MAT 2602 Tutorial sheet

1. According to the Federal Poverty Measure 12% of the U.S. population lives in poverty. The governor of a certain state believes that the proportion there is lower. In a sample of size 1,550, 163 were impoverished according to the federal measure.
    (a) Test whether the true proportion of the state's population that is impoverished is less than 12%, at the 5% level of significance.
    (b) Compute the observed significance of the test.
2. An insurance company states that it settles 85% of all life insurance claims within 30 days. A consumer group asks the state insurance commission to investigate. In a sample of 250 life insurance claims, 203 were settled within 30 days.
    (a) Test whether the true proportion of all life insurance claims made to this company that are settled within 30 days is less than 85%, at the 5% level of significance.
    (b) Compute the observed significance of the test.
3. At every setting a high-speed packing machine delivers a product in amounts that vary from container to container with a normal distribution of standard deviation 0.12 ounce. To compare the amount delivered at the current setting to the desired amount 64.1 ounce, a quality inspector randomly selects five containers and measures the contents of each, obtaining sample mean 63.9 ounces and sample standard deviation 0.10 ounce. Test whether the data provide sufficient evidence, at the 5% level of significance, to conclude that the mean of all containers at the current setting is less than 64.1 ounces.
4. A manufacturing company receives a shipment of 1,000 bolts of nominal shear strength 4,350 lb. A quality control inspector selects five bolts at random and measures the shear strength of each. The data are: 4320, 42904,360, 4350, 4320

    (a) Assuming a normal distribution of shear strengths, test the null hypothesis that the mean shear strength of all bolts in the shipment is 4,350 lb versus the alternative that it is less than 4,350 lb, at the 10% level of significance.
    (b) Estimate the *p*-value (observed significance) of the test of part (a).

Compare the *p*-value found in part (b) to α=0.10 and make a decision based on the *p*-value approach. Explain fully

5. Researchers wish to test the efficacy of a program intended to reduce the length of labour in childbirth. The accepted mean labour time in the birth of a first child is 15.3 hours. The mean length of the labours of 13 first-time mothers in a pilot program was 8.8 hours with standard deviation 3.1 hours. Assuming a normal distribution of times of labour, test at the 10% level of significance test whether the mean labour time for all women following this program is less than 15.3 hours.

6. A dairy farm uses the somatic cell count (SCC) report on the milk it provides to a processor as one way to monitor the health of its herd. The mean SCC from five samples of raw milk was 250,000 cells per millilitre with standard deviation 37,500 cell/ml. Test whether these data provide sufficient evidence, at the 10% level of significance, to conclude that the mean SCC of all milk produced at the dairy exceeds that in the previous report, 210,250 cell/ml. Assume a normal distribution of SCC.
7. Six coins of the same type are discovered at an archaeological site. If their weights on average are significantly different from 5.25 grams then it can be assumed that their provenance is not the site itself. The coins are weighed and have mean 4.73 g with sample standard deviation 0.18 g. Perform the relevant test at the 0.1% (1/10th of 1%) level of significance, assuming a normal distribution of weights of all such coins.
8. An economist wishes to determine whether people are driving less than in the past. In one region of the country the number of miles driven per household per year in the past was 18.59 thousand miles. A sample of 15 households produced a sample mean of 16.23 thousand miles for the last year, with sample standard deviation 4.06 thousand miles. Assuming a normal distribution of household driving distances per year, perform the relevant test at the 5% level of significance.
9. The recommended daily allowance of iron for females aged 19–50 is 18 mg/day. A careful measurement of the daily iron intake of 15 women yielded a mean daily intake of 16.2 mg with sample standard deviation 4.7 mg.
    (a) Assuming that daily iron intake in women is normally distributed, perform the test that the actual mean daily intake for all women is different from 18 mg/day, at the 10% level of significance.
    (b) The sample mean is less than 18, suggesting that the actual population mean is less than 18 mg/day. Perform this test, also at the 10% level of significance. (The computation of the test statistic done in part (a) still applies here.)
10. The target temperature for a hot beverage the moment it is dispensed from a vending machine is 170°F. A sample of ten randomly selected servings from a new machine undergoing a pre-shipment inspection gave mean temperature 173°F with sample standard deviation 6.3°F.
    (a) Assuming that temperature is normally distributed, perform the test that the mean temperature of dispensed beverages is different from 170°F, at the 10% level of significance.
    (b) The sample mean is greater than 170, suggesting that the actual population mean is greater than 170°F. Perform this test, also at the 10% level of significance. (The computation of the test statistic done in part (a) still applies here.)
11. The average number of days to complete recovery from a particular type of knee operation is 123.7 days. From his experience a physician suspects that use of a topical pain medication might be lengthening the recovery time. He randomly selects the records of seven knee surgery patients who used the topical medication. The times to total recovery were:

128, 135, 121, 142, 126, 151, 123

(a) Assuming a normal distribution of recovery times, perform the relevant test of hypotheses at the 10% level of significance.

(b) Would the decision be the same at the 5% level of significance? Answer either by constructing a new rejection region (critical value approach) or by estimating the *p*-value of the test in part (a) and comparing it to α.

(c) A small component in an electronic device has two small holes where another tiny part is fitted. In the manufacturing process the average distance between the two holes must be tightly controlled at 0.02 mm, else many units would be defective and wasted. Many times throughout the day quality control engineers take a small sample of the components from the production line, measure the distance between the two holes, and make adjustments if needed. Suppose at one time four units are taken and the distances are measured as 0.021, 0.019, 0.023, 0.020

(d) Determine, at the 1% level of significance, if there is sufficient evidence in the sample to conclude that an adjustment is needed. Assume the distances of interest are normally distributed.