1] (a) (i) $\hat{p} \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$n = 150,\ \alpha = 0.02,\ \hat{p} = \frac{30}{150} = 0.2$

$0.2 \pm Z_{0.01}\sqrt{\frac{0.2(1-0.2)}{150}}$

$0.2 \pm Z_{0.01}\sqrt{0.001066666}$

$0.2 \pm Z_{0.01}(0.032659863)$

$P(Z < Z_{0.01}) = 1 - 0.01$

$P(Z < Z_{0.01}) = 0.99$

$0.2 \pm 2.33(0.032659863)$

0.2 ± 0.076

$0.124 < P < 0.276$

(ii) $e = Z_{\alpha}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$n = \frac{Z_{\frac{\alpha}{2}}^2}{4e^2}$

$e = 0.04,\ Z_{\frac{\alpha}{2}}^2 = (2.33)^2$

$\therefore n = \frac{2.33^2}{4(0.04)^2}$

$\Rightarrow n = 848.265$

$\Rightarrow n = 849$

b) (i) $\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}$ $\alpha = 0.05, n = 14$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{x} = \frac{97+87+70+86+61+78+95+90+62+96+89+80+78+93}{14}$$

$$\bar{x} = 83$$

$$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2\right]$$

$$s^2 = \frac{1}{13}\left[\left(97^2+87^2+70^2+86^2+61^2+78^2+95^2+90^2+62^2+96^2+89^2+80^2+78^2+93^2\right) - \frac{1}{14}\left(97+87+70+86+61+78+95+90+62+96+89+80+78+93\right)^2\right]$$

$$s^2 = \frac{1}{13}\left[(98318) - \frac{1}{14}(1350244)\right]$$

$$s^2 = \frac{1}{13}\left[98318 - 96446\right]$$

$$s^2 = 144$$

$$\Rightarrow s = \sqrt{144} = 12$$

$$83 \pm t_{0.025, 13} \frac{12}{\sqrt{14}}$$

$$83 \pm 2.160\left(\frac{12}{\sqrt{14}}\right)$$

$$83 \pm 8.927$$

$$\Rightarrow 83 \pm 9$$

$$\Rightarrow \underline{\underline{74 < \mu < 92}}$$

(ii) $$\frac{s^2}{\chi^2_{\frac{\alpha}{2}, n-1}} < \sigma^2 < \frac{s^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}$$

$$\Rightarrow \sqrt{\frac{s^2}{\chi^2_{\frac{\alpha}{2}, n-1}}} < \sigma < \sqrt{\frac{s^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}}$$

$\alpha = 0.05$, $n = 14$, $s^2 = 144$ from (i)

$$\Rightarrow \sqrt{\frac{144}{\chi^2_{0.025, 13}}} < \sigma < \sqrt{\frac{144}{\chi^2_{0.975, 13}}}$$

$$\Rightarrow \sqrt{\frac{144}{24.736}} < \sigma < \sqrt{\frac{144}{5.009}}$$

$$\Rightarrow \underline{\underline{2.41 < \sigma < 5.36}}$$

(iii) It was assummed that the population for the team scores is normal.

2] (a)

i) A type II error can be committed by accepting of the null hypothesis when it is false

(ii) $n_1 = 500$ $\quad \hat{p}_1 = \frac{160}{500} = 0.32$

$n_2 = 300$ $\quad \hat{p}_2 = \frac{66}{300} = 0.22$

$\alpha = 0.01$

1. TEST PROBLEM

$H_0 : P_1 - P_2 = 0$

$H_1 : P_1 - P_2 > 0$

2. TEST STATISTIC

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$

3. CRITICAL REGION

Reject H_0 if

$$Z^* > Z_\alpha = Z_{0.01} = 2.23$$

4. OBSERVED VALUE

$$Z^* = \frac{\hat{P}_1 - \hat{P}_0}{\sqrt{\hat{P}(1-\hat{P})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\hat{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{160 + 66}{500 + 300} = 0.2825$$

$$Z^* = \frac{0.32 - 0.22}{\sqrt{0.2825(1-0.2825)\left(\frac{1}{500} + \frac{1}{300}\right)}}$$

$Z^* = 3.04$ ie $Z > 2.23$

5. CONCLUSION

Reject H_0 at the 0.01 level of significance. Therefore the percentage of men who play the lottery is higher than that of women.

iii) let $\hat{P}_1$ = percentage of men who play lottery in the sample.

$\hat{P}_2$ = percentage of women who play lottery in the sample

$n_1 = 500$ $\quad \hat{P}_1 = \frac{160}{500} = 0.32$

$n_2 = 300$ $\quad \hat{P}_2 = \frac{66}{300} = 0.22$

$\alpha = 0.02$

INTERVAL: $\hat{P}_1 - \hat{P}_2 \pm Z_{\alpha/2} \sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}$

$$0.32 - 0.22 \pm Z_{0.01} \sqrt{\frac{0.32(1-0.32)}{500} + \frac{0.22(1-0.22)}{300}}$$

$$0.1 \pm Z_{0.01} \sqrt{\frac{0.2176}{500} + \frac{0.1716}{300}}$$

$$0.1 \pm 2.33(0.0010072)$$

$$0.1 \pm 0.0023$$

$$\underline{\underline{0.098 < P_1 - P_2 < 0.102}}$$

(b)

(i) let n_1 = number of nonalcoholic men in the sample
n_2 = number of alcoholic men in the sample

$n_1 = 16$ $\bar{x}_1 = 4.6$ $s_1 = 0.8$

$n_2 = 13$ $\bar{x}_2 = 2.3$ $s_2 = 0.54$

$\alpha = 0.05$

1. TESTING PROBLEM

$H_0: \mu_1 - \mu_2 = 2$

$H_1: \mu_1 - \mu_2 > 2$

2. TEST STATISTIC

$$T = \frac{\bar{x}_1 - \bar{x}_2 - \mu_0}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

3. CRITICAL REGION

reject H_0 if

$t^* > t_{\alpha, n_1+n_2-2} = t_{0.05, 27} = 1.703$

4. OBSERVED VALUE

$$t^* = \frac{\bar{x}_1 - \bar{x}_2 - d_0}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

$$s_p^2 = \frac{(16-1)(0.8)^2 + (13-1)(0.54)^2}{16+13-2}$$

$$s_p^2 = 0.48516$$

$$t^* = \frac{4.6 - 2.3 - 2}{\sqrt{0.48516\left(\frac{1}{16} + \frac{1}{13}\right)}}$$

$$t^* = 1.153$$

ie $t^* < 1.703$

5. CONCLUSION

We fail to reject H_0 at the 0.05 level of significance and conclude that there is insufficient evidence to conclude that the average number of hours per week that nonalcoholic fathers spend with their children exceeds that of their alcoholic counterparts by 2 hours.

(ii) $n_1 = 16$ $\bar{x}_1 = 4.6$ $s_1 = 0.8$

$n_2 = 13$ $\bar{x}_2 = 2.3$ $s_2 = 0.54$

$\alpha = 0.1$

1. TESTING PROBLEM

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

2. TEST STATISTIC

$$F = \frac{s_1^2}{s_2^2}$$

3. CRITICAL REGION

reject Ho if

$f^* < f_{1-\frac{\alpha}{2}}(n_1-1, n_2-1)$ or $f^* > f_{\frac{\alpha}{2}}(n_1-1, n_2-1)$

$f^* < f_{0.95}(15, 12)$ or $f^* > f_{0.05}(15, 12)$

$f^* < \dfrac{1}{f_{0.05}(12, 15)}$ or $f^* > 2.62$

$f^* < \dfrac{1}{2.48}$ or $f^* > 2.62$

$f^* < 0.403$ or $f^* > 2.62$

4. OBSERVED VALUE

$$F^* = \frac{S_1^2}{S_2^2} = \frac{(0.8)^2}{(0.54)^2} = 2.19$$

ie $0.403 < F^* < 2.62$

5. CONCLUSION

Do not reject H_0 at 0.1 level of significance. The variances are not significantly different. Therefore we were justified in assuming that the variance were equal.