Resource Estimation 1:
(A review of):

Traditional Estimation Methods

Prof. Emmanuel K Chanda (UNI ADELAIDE/UNZA)
Prof. Serkan Saydam (UNSW)

MEA Mining Education Australia | Curtin University of Technology | The University of Queensland Australia | UNSW The University of New South Wales | Adelaide University Australia

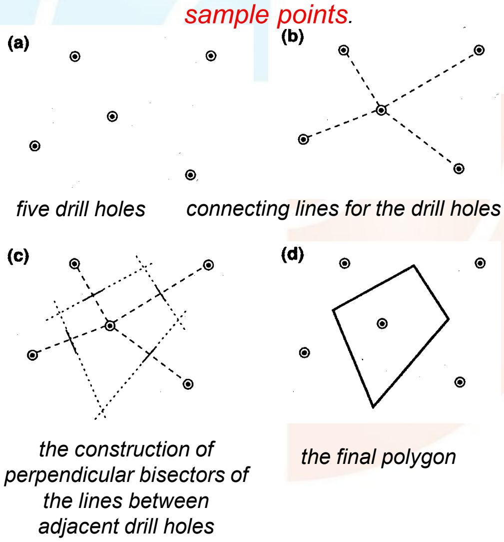
Traditional (Mineral) Resource Estimation Techniques

1. Polygon,
2. Triangulation,
3. **Inverse Distance Weighting (IDW)**
4. Cross-sectional methods (3D)

Polygon Method

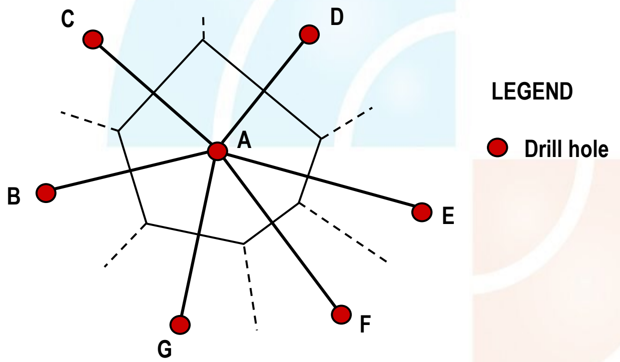
- The method of calculation by polygons is **often used with drill-hole data.** Polygons may be constructed on plans, cross sections, or longitudinal sections. The polygons, once constructed and ranked as to class of ore, are planimetered to determine the area of mineralization.
- In this method, the average grade of mineralization encountered by the sample point within the polygon is considered to accurately represent the grade of the entire volume of material within the polygon.

*The method **assumes** that the **area of influence of any sample point** extends halfway to the adjacent sample points.*



(a) *five drill holes*; (b) *connecting lines for the drill holes*; (c) *the construction of perpendicular bisectors of the lines between adjacent drill holes*; (d) *the final polygon*

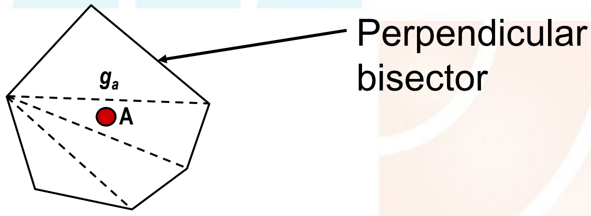
Polygonal method – perpendicular bisector



Aim: to estimate the tonnage and average grade within the outline polygon (solid lines)

Steps:

1. Connecting drill holes followed by perpendicular bi-section of the lines to form the polygonal influence around drill holes



2. Calculate area and volume of the polygon- The polygon can be divided into triangles for the calculation, as shown in the figure above

Steps cont'd:

3. Calculate the tonnage: volume x density

4. Report: Total tonnage and grade: T @ g_a

5. The polygon is termed the influence area (volume) of sample A.

- **Influence areas for all samples will have to be constructed for the resource calculation of the whole area.**

Polygon Method – Global Estimation

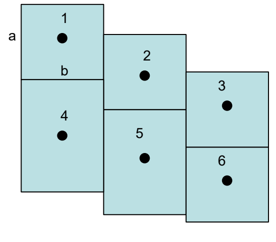
- Global estimate: total tonnage and average grade for the entire area
- If we have n numbers of polygons (i.e., drill-holes):

Total tonnage: $$T = \sum_{i=1}^{n} T_i$$

Global mean grade: $$\bar{g} = \frac{\sum_{i=1}^{n} T_i \cdot g_i}{\sum_{i=1}^{n} T_i}$$ **(weighted average)**

T_i and g_i are tonnage and grade for polygon i

Regular Blocks – a variation of polygonal method



Example: Drill hole 1

Area: A_1 = a x b

Volume = V_1 = A_1 x thickness = a x b x t_1

Tonnage = T_1 = V_1 x density

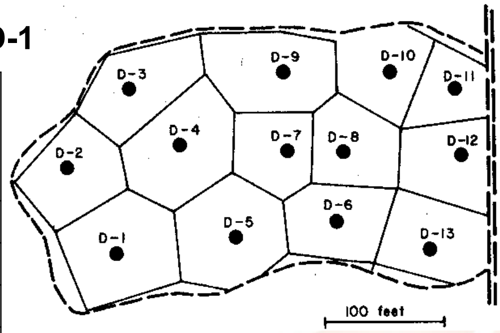
Global estimate: Total tonnage: $$T = \sum_{i=1}^{6} T_i$$

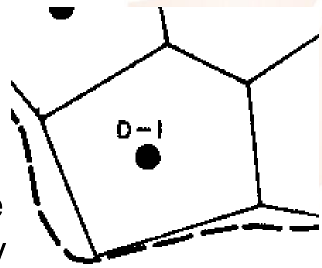
Global mean grade: $$\bar{g} = \frac{\sum_{i=1}^{6} T_i \cdot g_i}{\sum_{i=1}^{6} T_i}$$

Example – Sunshine copper deposit

Assay Data for Drill Hole D-1

Interval (depth) (m)	Thickness (m)	Grade % Cu
100-110	**10**	**0.47**
110-122	**12**	**0.72**
122-130	**8**	**0.96**
130-150	**20**	**1.04**
150-200	**50**	**0.82**
200-220	**20**	**0.54**
220-250	**30**	**0.42**



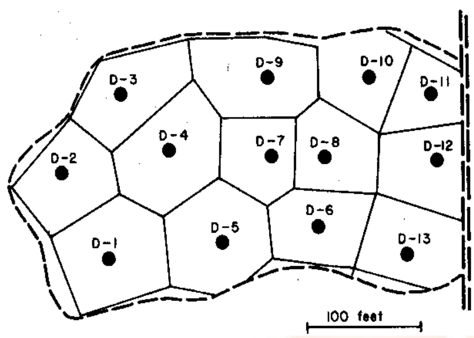


Exercise 1: Calculate the average ***grade*** for the polygon defined by D-1

Solution to Ex. 1:

Assay Data for Drill Hole D-1

Interval (m)	Thickness (m)	Grade % Cu	Grade * Thickness
100-110	10	0.47	**4.70**
110-122	12	0.72	**8.75**
122-130	8	0.96	**7.68**
130-150	20	1.04	**20.80**
150-200	50	0.82	**41.00**
200-220	20	0.54	**10.80**
220-250	30	0.42	**12.60**
sum	150		**106.43**

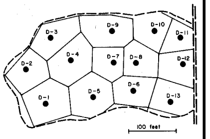


$$\bar{g} = \frac{\Sigma T_i g_i}{\Sigma T_i}$$ (weighted average – by mass)

Average Grade Drill Hole D-1 = 106.43/150 = **0.70% Cu** (weighted average)

Ore Reserves for Sunshine Copper Deposit

Polygon	Area (m^2)	Thickness (m)	Volume (m^3)	Density (ton/m^3)	Ton	Grade % Cu	Tons * Grade
D-1	5320	150	798,000	1/12.5	63,840	0.70	45,326
D-2	5300	135	715,500	1/12.5	57,240	0.66	37,778
D-3	4400	180	792,000	1/12.5	63,360	0.82	51,955
D-4	5520	175	966,000	1/12.5	77,280	0.75	57,960
D-5	6800	155	105,400	1/12.5	84,320	1.00	84,320
D-6	4960	180	892,800	1/12.5	71,424	0.97	69,281
D-7	4520	250	1,130,000	1/12.5	90,400	1.21	109,384
D-8	4640	240	1,113,600	1/12.5	89,088	1.36	121,159
D-9	5840	150	876,000	1/12.5	70,080	0.93	65,174
D-10	4840	135	653,400	1/12.5	52,272	0.87	45,476
D-11	3760	120	451,200	1/12.5	36,096	0.81	29,237
D-12	4270	165	637,200	1/12.5	50,976	0.75	38,232
D-13	4800	135	648,800	1/12.5	51,840	0.68	35,251
					858,216		790,553

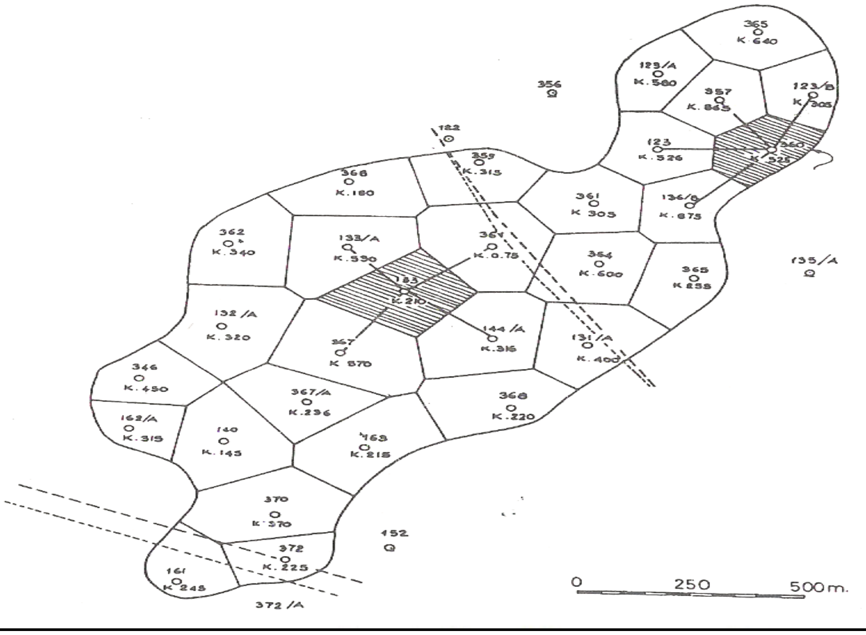


Global Estimate

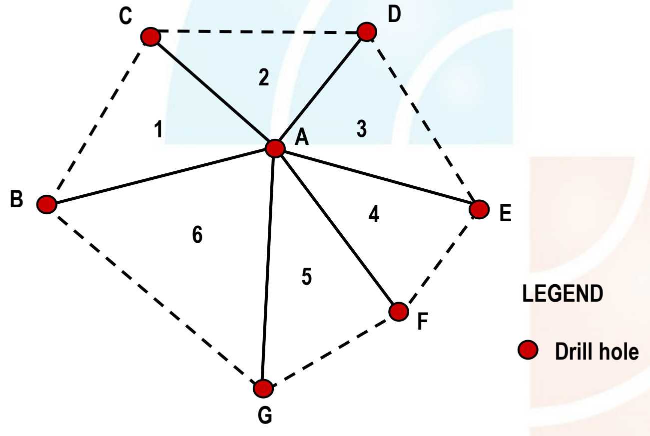
Average Grade Entire Deposit = 790,533/858,216 = **0.92% Cu**

Therefore; Total reserve: 0.86 mT @ 0.92% Cu

Another example …



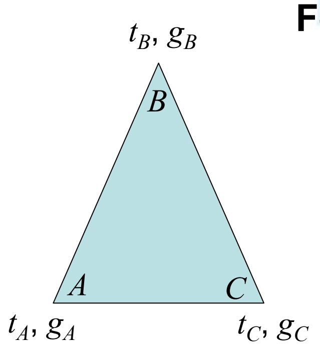
2. Estimation method – TRIANGLATION



Aim: to estimate the **tonnage** and average grade within the outline polygon (dotted lines) – consists of six triangles

Triangulation Technique

- A series of triangles is constructed with the drill holes at the apices.



For each triangle:

Average thickness:

$$\bar{t} = \frac{1}{3}(t_A + t_B + t_C)$$

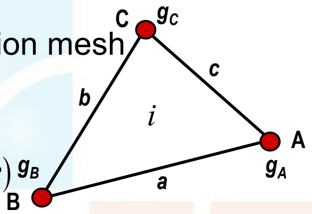
Average grade (weighted):

$$\bar{g} = \frac{t_A g_A + t_B g_B + t_C g_C}{t_A + t_B + t_C}$$

where: g_A – grade at drill hole A
t_A – thickness of mineralisation at drill hole A

Steps:

1. Join adjacent points to form a triangulation mesh
2. For individual triangle (e.g., triangle i):
   Estimate the area & (volume):



Area: $A_i = \sqrt{p(p-a)(p-b)(p-c)}$ where $p = \frac{1}{2}(a+b+c)$

Volume: $V_i = A_i \cdot H_i$; { $H_i = (t_A + t_B + t_C)/3$ }
where H_i is the average thickness of the triangle

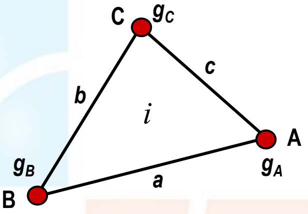
Estimate the weighted average grade of the triangle g_i

$$g_i = \{\frac{g_A t_A + g_B t_B + g_C t_C}{t_A + t_B + t_C}\}$$

Therefore, tonnage and grade for triangle *ABC* are:

Tonnage $T_i = V_i * d_i$ where d_i is the density of ore
Average grade = g_i

Steps con't :



3. Total tonnage of the polygon area (previous slide):

$$g_i = \frac{\{g_A t_A + g_B t_B + g_C t_C\}}{t_A + t_B + t_C}$$

Total tonnage $T = T_1 + T_2 + T_3 + T_4 + T_5 + T_6$

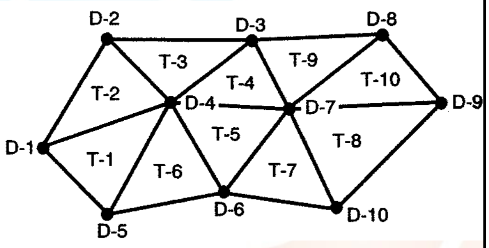
4. Overall average grade – weighted average:

$$g = \frac{g_1 T_1 + g_2 T_2 + g_3 T_3 + g_4 T_4 + g_5 T_5 + g_6 T_6}{T_1 + T_2 + T_3 + T_4 + T_5 + T_6}$$

5. Report: Total tonnage: T @ g

Triangulation Method

- It is a modification of the polygon method. A series of triangles is constructed with the drill holes at the apices.
- This method has the advantage in that the three points are considered in the calculation of the thickness and grade for each triangular block.



$$\bar{t} = \frac{1}{3}(t_A + t_B + t_C)$$

$$\bar{g} = \frac{t_A g_A + t_B g_B + t_C g_C}{t_A + t_B + t_C}$$

Triangulation Method – Global Estimation

Global estimate: total tonnage and average grade for the entire area

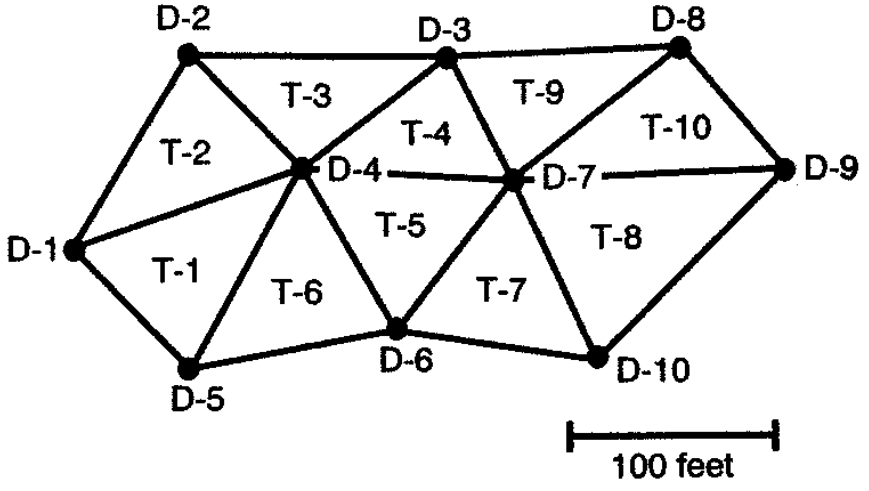
If we have n numbers of triangles:

Total tonnage: $T = \sum_{i=1}^{n} T_i$

Global mean grade: $\bar{g} = \frac{\sum_{i=1}^{n} T_i \cdot g_i}{\sum_{i=1}^{n} T_i}$ **(weighted average)**

T_i and g_i are tonnage and grade for triangle i

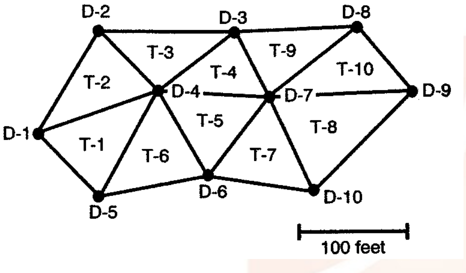
Triangulation method - Example



Assay Data (Copper Deposit)

Drill Hole	Thickness (m)	Grade % Cu
D-1	50	0.93
D-2	75	0.77
D-3	60	0.82
D-4	100	1.05
D-5	75	0.72
D-6	60	0.49
D-7	105	1.63
D-8	80	0.91
D-9	70	0.86
D-10	75	0.74

Triangulation for a given orebody



Given this data, the tonnage and grade calculation for triangle T-1 would be as follows:
Area = 4400 m^2 (by geometry)

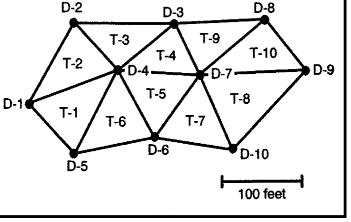
Drill Hole	Thickness (m)	Grade % Cu	Grade * Thickness
D-1	50	0.93	46.5
D-4	100	1.05	105.0
D-5	75	0.72	54.0
Total	225		205.5

Average grade = 205.5/225.0 = 0.91% Cu

Tonnage = area * average thickness * density

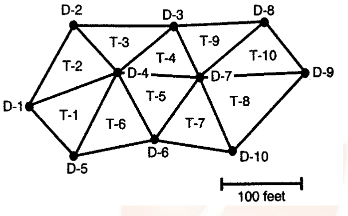
= 4400 *(225/3)/12.5

= 26,400 tons.



Ore reserve for the copper deposit

Triangle	Drill Holes	Ton	Avg. Grade % Cu	Tons * Grade
T-1	D-1, D-4, D-5	26,400	0.91	24,024
T-2	D-1, D-2, D-4	26,400	0.94	24,816
T-3	D-2, D-3, D-4	22,500	0.91	20,475
T-4	D-3, D-4, D-7	22,260	1.23	27,380
T-5	D-4, D-6, D-7	18,550	1.15	21,332
T-6	D-4, D-5, D-6	27,260	0.79	21,535
T-7	D-6, D-7, D-10	26,240	1.07	28,076
T-8	D-7, D-9, D-10	40,500	1.15	46,575
T-9	D-3, D-7, D-8	24,418	1.20	29,301
T-10	D-7, D-8, D-8	28,917	1.19	34,411
Total		**263,445**		**277,927**



Average Grade:

= 277,927/263,445
= 1.05 % Cu

Total reserve:
= 0.26 mT @ 1.05% Cu

Inverse Distance Weighting Technique

3. POINT ESTIMATION

- Estimate unknown values at specific locations.
- Need to use weighted linear combinations; weights need to account not only for possible clustering but also distance to nearby samples.
- Consider the sample configuration as shown in the next slide.

Table 11.1: Distance to sample values from (65E,137N)

	Sample ID	X	Y	ppm	Distance from (65E,137N)
1	DDH225	61	139	447	4.5
2	DDH437	63	140	696	3.6
3	DDH367	64	129	227	8.1
4	DDH52	68	128	646	9.5
5	DDH259	71	140	606	6.7
6	DDH436	73	141	791	8.9
7	DDH366	75	128	783	13.5

(i) Inverse Distance Techniques

- This technique applies a weighting factor which is based on linear or exponential distance function, to each sample surrounding the central point of an ore block.
- The weighting factor is the inverse of the distance between each sample and the block centre, raised to power 'n' (some books; p, m), where 'n' usually varies between 1 and 3.
- Only samples falling within a **specified radius** or volume are weighted this way.

(i) Inverse Distance Techniques

Fundamental formula

$$Estimate = \frac{\sum_{i=1}^{n} \frac{v_i}{d_i^m}}{\sum_{i=1}^{n} \frac{1}{d_i^m}}$$

(i) Inverse Distance Techniques

- Note that the weights, W_i is equivalent to:

$$w_i = \frac{1/d_i^m}{\sum_{i=1}^{n} 1/d_i^m}$$

(i) Inverse Distance Techniques

- *Estimate* = estimated value for the centre of the block (point estimate)
- V_i = value of sample i within the search area
- d_i = distance from point to be estimated to sample i, for i =1,……n
- m = exponent. Different choices of m will result in different estimates. ***m =2 is very commonly used***
- n = number of samples within the search area.

(i) Inverse Distance Techniques

The distance d, may be calculated as follows

- Using Pythagoras theorem and knowing the X,Y,Z coordinates of the sample locations
- Using the scale of the map to measure distances between samples.
- ***Because the method is laborious and repetitive, it is usually computerised***.

(ii) Inverse Distance Interpolation Rules

• Samples should be within the same geological domains.

• Search radius – 2D or 3D.

• Other rules as recommended by the geologist

(ii) Inverse Distance Interpolation Rules

• Circular Search Radius

- In this case, the computer checks the distance of each sample in the dataset from the current block centre and rejects those that are outside the specified radius.

(ii) Inverse Distance Interpolation Rules

• Elliptical Search Radius

- Where circular search areas are not deemed to be suitable on the basis of the drilling pattern or knowledge of the mineralisation, an elliptical search radius may be defined.

(ii) Inverse Distance Interpolation Rules

• 3D Search Volumes

- Applicable to large equi-dimensional bodies such as porphyry copper-molybdenum deposits.

- This method obviates the need to calculate bench composite grades.

- The search volume selected can be varied by the geologist.

(ii) Inverse Distance Interpolation Rules

• Cross-validation of Inverse Distance Weighting

- Used to test how well IDW methods are able to estimate points within an or body.

- It works by removing known (sampled) points within the ore body and then estimating their values using nearby samples

- The calculated values are then compared to actual sample values by calculating the correlation coefficient and plotting a regression line. If all is well the slope of this line will turn out to be 45 degrees.

(iii) Inverse Distance Weighting techniques – Example using Table 11.2 dataset

• Assuming the exponent ***m=2*** (inverse distance squared method):

• The weights are calculated as follows:

$w_1 = 0.2555$

$w_2 = 0.3993$

$w_3 = 0.0789$

$w_4 = 0.0573$

$w_5 = 0.1153$

$w_6 = 0.0653$

$w_7 = 0.0284$

(iii) Inverse Distance Weighting techniques – Example using Table 11.2 dataset

- Hence, the estimate for point (65E, 137N) is:

=
(0.2555x477)+(0.3993x696)+(0.0789x227)+(0.0573x646)
+(0.1153x606)+(0.0653x791)+(0.0284x783)

= **598ppm**

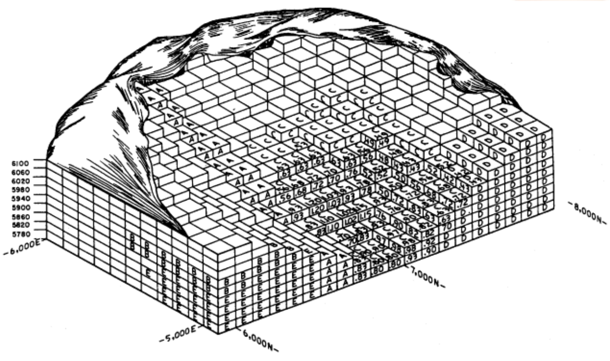
- **Can you think of any drawbacks of this method?**

Resource Model

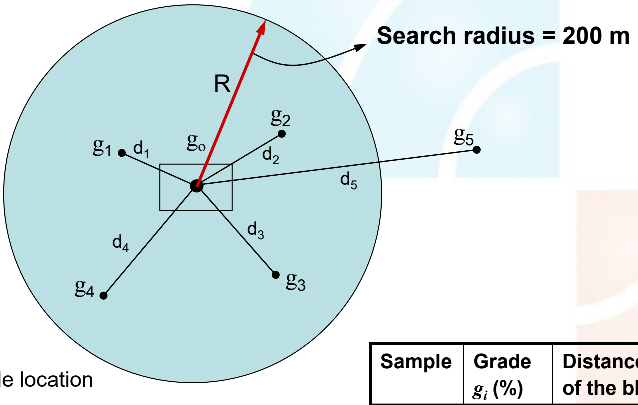
Block model (used for mine design/planning) can be created by:

- Inverse Distance
- Kriging (geostatistics)

but not by polygon or triangulation method



Distance Weighting Calculations - example



• = Drill hole location

g_o = Estimate grade @ center of block

g_i = Grade @ drill hole *i*

d_i = Distance from drill hole to center of block

R = Radius of search neighbourhood

Sample	Grade g_i (%)	Distance to centre of the block, d_i (m)
1	0.5	60
2	0.6	90
3	0.8	120
4	0.5	150
5	0.9	210

INVERSE DISTANCE METHOD

$$g_0 = \frac{\sum \frac{g_i}{d_i}}{\sum \frac{1}{d_i}} = \frac{\frac{g_1}{d_1} + \frac{g_2}{d_2} + \frac{g_3}{d_3} + \frac{g_4}{d_4}}{\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \frac{1}{d_4}}$$

$$g_0 = \frac{\frac{0.5}{60} + \frac{0.6}{90} + \frac{0.8}{120} + \frac{0.5}{150}}{\frac{1}{60} + \frac{1}{90} + \frac{1}{120} + \frac{1}{150}} = 0.58$$

COMPARISON OF WEIGHTING FACTOR

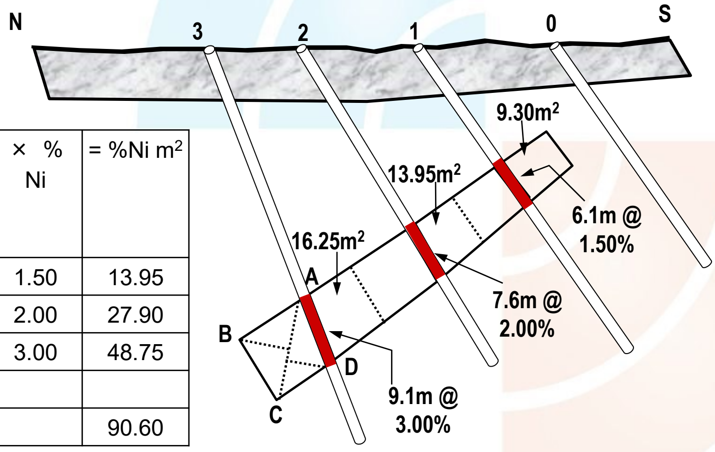
In general: $g = \frac{\sum_{i=1}^{n} \frac{g_i}{d_i^{\,p}}}{\sum_{i=1}^{n} \frac{1}{d_i^{\,p}}}$ $\quad p \geq 0$ $\qquad$ Weight: $w_i = \frac{\frac{1}{d_i^{\,p}}}{\sum_{j=1}^{n} \frac{1}{d_j^{\,p}}}$ (≤ 1)

(weights)

	60 m	90 m	120 m	150 m	
INVERSE DISTANCE (power of 1)	0.389	0.259	0.195	0.156	
	(38.9%)	(25.9%)	(19.5%)	(15.6)	Σ=100%
INVERSE SQUARE DISTANCE (power of 2)	0.539	0.239	0.135	0.086	

Higher power for distance d_i means the mineralisation is more continuous and therefore more weight will be given to samples closer to the block whose grade value is to be estimated.

3D Estimation - Cross-sectional Method



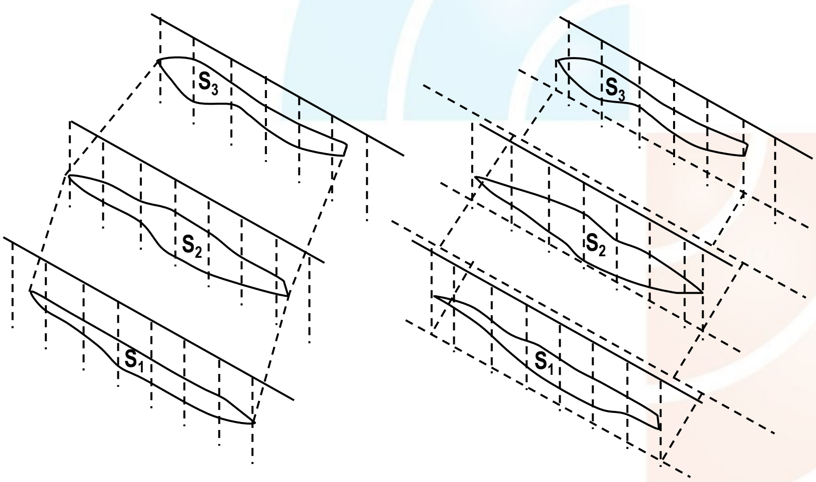
Drillhole and Area No.	Area in m^2	× % Ni	= %Ni m^2
1	9.30	1.50	13.95
2	13.95	2.00	27.90
3	16.25	3.00	48.75
Total	39.50		90.60

$$\text{Average grade} = \frac{90.60}{39.50} = 2.29\%\ \text{Ni}$$

Area and average grade of Section A = 39.5m^2 averaging 2.29% Ni

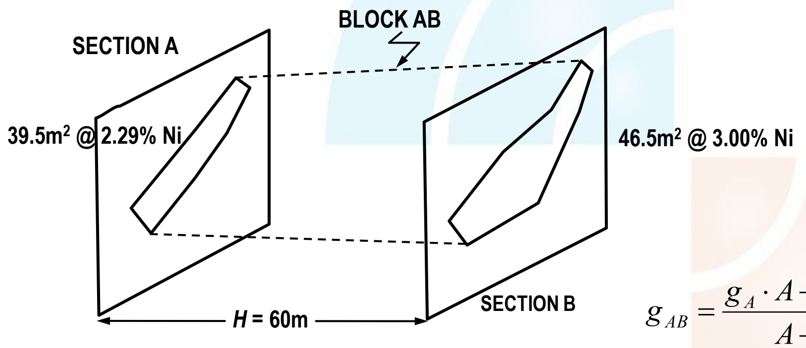
Cross-section Method – two approaches

- extended to three dimensions -



Gradual change **Nearest points**

Cross-section Method – gradual change



$$g_{AB} = \frac{g_A \cdot A + g_B \cdot B}{A + B}$$

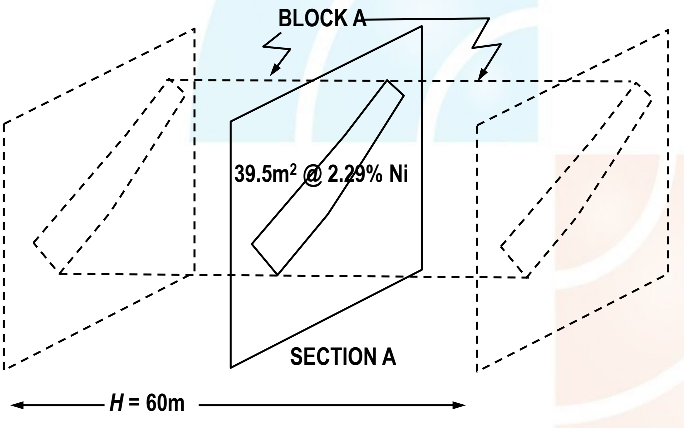
Grade of Block AB = the average of two sections weighted by their areas

Tonnage of Block AB $= \left(A + B + \sqrt{AB}\right) \times sg \times \frac{H}{3}$

= 10,410 tons averaging 2.67% Ni

(sg – density of ore)

Cross-section Method – nearest points



$$\begin{aligned} \text{Tonnage of block A} &= \text{Area of section } A \times H \times \textit{specific gravity} \\ &= 39.5 \times 60 \times 4.04 \\ &= 9{,}575 \text{ tonnes averaging } 2.29\% \text{ Ni} \end{aligned}$$

Pros and cons of traditional estimation methods

Pros: Simple & easy to apply

Cons: Model is subjective
Model & assumption are difficult to verify
Reliability of estimation is hard to quantify

Students to do Exercise 3A (Handout) in Groups

Q1 – Metal accumulation values (centimetre-gram/tonne) are available at five locations within a study area. Table 1 shows the locations and value of each of the five samples. Estimate the value at the location where X = 25; Y = 165m using the (a) Inverse distance technique and (b) Inverse distance squared technique. [**20 marks**]

Table 1. Metal accumulation sample data

X-Coordinate (m)	Y-Coordinate (m)	Metal accumulation (cmg/t)
24.1	184.8	1476
10.8	175.1	1402
19.0	171.9	1364
38.6	158.5	1735
36.7	150.9	1865

Q2 – Boreholes have been drilled in a sedimentary iron ore deposit. We wish to estimate the likely value at an (as yet) unsampled location P at Easting = 250m and Northing = 150m. Refer to Table 2.

Table 2. Drill-hole sample data for an Iron ore deposit

Sample No.	Easting (m)	Northing (m)	Fe content (%)
DDH1	285	110	35.3
DDH2	260	115	33.2
DDH3	335	170	27.4
DDH4	240	185	30.2
DDH6	325	195	33.9
DDH7	290	230	39.9

- Plot the location of drill holes indicating Fe content on a plan view using graph paper. Scale 1cm ≈ 20m [**5 marks**]
- Using the scale measure the distance di from each borehole t the point P [**5 marks**]
- Use the Inverse Distance Technique with m=1.5 to estimate the grade at location P.[**5 marks**]
- Compare your result in (c) if m = 2.0, 3.0. Comment on the comparison. [**5 marks**]