# The University of Zambia

Department of Mathematics and Statistics

**MAT2602: INTRODUCTION TO STATISTICS**

**Tutorial Sheet 3** **September 2020**

1. To study the effect of a special study programme, 14 students were selected and paired according to IQ and scholastic performance. One student from each pair was randomly selected to participate in the special programme, while the other student participated in the standard programme. Shortly thereafter, the students took the national exam and obtained the following scores:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Special programme | 66 | 82 | 96 | 72 | 78 | 82 | 67 |
| Standard programme | 60 | 79 | 92 | 73 | 75 | 80 | 69 |

Assume normality for the population of differences.

(a) Find a 95% confidence interval for the population mean difference of the scores under the two programmes.

(b) Based on your answer in (a), is there a difference in the mean scores under the two programmes? Why?

2. An administrator at a large university stated that there was a difference in the mean grade point average of graduating males and females. A random sample of 45 graduating males gave a mean grade point average of 2.10 and a variance of 0.64, while a random sample of 50 graduating females gave a mean grade point average of 2.45 and a variance of 0.70.

(a) Construct a 95% confidence interval for the difference in the mean grade point average of all male and female graduating students at the university.

(b) Using the confidence interval in part (a), do the data support the administrator's belief?

3. A typing instructor wanted to compare two methods of teaching, methods A and B. The instructor tested her 10 students under the current method A. Another typing test was given following 4 weeks of instruction under a new method B. The results, measured in words per minute, are as follows:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Method A | 58 | 63 | 66 | 69 | 70 | 70 | 70 | 76 | 77 | 86 |
| Method B | 60 | 64 | 67 | 69 | 71 | 72 | 74 | 76 | 75 | 85 |

(a) Find a 90% confidence interval for $\mu_B - \mu_A$, where $\mu_A$ and $\mu_B$ are the population means for methods A and B respectively.

(b) Are the two methods different? Why?

(c) State the assumption(s) required in order for (a) and (b) to be valid.

4. A random sample of size $n_1 = 25$ taken from a normal population with a standard deviation $\sigma_1 = 5$ has a mean $\bar{x}_1 = 80$. A second random sample of size $n_2 = 36$, taken from a different normal population with a standard deviation $\sigma_2 = 3$, has a mean $\bar{x}_2 = 75$. Find a 94% confidence interval for $\mu_1 - \mu_2$.

5. The following data, recorded in days, represent the length of time to recovery for patients randomly treated with one of two medications to clear up severe bladder infections:

| | n | $\bar{x}$ | $s^2$ |
|---|---|---|---|
| Medication 1 | 14 | 17 | 1.5 |
| Medication 2 | 16 | 19 | 1.8 |

Find a 99% confidence interval for the difference $\mu_2 - \mu_1$ in the mean recovery time for the two medications, assuming normal populations with equal variances.

6. A certain geneticist is interested in the proportion of males and females in the population that have a certain minor blood disorder. In a random sample of 1000 males, 250 are found to be afflicted, whereas 275 of 1000 females tested appear to have the disorder. Compute a 95% confidence interval for the difference between the proportion of males and females that have the blood disorder.

7. A study compared fuel economies for two types of trucks. Twelve Volkswagen trucks used in the study gave an average of 16km per litre with a standard deviation of 1.0km per litre, while 10 Toyota trucks gave an average of 11km per litre with a standard deviation of 0.8km per litre. Assume that the distances per litre for each truck model are approximately normally distributed.
   (a) Construct a 90% confidence interval for the difference between the average kilometres per litre of these two trucks assuming the population variances are equal.
   (b) Construct a 98% confidence interval for $\frac{\sigma_1}{\sigma_2}$, where $\sigma_1$ and $\sigma_2$, are respectively, the standard deviations for the distances obtained per litre of fuel by the Volkswagen and Toyota trucks.
   (c) Were we justified in assuming that the population variances are equal in part (a)? Why?

8. A taxi company is trying to decide whether to purchase brand A or brand B tires for its fleet of taxis. To estimate the difference in the two brands, an experiment is conducted using 12 tires of each brand. The tires are run until they wear out. The results are:
   Brand A: $\bar{x}_1 = 36{,}300$ km  $s_1 = 5000$ km
   Brand B: $\bar{x}_2 = 38{,}100$ km  $s_2 = 6100$ km
   Compute a 95% confidence interval for $\mu_1 - \mu_2$, assuming the populations to be approximately normally distributed.

9. A railroad company used two types of wheel mounts that differ in the way they handle track irregularities: Type A (spring equalised) and Type B (frame equalised). The following data give the repair records for the two types over a 1-year period:

| | Number of cars | Number needing Service |
|---|---|---|
| Type A | 150 | 20 |
| Type B | 180 | 18 |

   (a) Find a 95% confidence interval for the difference in the population proportions $p_A - p_B$.
   (b) Based on your results in part (a), do you think there is a difference between $p_A$ and $p_B$.

10. The following data represent the running times of films produced by two motion picture companies.

| Company | Time (in minutes) | | | | | | |
|---|---|---|---|---|---|---|---|
| I | 103 | 94 | 110 | 87 | 98 | | |
| II | 97 | 82 | 123 | 92 | 175 | 88 | 118 |

   Assume that the running times are approximately normally distributed.
   (a) Compute a 90% confidence interval for the difference between the average running times of the films produced by the two companies.
   (b) Construct a 90% confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$.
   (c) Should we have assumed $\sigma_1^2 = \sigma_2^2$ in constructing a confidence interval for $\mu_{II} - \mu_I$ ? Why?