3. Statistical Hypothesis Testing

3.1 Introduction
Hypothesis testing is important because it provides an objective framework for making decisions using probabilistic methods rather than relying on subjective impressions.

3.1.1 Defn
A statistical hypothesis is an assertion or conjecture concerning one or more populations.

Note
1. A hypothesis is usually stated in terms of a population parameter.
2. There are 2 types of hypotheses; the null hypothesis (denoted by H_0) and the alternative hypothesis (denoted by H_1 or H_a).
3. Based on information given by a sample, a choice is made between the null hypothesis (H_0) and the alternative hypothesis (H_1).

3.1.2 Defn
The null hypothesis is a statement asserting no change or no difference or no effect. It usually takes the form of a statement about a population paramater (or parameters) containing an equal sign.

3.1.3 Defn
The alternative hypothesis is a statement that might be true instead of the null hypothesis. It usually contains the sign $>$ or $<$ or $\neq$.

Note
1. The procedure for choosing between hypotheses is called hypothesis testing.
2. In hypothesis testing, the idea is to give the benefit of the doubt to the null hypothesis. The null hypothesis will be rejected (and the alternative accepted) only if the sample data suggest beyond any reasonable doubt that the null hypothesis is false.

Example
A food company produces packets of peanuts weighing 336g (on the average). Periodically, the quality control department takes samples of peanut packets to determine whether the packaging process is under control.

The hypotheses of interest are

$$H_0: \mu = 336g \ (process\ is\ under\ control)$$

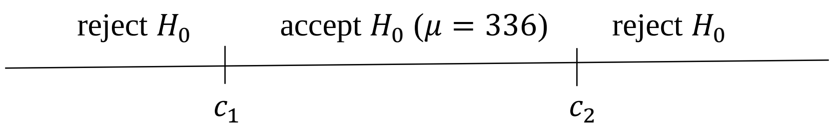
$$H_1: \mu \neq 336g \ (process\ is\ not\ under\ control)$$

To decide between the two hypotheses we

- Choose a random sample of size n.
- Calculate $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$.
- Reject H_0 if $\bar{x}$ is “too large” or “too small” compared to 336.
- How large or small should $\bar{x}$ be?

- Find c_1 and c_2 such that

  if $\bar{x} < c_1$ reject H_0
  if $c_1 \leq \bar{x} \leq c_2$ accept H_0
  if $\bar{x} > c_2$ reject H_0



  c_1 and c_2 are called critical points. $(-\infty, c_1)$ and (c_2, ∞) constitute what is called a critical region, $[c_1, c_2]$ is the acceptance region.

Two types of errors can be made in hypothesis testing

- rejecting H_0 when H_0 is true.
- accepting H_0 when H_0 is false.

3.1.4 Defn
Rejection of the null hypothesis when it is true is called a type I error.

3.1.5 Defn
Acceptance of the null hypothesis when it is false is called a type II error.

Note

1. We represent the probability of a type I error by α
$$\alpha = P(Type\ I\ error)$$
and the probability of a type II error by β
$$\beta = P(Type\ II\ error)$$
2. We can summarise the possible situations as follows:

		Reality	
		H_0 is true	H_0 is false
Decision	Reject H_0 Accept H_0	Type I error (α) Correct decision	Correct decision Type I error (β)

Note that $1 - \beta$ is the probability of rejecting H_0 when H_0 is false. We call $1 - \beta$ the power of the test or decision rule.

3. In the above example, a type I error is made if it is decided that the process in not under control ($i.e.\ \mu \neq 336g$) when infact it is under control ($i.e.\ \mu = 336g$). A type II error is committed if it is decided the process is under control when it is not.

4. The probability of a type I error is often referred to as the level of significance.

Choosing H_0 and H_1
Usually a hypothesis will take the form of a claim, a belief or a suspicion.

Examples

1. A nutritionist claimed that a food company's cans of soup contained more than 900mg of sodium on average. We test the hypotheses
$$H_0: \mu = 900$$
$$H_1: \mu > 900 \quad \leftarrow claim$$

2. An auto company claims that the mean weight of its pickup trucks is 2 tonnes. Test the hypotheses
$$H_0: \mu = 2 \leftarrow claim$$
$$H_1: \mu \neq 2$$

3. The proportion of female students at UNZA is less than 0.4 i.e.
$$H_0: p = 0.4$$
$$H_1: p < 0.4$$

Note:

1. Examples 1 and 3 are known as one sided tests (or alternatives) while 2 is an example of a two sided test.
2. The direction of the alternatives in Examples 1 and 3 are clearly given in the statement. The sign used in the alternative for Example 2 is simply the opposite of the null hypothesis i.e. if we reject the claim that the $\mu = 2$ then it should be $\mu \neq 2$.

Steps in hypothesis testing

1. Testing problem
   State the null hypothesis H_0
   State the alternative hypothesis H_1

2. Test statistic
   Identify the test statistic to be used

3. Critical region
   Determine the critical region (clearly indicate whether it is one or two sided)

4. Observed value
   Calculate the observed value of the test statistic and determine if it falls in the critical region.

5. Conclusion
   Make your decision and state the conclusion.