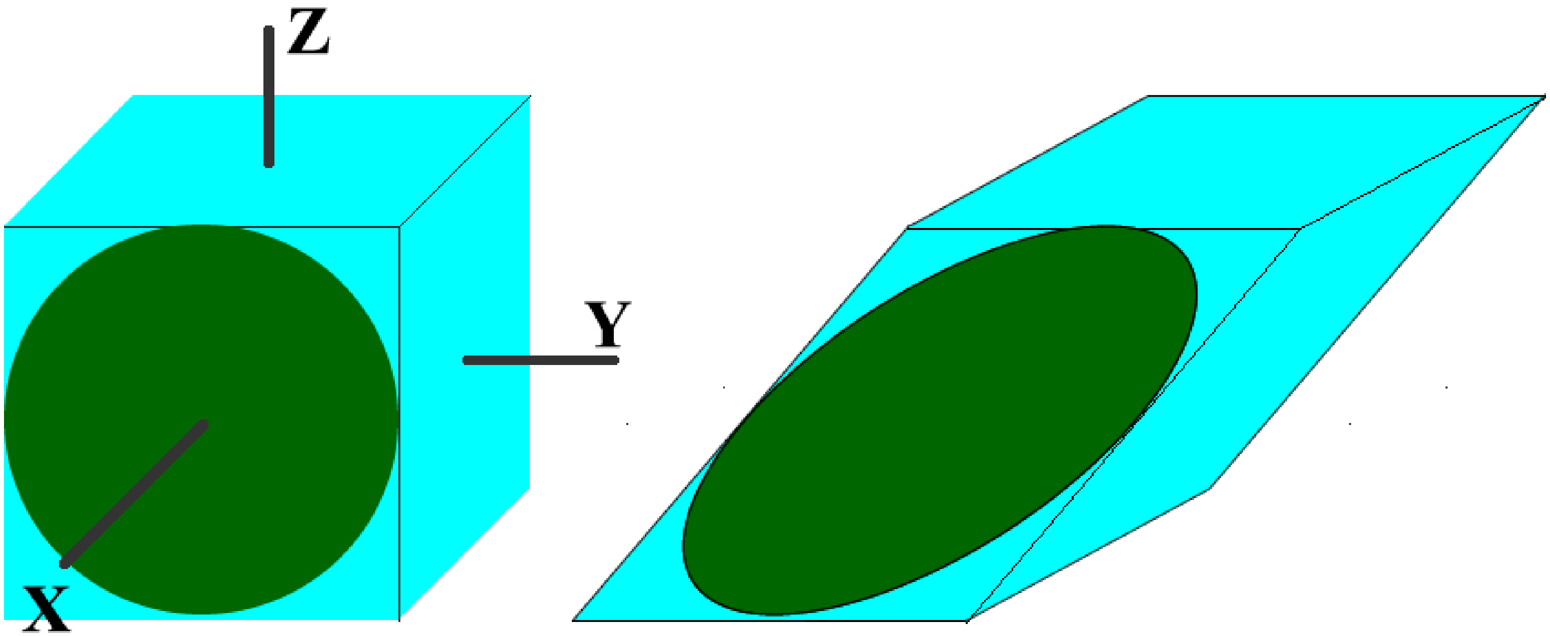
1. **Straight lines that exist in the non-rigid body before deformation remain straight after deformation**
2. **Lines that are parallel in the body before deformation remain parallel after deformation.**

For these conditions to hold, the strain must be systematic and uniform across the body that has been deformed. A simple test for homogeneity will become obvious; homogeneous deformation transforms:

- **perfect circles into perfect ellipses, and**
- **perfect spheres into perfect ellipsoids.**

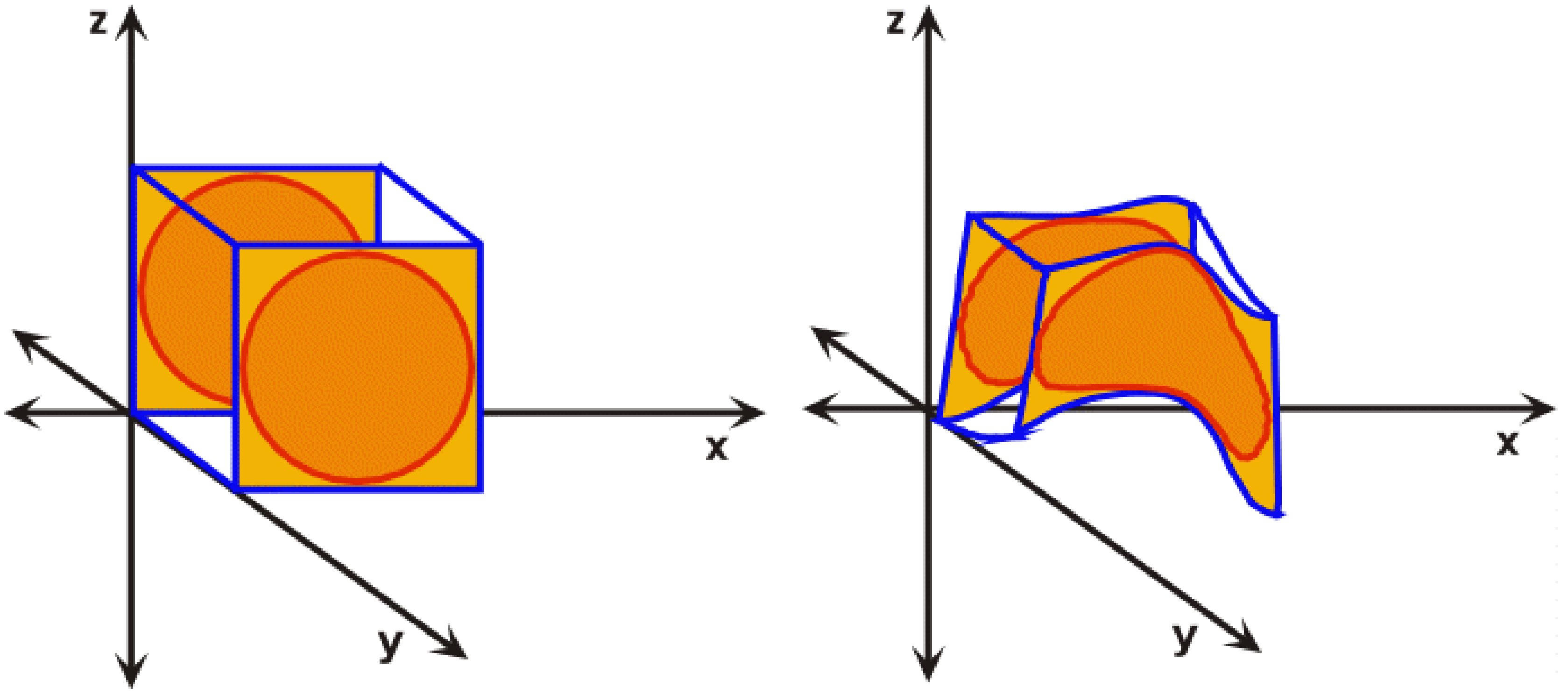
Strained rocks that we find in nature typically depart from the rules of homogeneous deformation.

Homogeneous Strain



Heterogeneous or Inhomogeneous strain

Leads to distorted complex forms



Measurement of strain

General concept

1. Change of length (extension and stretch (e and s)
2. Angular shear (Psi ψ),
3. Shear strain (gamma γ)

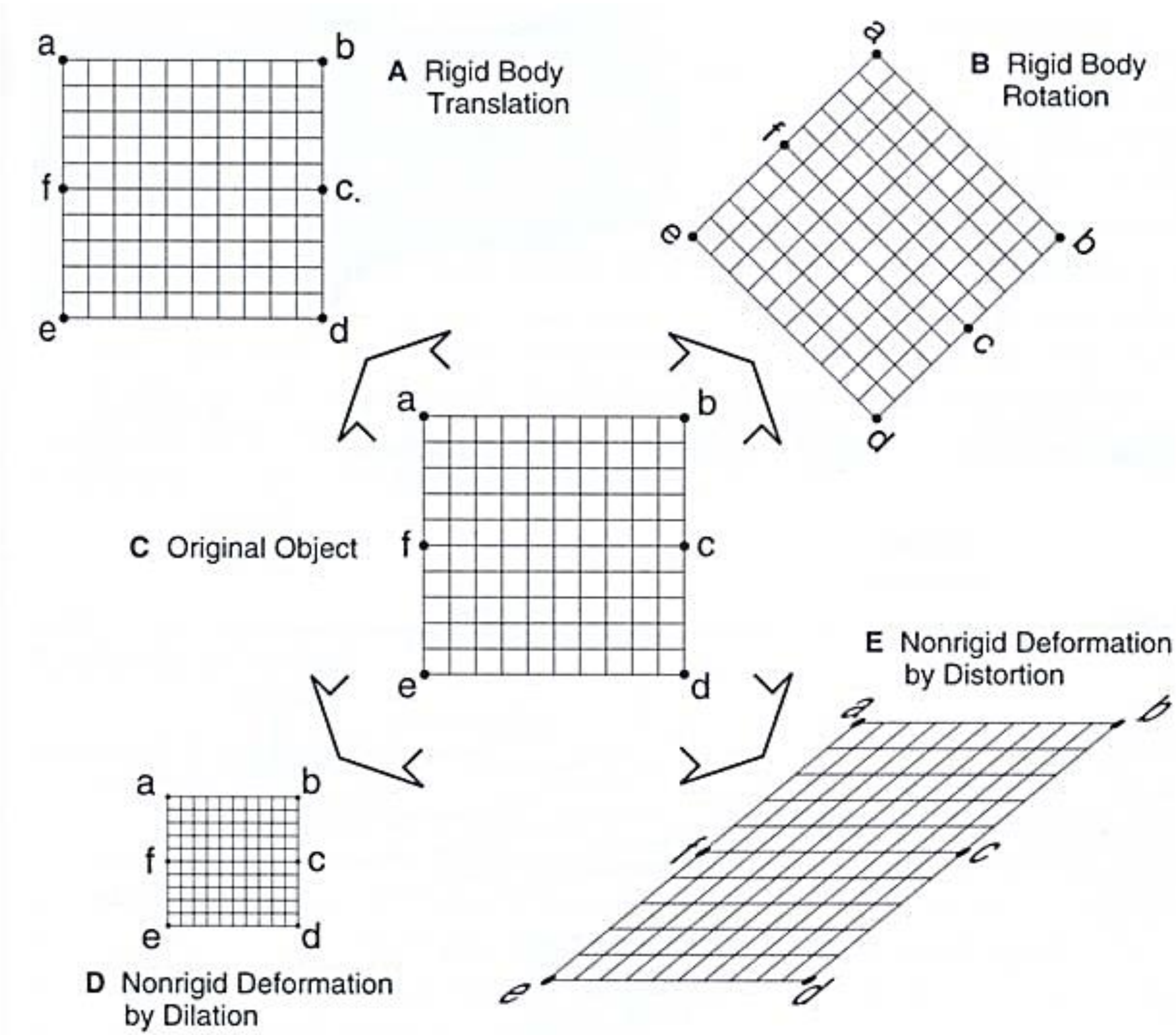
Strain produces non-rigid body deformation

1. **dilation, a change in size, and**
2. **distortion, a change in shape.**

Points within strained bodies do not retain their original spacing and configuration relative to one another. The original spacing of points within the body is changed.

Where pure dilation takes place without change in shape, internal points of reference spread apart or pack closer together in such a way that line lengths between points become uniformly longer or shorter. Overall shape remains the same.

During distortion, the changes in spacing of points in a body are such that the overall shape of the body is altered, with or without a change in size/volume (figure 2.2).



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The ground rules

We simplify our work in strain analysis by studying the theory in two, and not three, dimensions. A customary simplification is to restrict strain analysis to the description of *homogeneous deformation*.

It is almost impossible to apply mathematical theory to unwieldy, irregularly deformed, *heterogeneous* structural systems. Instead, strain analysis focuses on the properties of bodies, or parts of bodies, that have deformed in a regular, uniform manner.

The chief constraints that the “homogeneous deformation” clause imposes are:

1. **Straight lines that exist in the non-rigid body before deformation remain straight after deformation**
2. **Lines that are parallel in the body before deformation remain parallel after deformation.**

For these conditions to hold, the strain must be systematic and uniform across the body that has been deformed. A simple test for homogeneity will become obvious; homogeneous deformation transforms:

- **perfect circles into perfect ellipses, and**
- **perfect spheres into perfect ellipsoids.**

Strained rocks that we find in nature typically depart from the rules of homogeneous deformation.

The strain ellipse

Given the perfection of ellipses derived from homogeneous deformation (distortion) of circles and ellipses, it is no wonder that the strain within geologic bodies is conventionally described through the image of a strain ellipse.

A strain ellipse pictures the distortion accommodated by a geologic body.

It pictures how the shape of an imaginary circular reference object, or perhaps a not-so-imaginary circular geologic object, would be changed as a result of distortion.

We distinguish two kinds of strain ellipse:

- **the *instantaneous* ellipse and**
- **the *finite strain* ellipse.**

An instantaneous strain ellipse is used to portray how a circle is affected by a tiny increment of deformation. The ellipse should be nearly circular because it represents infinitesimally small amounts of strain.

The finite strain ellipse represents the total strain experienced by a circle that has been deformed.

It is the final result of deformation, which we normally see as geologists. It is the summation of all the incremental components.

Looking at Lines inside an Ellipse

The shape and orientation of an ellipse can be constructed on the basis of orientations and lengths of lines that pass through the centre of the ellipse and connect with the perimeter.

Let us take an ellipse of our choice (fig. 2.27A). and construct within it a set of perpendicular lines that intersect at the "centre" of the ellipse (figure 2.27B).

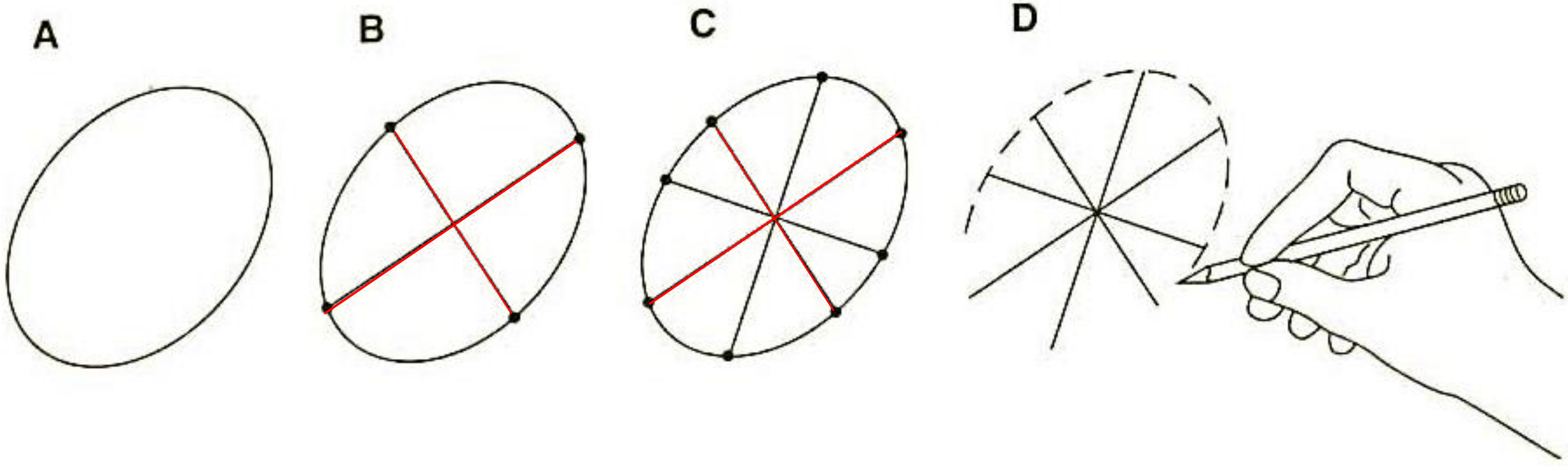


Fig. 2.27. (A) A perfect ellipse “contains” (B) sets of mutually perpendicular lines of just the right lengths and orientations. (c) If we knew the lengths and orientations of just two such sets of mutually perpendicular lines, (D) we could reasonably draw the ellipse.

Of course, each of these two lines has the perfect length just to touch, but not extend beyond, the perimeter of the ellipse.

If we were given the two sets of mutually perpendicular lines (figure 2.27C), we could construct the size and shape and orientation of the ellipse within which they lie (figure 2.27D).

The mastery of strain analysis requires keeping track of changes in the orientations and length of lines and keeping track of the angles between lines.

If we learn to this, we do not have to rely exclusively on the relatively uncommon occurrence of perfect circular or spherical objects being transformed and preserved in the geologic record as perfect ellipses or ellipsoids.

Instead, we can describe the state of strain in a deformed body based on a surprisingly small amount of information bearing on changes in lengths in lines and changes in the angles between lines.

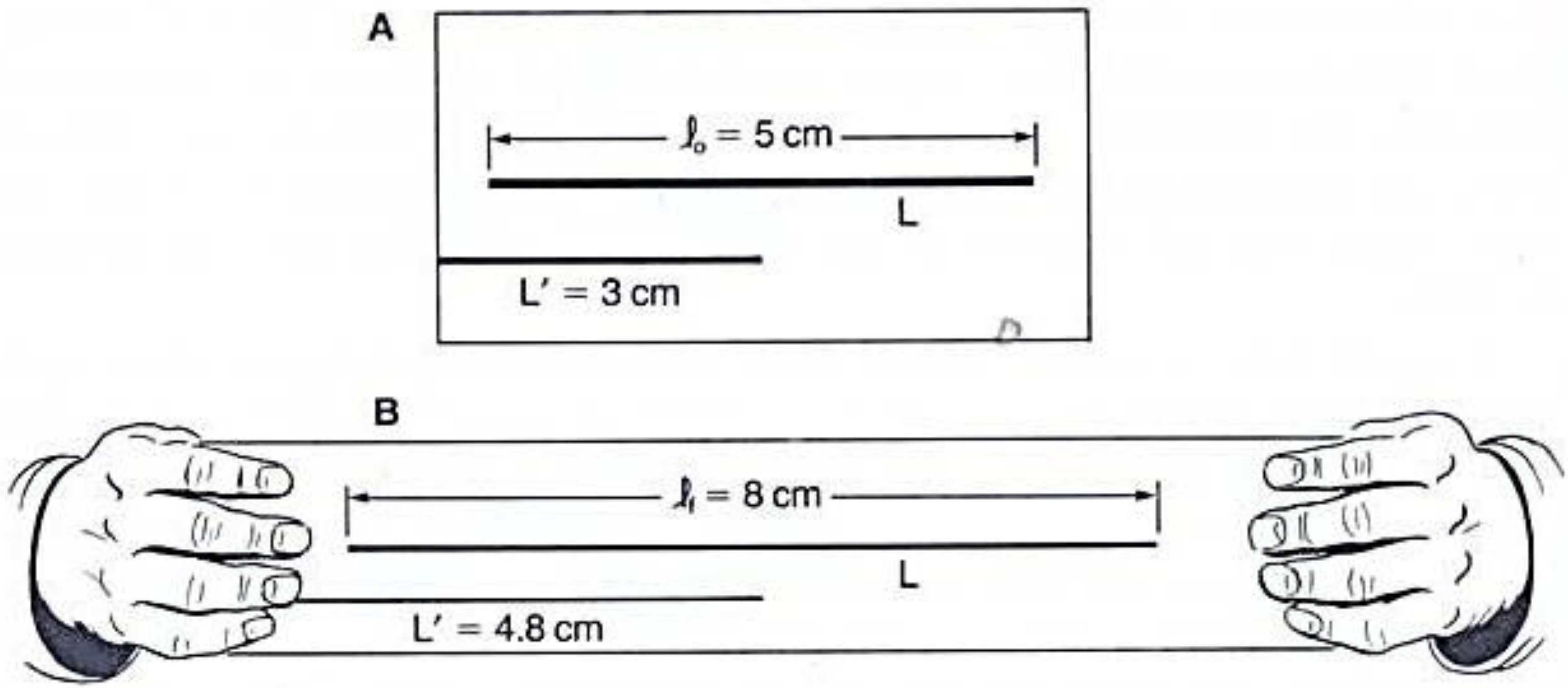
In fact, the changes in lengths of lines, and the change in angles between lines that were originally perpendicular, are sufficient to convey the magnitudes and directions of greatest shortening or stretching in the rock body as a whole!

Describing changes in lengths of lines

There are two parameters that permit changes in lengths of lines to be described easily,

1. **extension, symbolized by (*e*);**
2. **stretch, symbolized by (*s*).**

Consider line L whose original length (l_o) is 5 cm (figure 2.29A).



$$e = \frac{8\text{ cm} - 5\text{ cm}}{5\text{ cm}} = 0.6$$

Figure 2.29. (A) Lines L and L' before stretching. (B) Lines L and L' after stretching

During deformation, the non-rigid body in which L is contained changes shape and or size such that the line stretches to a final length (l_f) of 8 cm (figure 2.29B). the change in length (Δl) is 3 cm.

The magnitude of extension (*e*) in the direction of lengthening is the change in unit length of the line.

$$e = \frac{l_f - l_o}{l_o} \qquad (2.1)$$

$$e = \frac{8\ cm - 5\ cm}{5\ cm} = 0.6$$

A 0.6 value for extension *e* corresponds to a 60% lengthening of the line, percent lengthening (or percent shortening) is determined by multiplying *e by 100%.*

A second way to describe the magnitude of the change in length of line L is in terms of the stretch (*S*).

Stretch is equal to final length (l_f) divided by original length (l_o), which is also equal to the value of extension plus one (i.e., 1+ *e*).

Stretch tells us the final length of a line originally of unit length. A stretch of 3 means that a line was lengthened 3 x.

For the example we are considering,

$$S = \frac{8cm}{5\ cm} = 1.6$$

Here is how this relationship is derived:

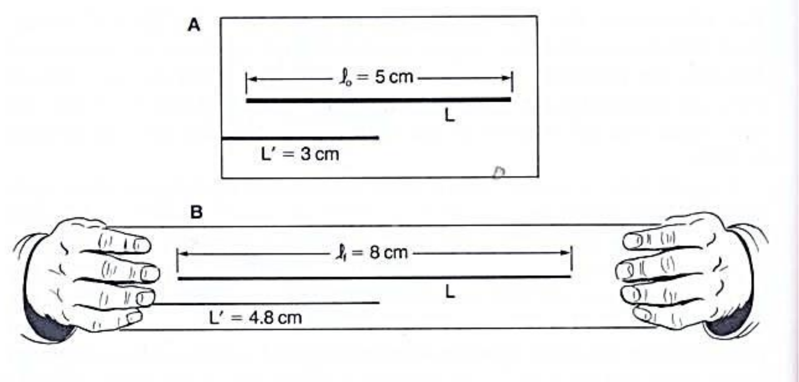
$$e = \frac{l_f - l_o}{l_o} \qquad (2.1)$$

$$e = \frac{l_f}{l_o} - 1$$

$$e + 1 = \frac{l_f}{l_o}$$

$$S = \frac{l_f}{l_o} = 1 + e \qquad (2.2)$$

If line L in figure 2.29B lies within a body that has undergone homogeneous deformation, the values of *e* = 0.6 and *S* = 1.6 must hold for all the lines in the body that are parallel to L. Line L' is such a line (see figure 2.29A).



$$e = \frac{8\text{ cm} - 5\text{ cm}}{5\text{ cm}} = 0.6$$

Figure 2.29. (A) Lines L and L' before stretching. (B) Lines L and L' after stretching

If the length of L' before deformation is 3 cm, its length after deformation can be determined by using Equation 2.1:

$$e = \frac{l_f - l_o}{l_o}$$

$$0.6 = \frac{l_f - 3\ cm}{3\ cm}$$

$$l_f = (0.6)\ (3\ cm) + 3\ cm$$

l_f = 4.8 cm (see figure 2.29B).

An even simpler way to compute l_f for this line is to multiply l_0 by the stretch (*S*).

l_f = 1.6 x 3 cm = 4.8 cm (figure 2.29B).

Percent lengthening or shortening is determined by multiplying 100% times (*S*- 1.0).

Angular Shear

Although strain parameters of extension and stretch effectively describe changes in length of lines in deformed bodies, they provide no information regarding changes that take place in the angles between lines.

A parameter known as angular shear, symbolized by the Greek letter psi (ψ), comes to the rescue.

To determine the angular shear along a given line, *L*, in a strained body, it is essential to identify a line that was originally perpendicular to *L*.

Angular shear describes the departure of this line from its perpendicular relation with L (fig. 2.35).

The full description requires a sign (positive equals clockwise; negative equals counterclockwise) and a magnitude expressed in degrees.

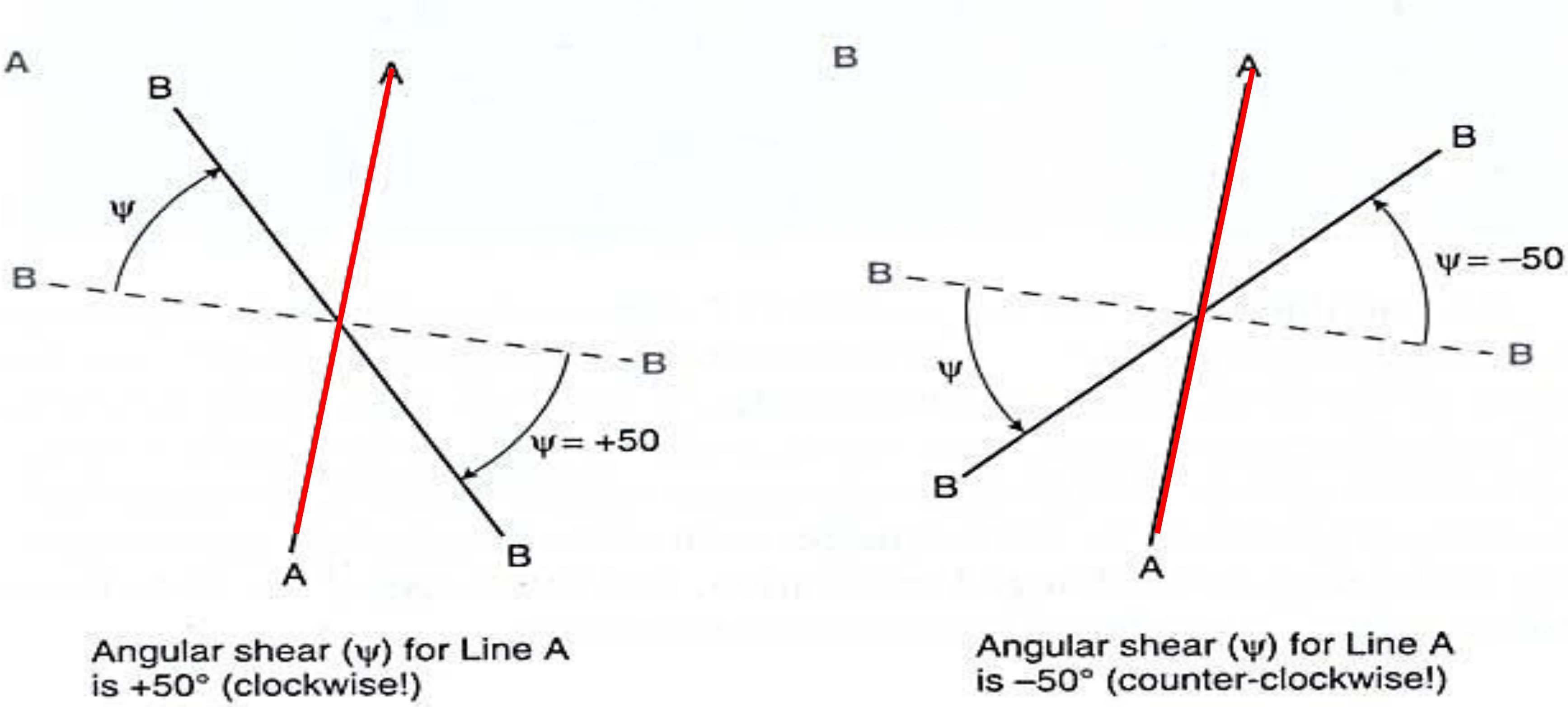


Figure 2.35. Sign conventions for angular shear. (A) Determination of the angular shear of line A requires identifying a line, in this case B, that was originally perpenducular to A. The original orientation of line B relative to line A is shown by the dash line. Angular shear is the shift in angle of B original versus B final. Because the shift is clockwise, the angular shear is positive (+). (B) In this example, the angular shear of line A is -50°. A counterclockwise shift is denoted by a negative (-) sign.

We can illustrate the measurement of angular shear by first fashioning a block of material that we will deform by flattening, and "paint" on the front of the block four reference circles (1-4), each containing two sets of mutually perpendicular lines (a-b, c-d, e-f, g-h) (fig. 2.36A).

After deformation, the block has shortened vertically and lengthened horizontally (Fig. 2.36B).

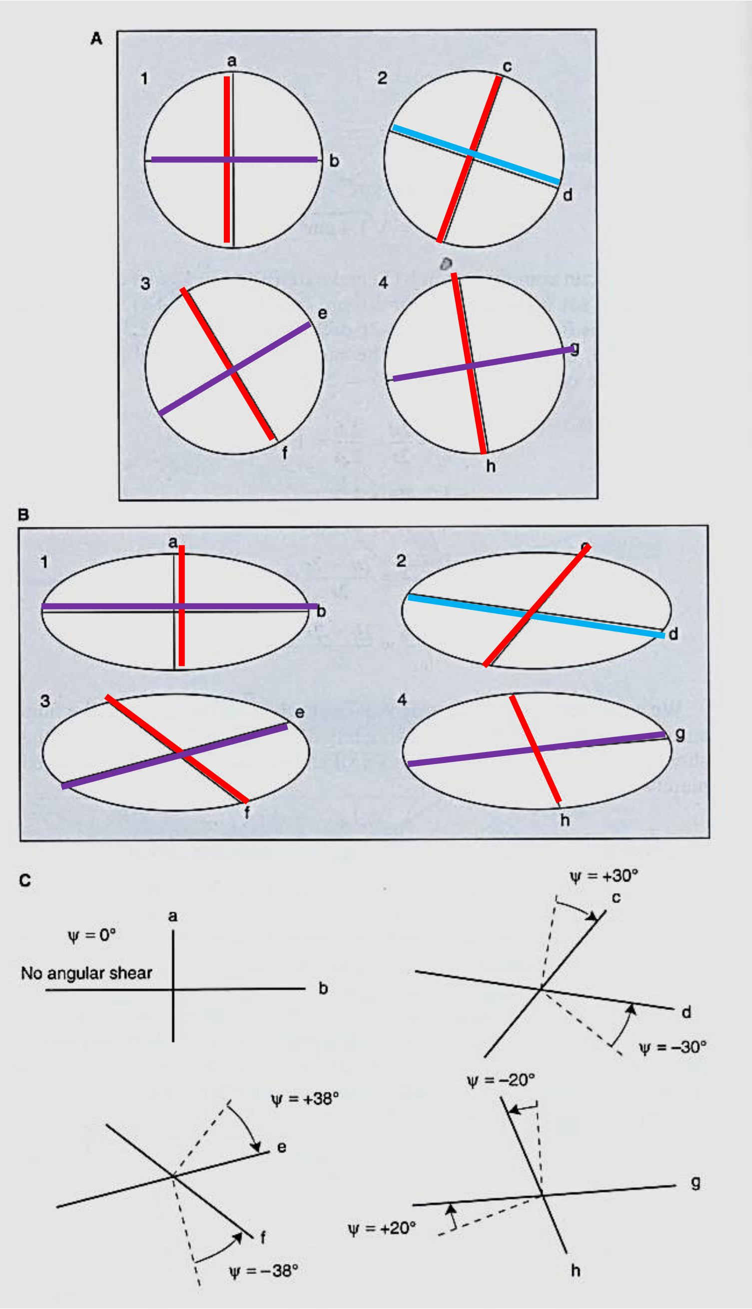


Figure 2.36. (A) Block containing reference circles and lines before deformation. (B) Shape of the block after deformation. Original reference circles now are ellipses. The originally mutually perpendicular reference liens have all changed length, and most have changed orientation as well (C) Angular shear along any line can be determined by first identifying a line originally perpendicular to it, and then measuring the angular shift. Remember, clockwise shifts are positive I+): counterclociwise shifts are negative (-).

- **In ellipse 2, the angular shear along c is -30° and the angular shear along d is +30° (see fig. 2.36C).**
- **For ellipse 3, the angular shear along e is +38° and the angular shear along f is -38° (see fig. 2.36C).**
- **Finally, for ellipse 4, the angular shear along g is -20° and the angular shear along h is +20°.**

Furthermore,

- each of the lines has changed length,
- six of the lines have changed orientation, and
- three sets of the lines have moved out of the original right-angle relationship.

It would not be difficult to calculate the extension and stretch values for each of the lines in the flattened block. We already know how to do it.

We can describe the angular shear (ψ), for any given line by identifying a line that was originally perpendicular to it, then measuring the angle through which the perpendicular line moved during deformation.

Within ellipse 1, there is no angular shear along the line a, nor is there any along line b, for the original perpendicular relationship remains after deformation (fig. 2.36C).

In ellipse 2, the angular shear along c is -30° and the angular shear along d is +30° (see fig. 2.36C).

For ellipse 3, the angular shear along e is +38° and the angular shear along f is -38° (see fig. 2.36C).

Finally, for ellipse 4, the angular shear along g is -20° and the angular shear along h is +20°.

If we keep our eyes open, we will see expressions of angular shear (2.37 A,B). Some fossils contain perpendicular lines in the makeup of their shells.

The modification of such originally perpendicular lines by distortion can be used as means of determining angular shear.

The distorted trilobite featured in fig. 2.37 C readily lends itself to appraisal of angular shear.

Lines parallel to the original length (line *L-L'*) and to the original width (line *W-W'*) of the trilobite are assumed to have been perpendicular before deformation. Now they intersect at 60°.

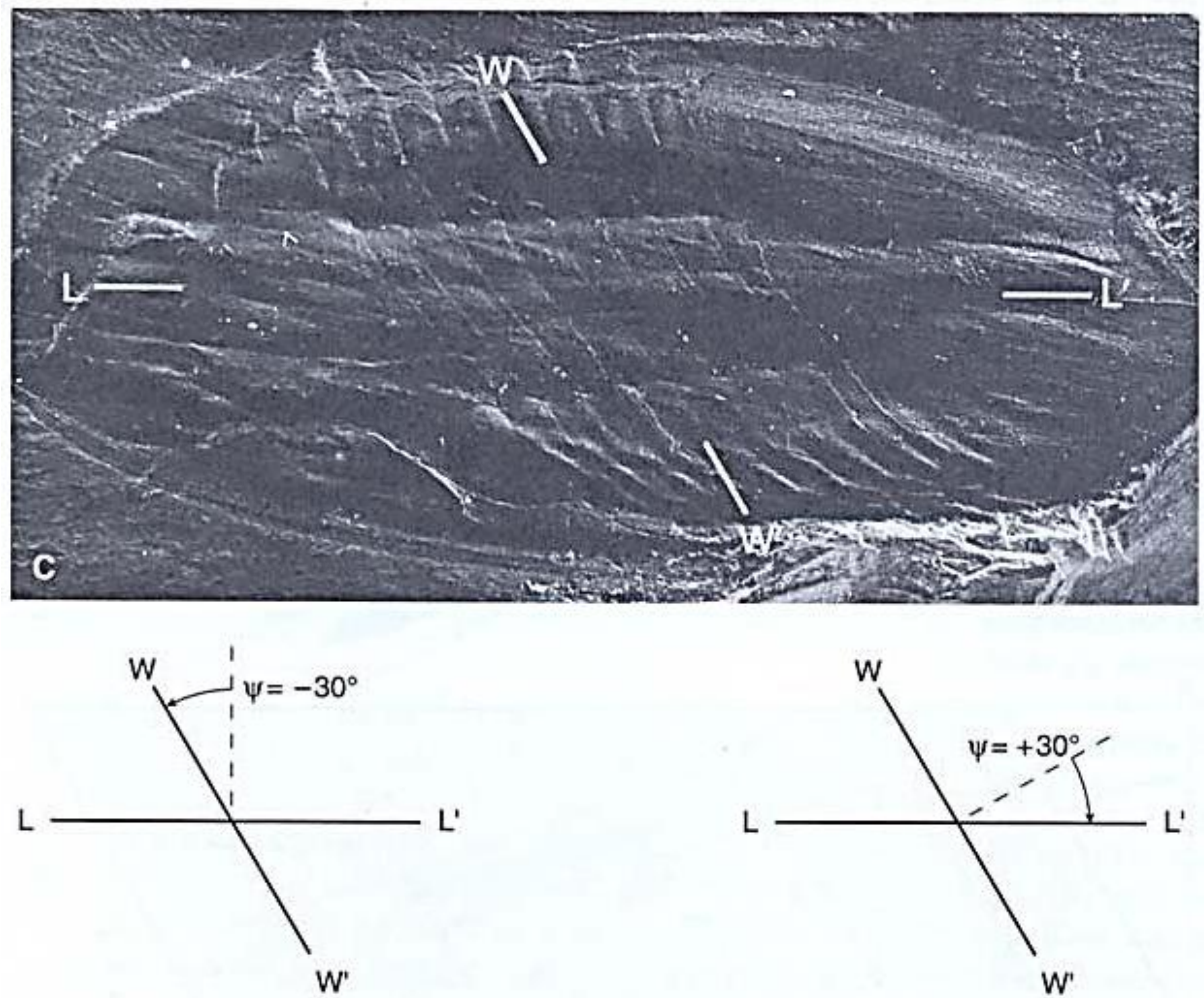


Fig. 2.37: Cambrian shale, Caernarvonshire, Wales. Angular shear of rock within which this fossil is found can be determined by measuring the angular relationship between lines *L- L'* and *W –W',* Lines that were perpendicular before the deformation. *(from The Minor Structures of Deformed Rocks: A petrographic Atlas by L.E. Weiss. Published with permission of Springer-Verlag. New York, copyright* © *1972*). Angular shear along L –L' is -30^0. Angular shear along W –W; is +30^0

The angular shear along line *L-L’* is -30° (fig 2.37 C). This value is determined by focusing on the line W-W’, which was originally perpendicular to *L-L’*, and describe the sense and amount of deflection of that line.

In the same fashion, the angular shear along W-W’ is found to be +30° (fig. 2.37C). In this case, we focus on line *L-L’*, which was originally perpendicular to *W-W’*, and we measure the magnitude and sense of rotation of *L-L’* with respect to W-W’.

Shear Strain (γ)

Let us consider how points on a line move as a response to angular shear. Points 1 to 4 on line A_0 in Fig. 2.38A are translated by various distances as a result of the rotation of the line on which they reside.

Line A_0 is the locus of points 1-4. Line A_f is the locus of the same points in their deformed locations (fig. 2.38B).

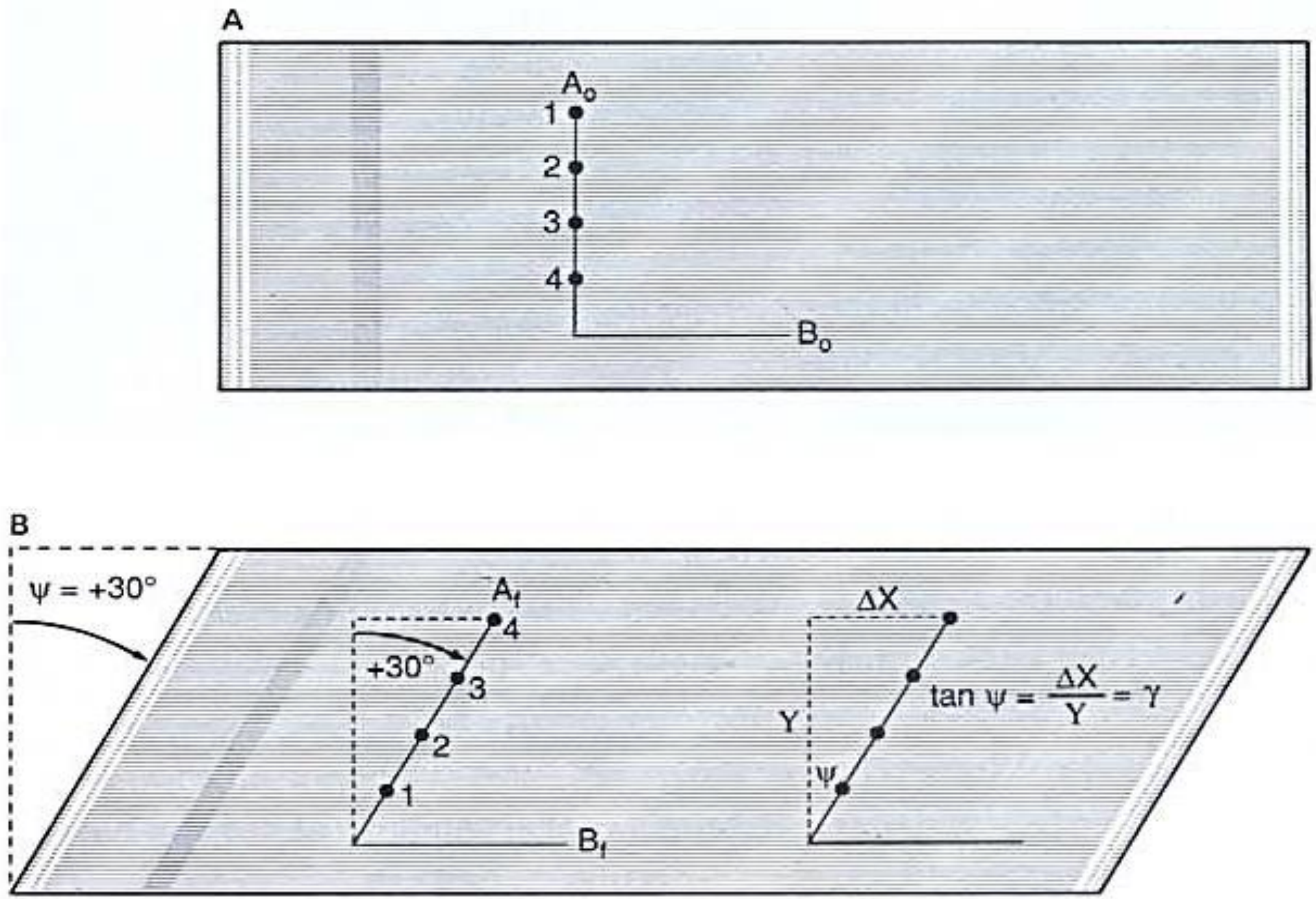


Fig. 2.38. Simulation of the shearing of a computer card deck. (A) Deck embossed with lines A0 and B0 and points 1 to 4 before deformation. (B) Configuration of the deck, including the reference lines and points, after shearing.

Since angular shear was systematic and deformation was homogeneous, lines A_0 *and* A_f are straight.

Points 1-4 move a distance that is directly related to the angular shear and to the distance of each point above the point of intersection with the complementary line.

If the distance of each point above the intersection is denoted as *y* (fig. 2.38B), the horizontal distance of translation can be found as follows (Ramsay, 1967):

$$\tan \psi = \frac{\Delta x}{y}$$

$$\Delta x = y \tan \psi$$

Thus tan ψ is another way of describing relative shifts in orientations of lines that were originally perpendicular. It is called shear strain, symbolized by the Greek letter gamma (γ),

γ (gamma) = tan ψ

(= Δ*x*/*y*. If we know the value of Δx and y, we can find the value of shear strain)

Shear strain along a line may be positive or negative, depending on the sense of rotation (deflection) of the line originally perpendicular to it. The range of shear strain is zero to infinity.

For example shown in figure 2.38B the shear strain of line B_f is + tan30° or +0.58.

The shear strain of line A_f is - tan30° or -0.58.

For the example of distorted trilobite shown in fig. 2.37C, the shear strain of line L- L' is - tan30° or - 0.58°; and the shear strain of line W-W' is + tan30° or + 0.58°

Pure shear and simple shear

These are two types of homogeneous deformation for which it is usual to specify both

1. **the type of strain and**
2. **the relative orientations of certain lines in the deformed and undeformed states.**

They represent two special end members of plane strain.

- **Pure shear is nearly synonymous with co-axial strain.**
- **Simple shear is nearly synonymous with non-coaxial strain.**

Both can be illustrated by examining the front face of a cube of material, flattened or sheared in such a way that there is no change in volume and neither shortened nor stretched in a direction perpendicular to the front face (fig. 2.59).

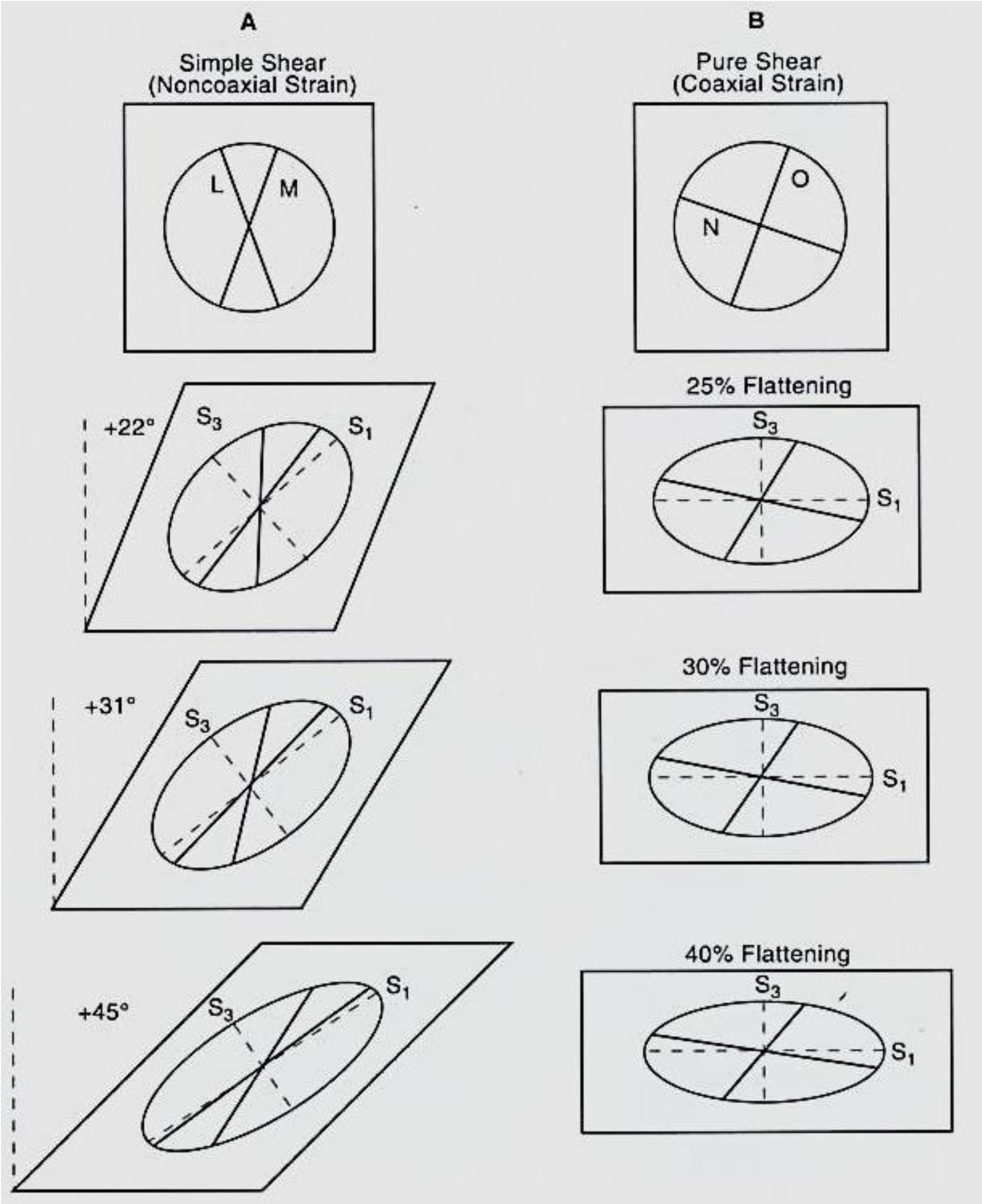


Figure 2.59 (*A*) Progressive deformation of a reference circle and two lines (*L* and *M*) by noncoaxial strain, which in this example is a simple shear. The principal strain axes rotate continuously to different orientations during the deformation. Line *L* at first shortens and then lengthens. Line *M* lengthens throughout the deformation. (*B*) Progressive deformation of a reference circle and two lines (*N* and *O*) by coaxial strain, which in this example is a pure shear. The orientations of the principal strain axes remain fixed throughout the deformation. Line *N* continuously lengthens. Line *O* continuously shortens.

Pure shear

It is a homogeneous deformation involving either a plain strain or a general strain, in which lines of particles that are parallel to the principal axes of the strain ellipsoid have the same orientation before and after deformation.

examples are shown in figure 1.12a to c and figure 1.13a, b, c and f.

Because there is no rotation of material lines along principal directions, a pure shear is referred to as an irrotational deformation or, loosely, as an irrotational strain.

In pure shear, the cube of rock is shortened in one direction and extended in the perpendicular direction within the plane of strain (figure 2.59B).

The square is converted to a rectangle, and the original sides of the square remain parallel after deformation. The finite stretching axes do not rotate.

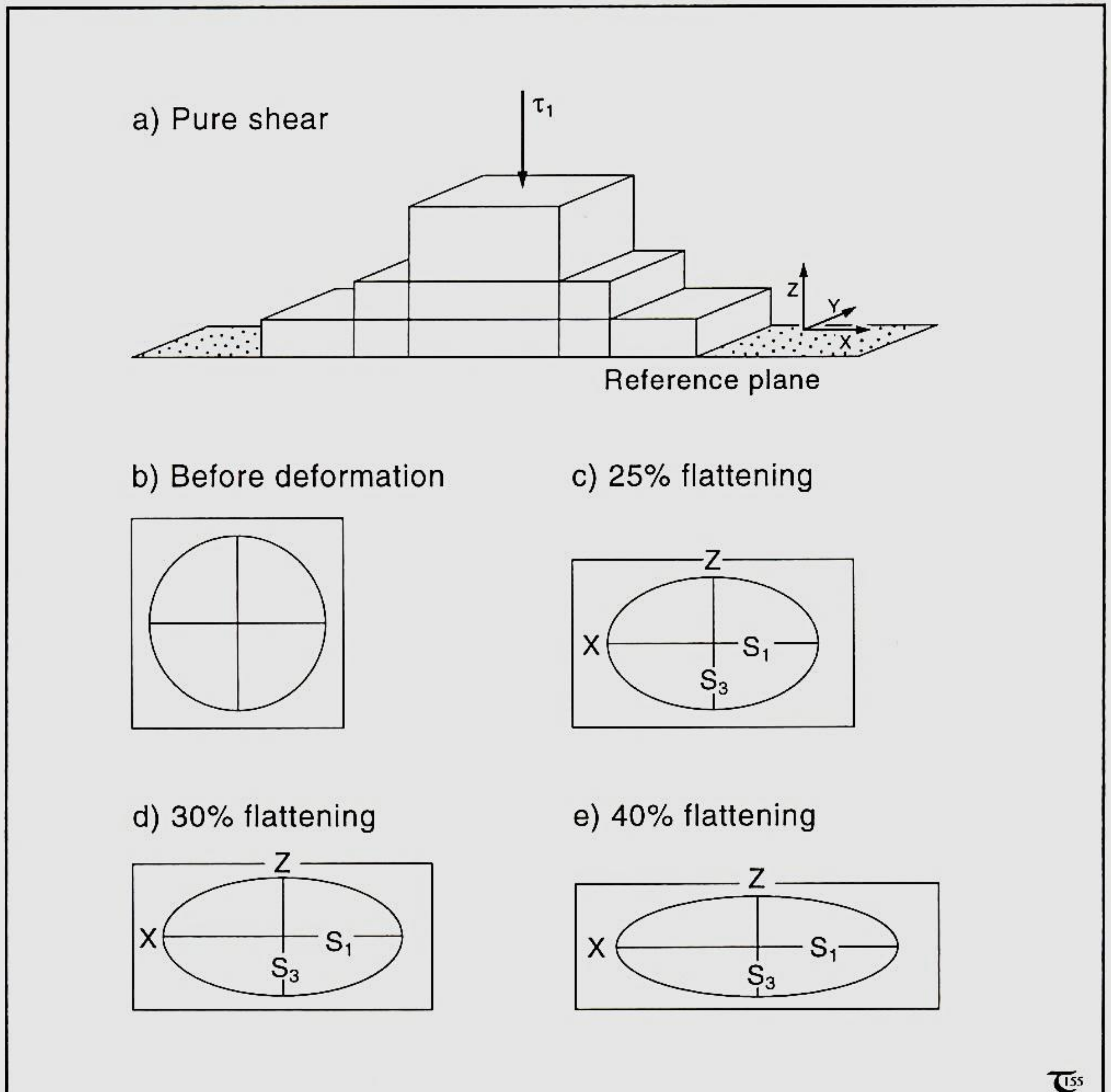


Figure 12-6: *a) to e) Principle sketch of pure shear deformation. The deformation is always by plane strain without rotation; this is a coaxial deformation.*

Simple shear

It is a constant volume, homogeneous deformation involving plane strain, in which a single family of parallel material planes is undistorted in the deformed state and parallel to the same family of planes in the undeformed state.

Examples are shown in figures 1.12a and b and 1.13d.

Simple shear involves a change in orientation of material lines along two of the principal axes (λ_1 and λ_3 , principal strain axes).

Simple shear is, therefore, referred to as a rotational deformation or, loosely, as a rotational strain.

In simple shear, the cube of rock is sheared like a deck of cards (fig. 2.59A).

The square is converted to a parallelogram. The sides of the parallelogram progressively lengthen as deformation proceeds, but the top and bottom surfaces neither stretch or shorten.

Instead, they maintain their original length, which is the original length of the edge of the cube.

The finite stretching axes rotate during the deformation.

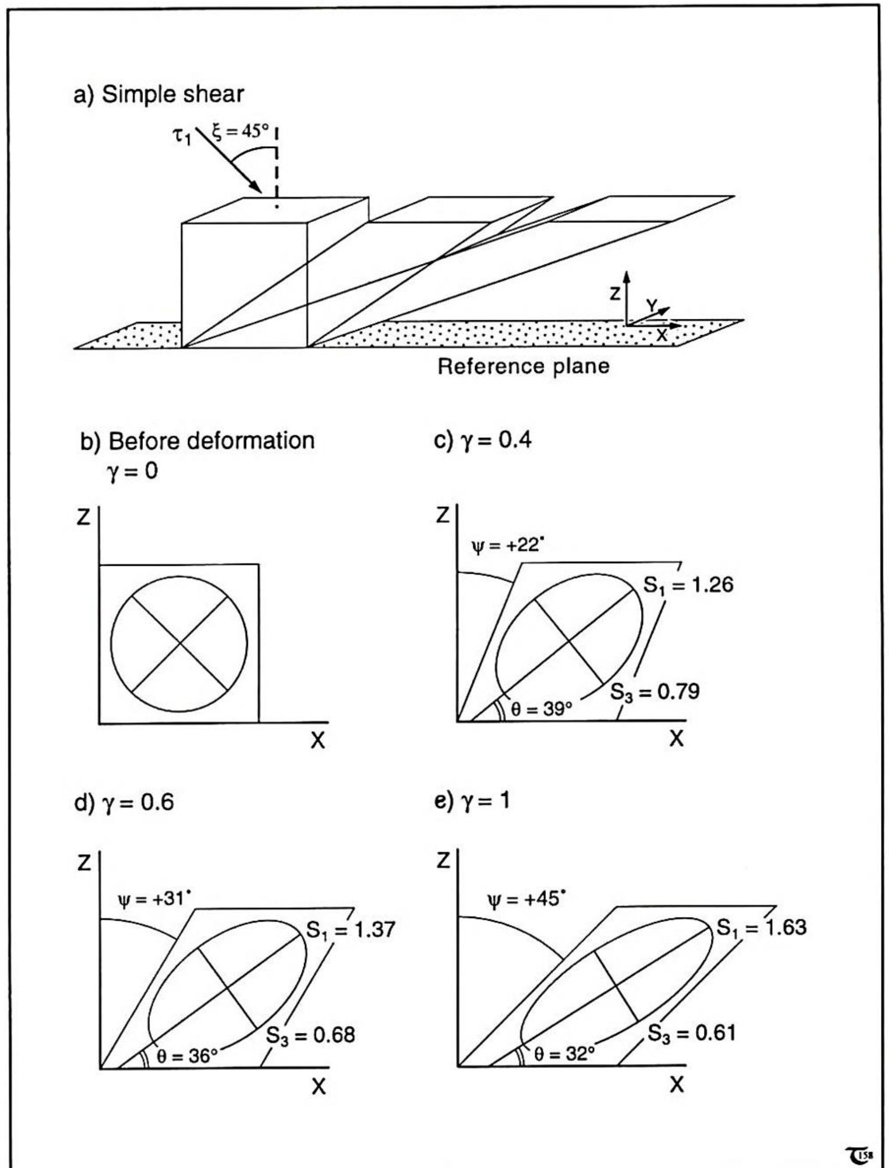


Figure 12-8: *a) to e) Simple shear deformation. This deformation is a special case of non-coaxial, plain strain, where the height of the sheared cube remains constant.*

Finite strain ellipsoid and plane strain

The most complete strain analyses are three-dimensional. The three dimensional counterpart of the strain ellipsoid is called the strain ellipsoid.

It pictures how the shape of an imaginary spherical reference object would be changed as a result of distortion. It is defined by three mutually perpendicular finite stretching axes (Fig. 2.52).

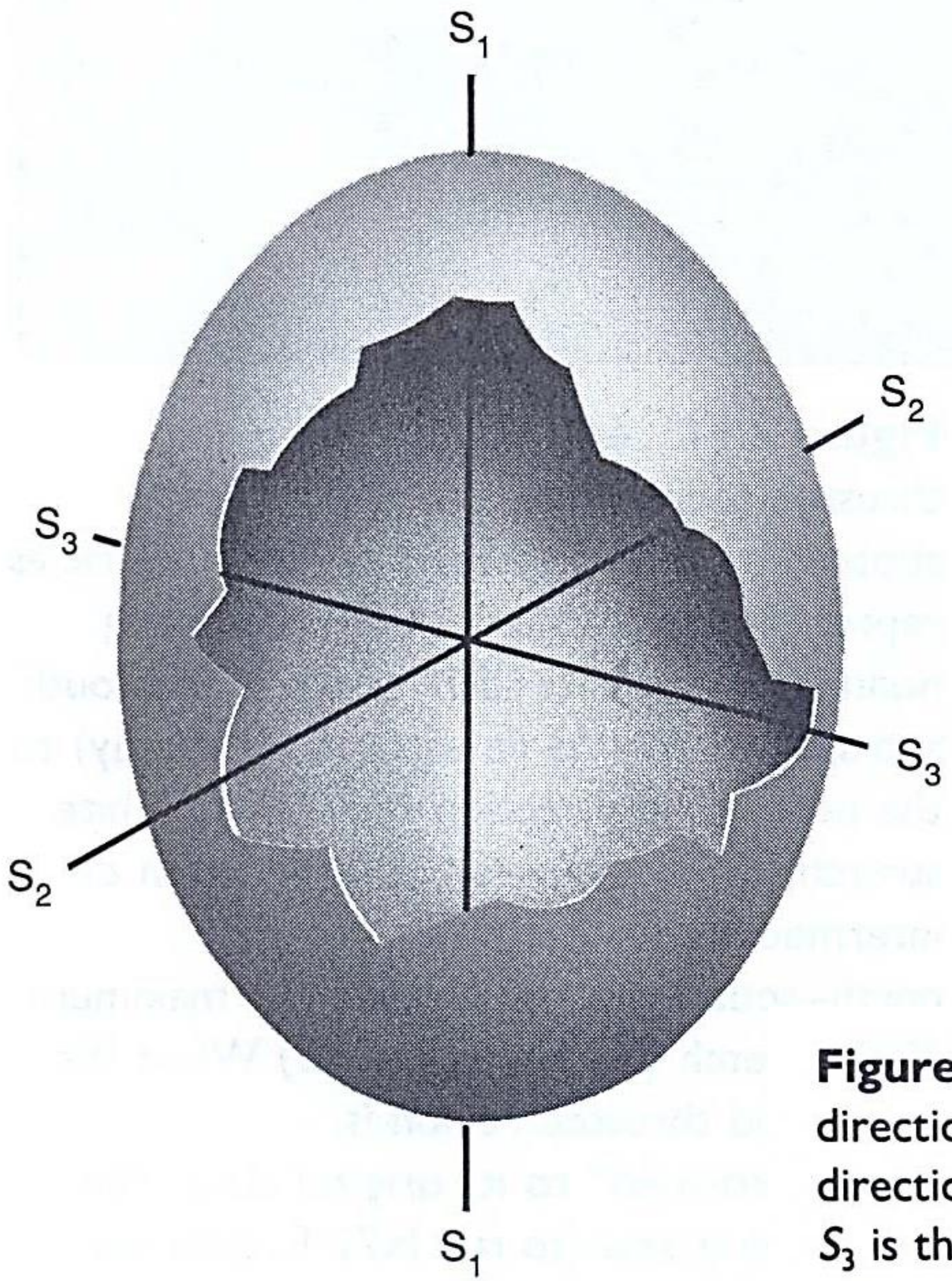


Figure 2.52 The strain ellipsoid: S_1 is the direction of maximum finite stretch, S_2 is the direction of intermediate finite stretch, and S_3 is the direction of minimum finite stretch.

If stretching in the s_1 direction is perfectly compensated by the shortening in the S_3 direction, and there is no change in length along the S_2 direction, there will be no change in volume of the deformed body.

Under such conditions and assuming homogenous deformation, a perfect sphere of a certain size will be transformed to a perfect ellipsoid of the same volume.

This special state of strain is known as plane strain.

However, it is very common for strain to be truly three-dimensional – that is, the intermediate finite stretch (S_2) is not always one.

Thank you

End of Lecture-4