Quiz 1 Key MAT 2602

1. The variance of sampling distribution is:

$$Var(\bar{x}) = Var\left(\frac{\sum_{1}^{n} x_i}{n}\right)$$

$$= \frac{1}{n^2} Var\left\{\sum_{1}^{n} x_i\right\}$$

$$= \frac{1}{n^2}\{Var(x_1) + Var(x_2) + ... + Var(x_n)\}$$

$$= \frac{1}{n^2}\{\sigma^2 + \sigma^2 + ... + \sigma^2\}$$

$$= \frac{1}{n^2}.n\sigma^2 = \frac{\sigma^2}{n}$$

Therefore, $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$, This is a biased estimator of the population variance since $\sigma_{\bar{x}}^2 \neq \sigma^2$

2. (a) We know that $\bar{x} = \frac{\sum_{i}^{n} x_i}{n}$ implies $\sum x = n\bar{x}$

Therefore, $\sum x_A = n_A \bar{x} = 15x67 = 1008$ and

$\sum x_B = n_B \bar{x} = 18x58 = 1044$

The combined samples is:

$\sum x = 1005 + 1044 = 2049$ and n = 15 + 18 = 33

The combined mean of the two samples is:

$\bar{x} = \frac{2049}{33} = 62.1$ note that we have calculated the weighted mean.

(b) We know that $S^2 = \frac{\sum x}{n} - \bar{x}$, which we can rearrange to be

$\sum x^2 = n(\bar{x}^2 + S^2)$, so for the two samples separately we have

$\sum x_A^2 = n_A(\bar{x}_A + S_A^2) = 15(67^2 + 5^2) = 67710$

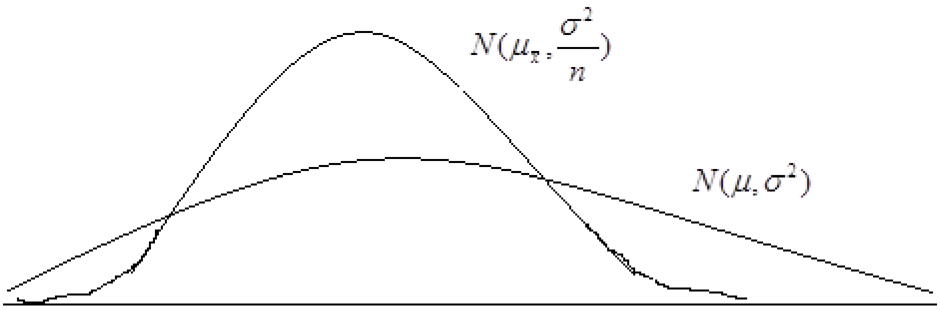
$\sum x_B^2 = n_B(\bar{x}_B + S_B^2) = 18(58^2 + 8^2) = 61704$ So for the combined we have

$$S^2 = \frac{\sum x^2}{n} - \bar{x}^2 = \frac{129414}{33} - \left(\frac{2049}{33}\right)^2 = 66.335$$ and s.. for the combined sample is 8.15.

3. (i) Theorem: In probability theory, the central limit theorem (CLT) states that the distribution of a sample variable approximates a normal distribution (i.e., a “bell curve”) as the sample size becomes larger, assuming that all samples are identical in size, and regardless of the population's actual distribution shape.

   (ii) The population distribution is $N(\mu, \sigma^2)$ shown together with a sampling distinction of sample size n.

$N(\mu_{\bar{x}}, \frac{\sigma^2}{n})$



4. We know that the sample mean is an unbiased estimator of the population mean, hence,

$$\mu = \frac{\sum_{1}^{12} x_i}{12} = \frac{5472}{12} = 456$$ and the variance is estimated as:

$$\hat{S}^2 = \frac{\sum_{1}^{12} (x - \bar{x})^2}{n-1}$$

Using the equation $\sum (x_i - 450)^2 = 1620$ we find summation of x squared as follows;

$$\sum_{1}^{12}(x_i - 450)^2 = 1620$$

$$\sum_{1}^{12}\left[x^2 - 2(450)x + 450^2) = 1620\right]$$

$$\sum_{1}^{12}x^2 - 900\sum_{1}^{12}x + \sum_{1}^{12}450^2 = 1620$$

$$\sum_{1}^{12}x^2 = 2496420$$

$$\hat{S}^2 = \frac{\sum_{1}^{12}(x-\bar{x})^2}{n-1}$$

$$= \frac{1}{n-1}\left[\sum_{1}^{12}\left[x^2 - 2\bar{x}x + \bar{x}^2\right]\right]$$

$$\hat{S}^2 = 108$$

(ii) This is a sample size since n = 12. Therefore, our CI is given by:

$\bar{x} \pm t_{v,n-1}\dfrac{\hat{S}}{\sqrt{n}}$, where alpha is 5%, so our critical value for $t_{0.025,11} = 2.201$ from the t-distribution table, with $\bar{x} = 456$, $\hat{S} = 10.4$ and n = 12. Substituting in the expression we have

$$456 \pm 2.201\left(\frac{10.4}{\sqrt{12}}\right)$$

$$456 \pm 6.6$$

$$\left[449.4, 462.6\right]$$

We are 95% confident that the true mean is between 449.4 and 462.6.