UNIVERSITY OF ZAMBIA,
GEOLOGY DEPARTMENT

GGY_3049: STRUCTURAL GEOLOGY

INTRODUCTION TO STRESS

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2.1. Definition of stress

2.2. Normal stress and shear stress

2.3. Unit of stress

2.4. Calculation of stress

2.0. Definition of Stress

- Imagine a **vertical column of material**.
- Along any imaginary horizontal plane within this column, the material **above the plane**, pushes downward on the material below the plane, because of its weight,
- Similarly, the part of the **column below the plane** pushes upward with an equal force on the material above the plane.
- The mutual **action** and **reaction** along a surface constitute a **stress**.

2.1. Normal & Shear Stress

- **Moreover,** along any imaginary plane within the column there are similar **actions** and **reactions**.

- The imaginary plane may be:
  - Horizontal,
  - Vertical, or
  - **Inclined** at any angle.

- The **force**, due to the **weight** of that part of the column that lies **above the plane**, acts in a **vertical direction.**

- However, along an **inclined plane**, the vertically directed force would be resolved into:
  - A **normal** component and
  - A **tangential** component.

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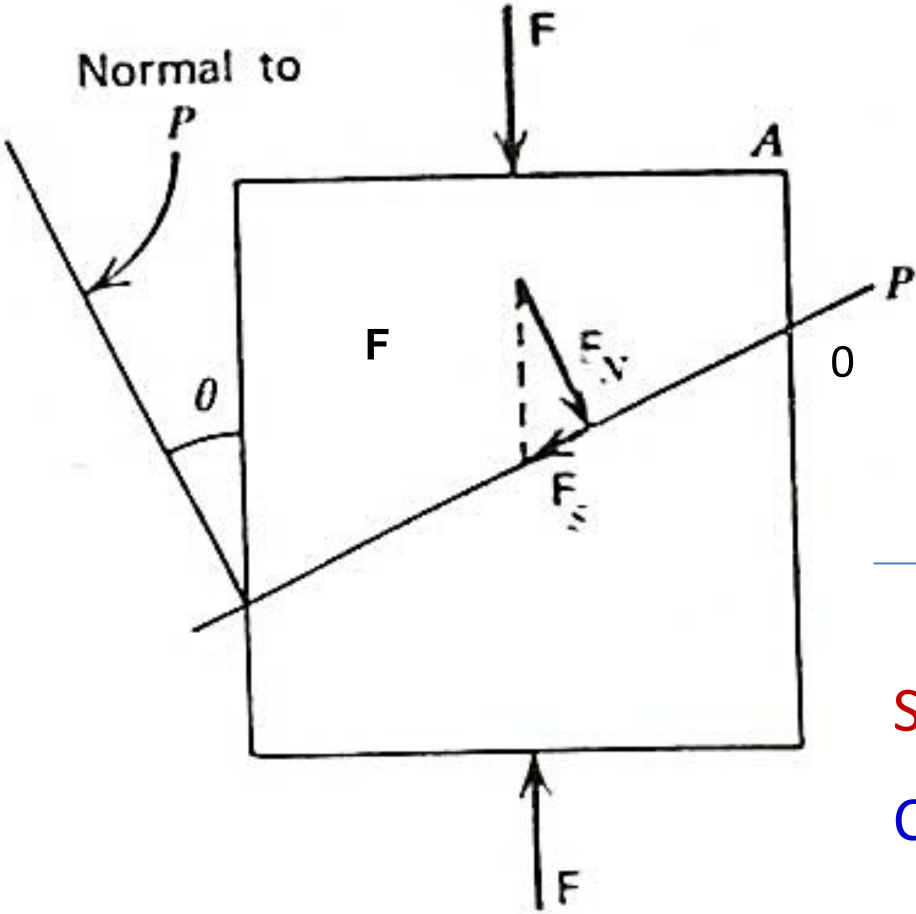
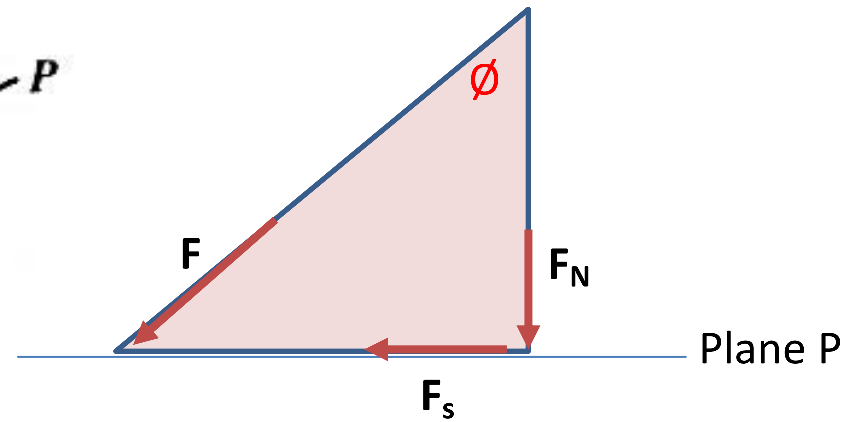


$|F_N| = F \cos \theta$

$|F_S| = F \sin \theta$



$\sin ø = F_s / F \quad (F_S = F \sin ø)$

$\cos ø = F_N / F \quad (F_N = F \cos ø)$

- A **Force vector F** acting on a surface can be decomposed into **a normal (**F_N**)** and **a shear (**F_s**)** component by simple vector addition.
- The **stress vector**, **sigma (σ) can not be decomposed in this way.**
- **This is** because the stress vector **depends on the area across which the force acts**.
- Trigonometric expressions for the components σ_N and σ_S are derived **[Fig. 4.1].**

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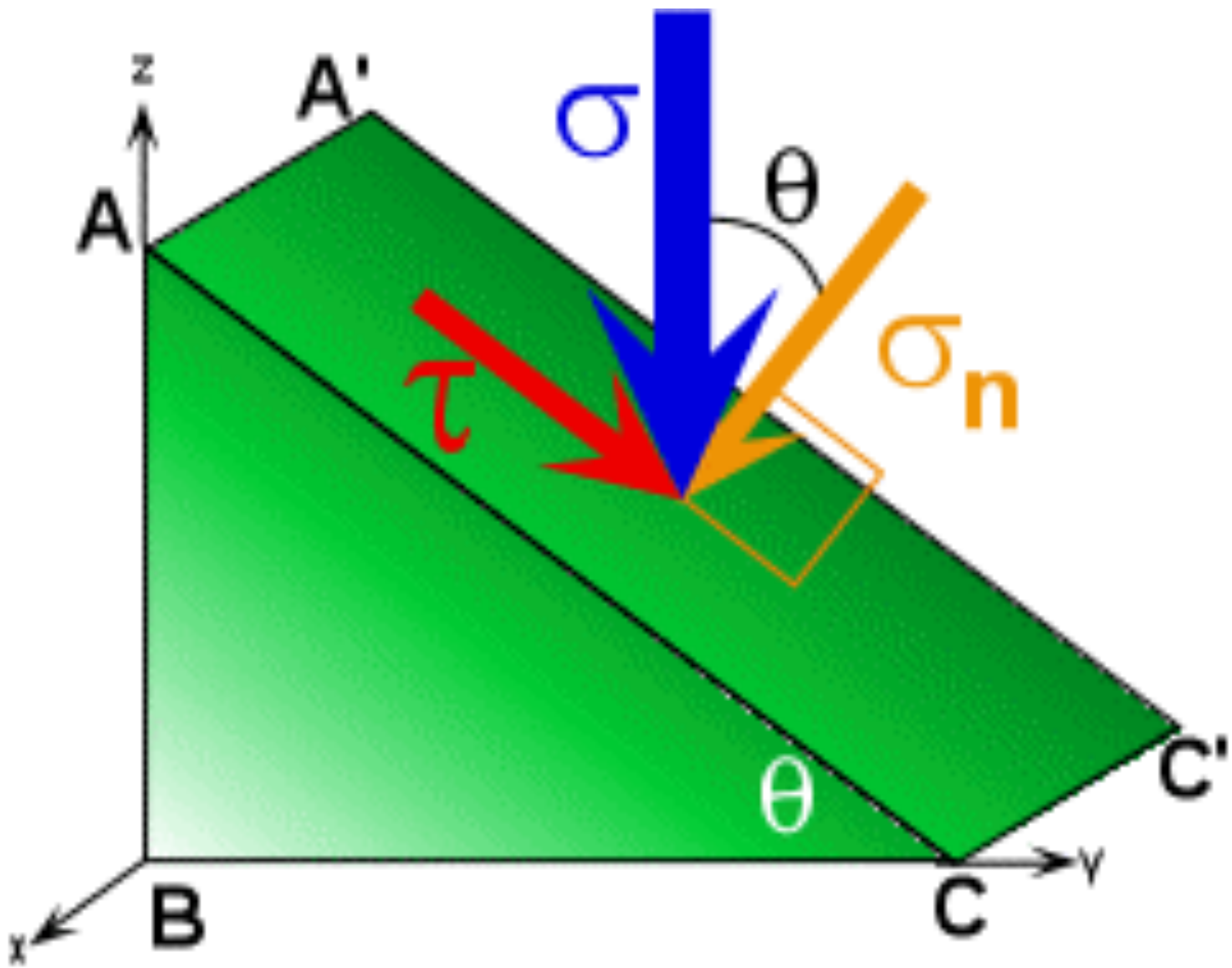
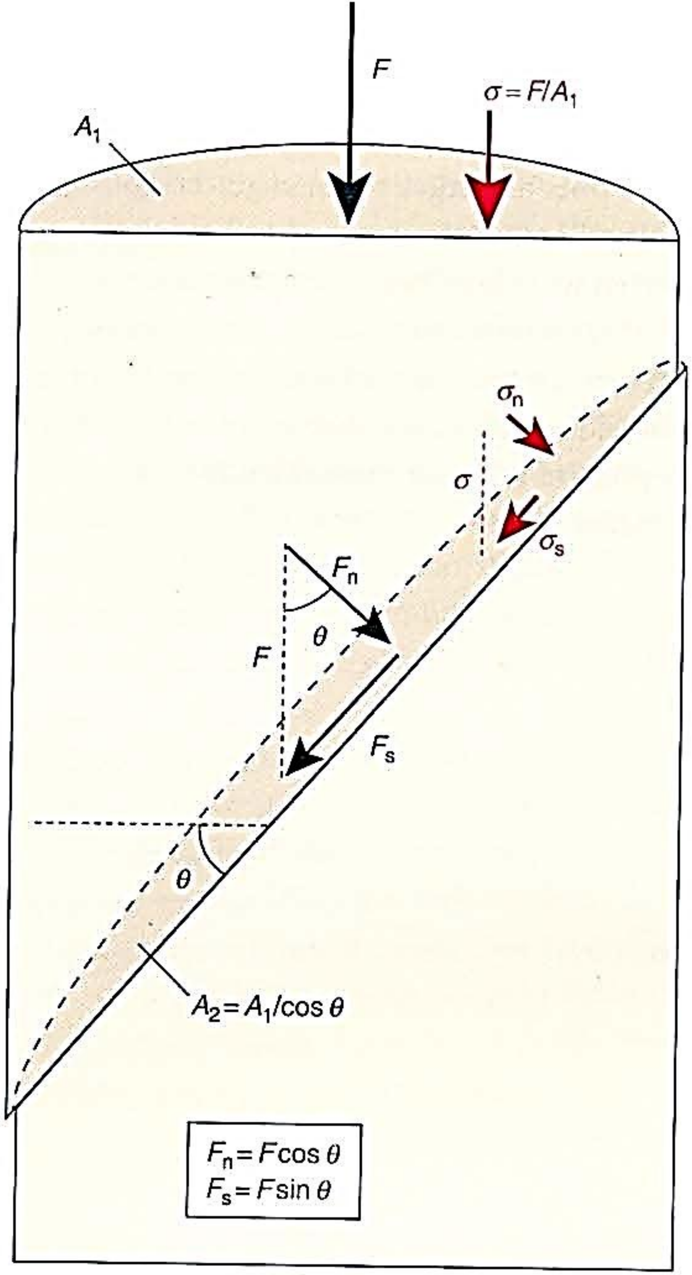
Normal and Shear Stress





$\sigma_n = F_n/A_2 = F\cos\theta/A_2 = F\cos^2\theta/A_1 = \sigma\cos^2\theta$

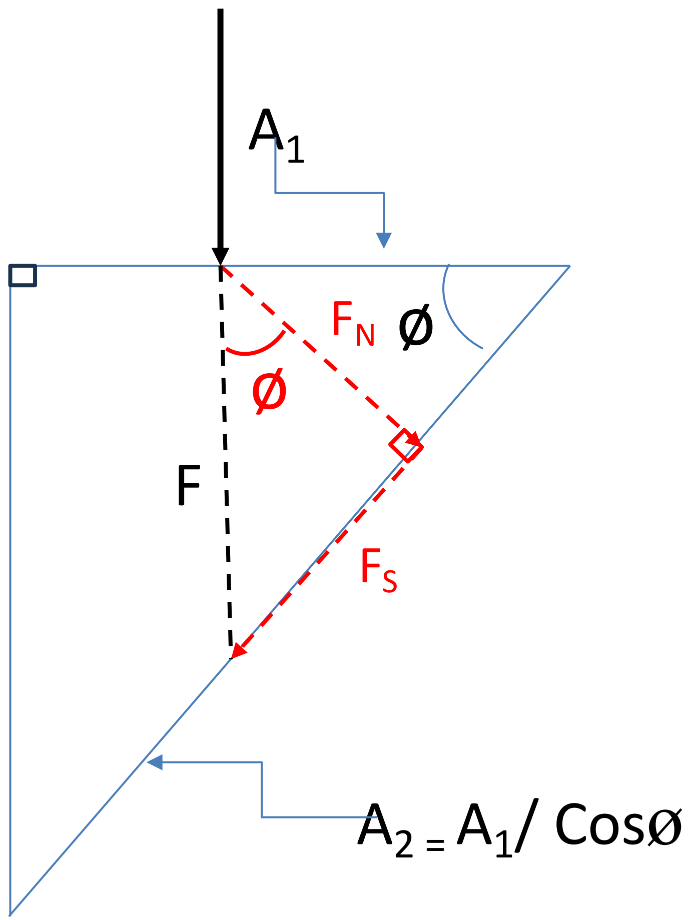
$\sigma_s = F_s/A_2 = F\sin\theta/(A_1/\cos\theta)$

$= F\sin\theta\cos\theta/A_1 = \sigma\sin\theta\cos\theta = \sigma 1/2 \sin 2\theta$

$\sigma = F/A_1$

Calculate the value of σ_N

Calculate the value of σ_S



$\sigma = F/A_1$

Calculate the value of σ_N

$$\sigma_N = F_N/A_2$$
$$= (F\cos ø)/A_2$$
$$= (F\cos ø)/(A_1/\cos ø)$$
$$= F\cos^2 ø/A_1$$
$$= (F/A_1)\cos^2 ø$$
$$= \sigma \cos^2 ø$$

$\sigma = F/A_1$

Calculate the value of σ_S

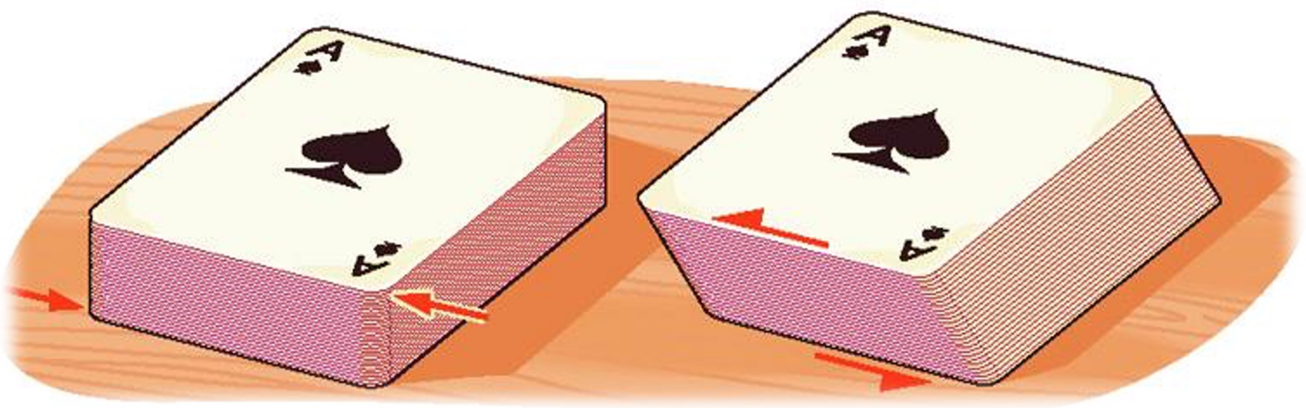
$$\sigma_S = F_S/A_2$$
$$= F\sin ø/(A_1/\cos ø)$$
$$= (F\sin ø \cos ø)/A_1$$
$$= \sigma \sin ø \cos ø$$
$$= \sigma \, {}^1/_2 \sin 2ø$$

- The **normal** component is a compressive **stress** if it tends to **push together** the material on opposite sides of the plane.

- The **normal** component is a **tensile** stress if it tends to **pull apart** the material on opposite sides of the plane.

- The tangential component is generally called a **shearing stress or shear.**

Shear Stress



Shear stress: **stress that acts parallel to a plane**

- It is essential to distinguish between the external force that is applied to a body and the resulting internal actions and reactions that constitute the stress.

2.3. Unit of Stress

- There are several ways to express the magnitude of stress.

- When describing stress levels in the **Earth's crust** or in **specimens** subjected to deformation in the laboratory, the preferred unit of major is the Pascal (Pa):

- *A stress of one pascal is created by the force of one Newton [1N] acting on an area of one square meter [m^2].*

- Recall that a newton is the force required to impart an **acceleration** of **one meter per second per second [1m/s^2]** to a body of **one kilogram[1kg] mass**.

- Because of the small magnitude of a single pascal in comparison to the greater magnitude of stresses in the earth, we commonly precede the term pascal with the prefix **kilo-**, **mega**-, or **giga**-, where:

❖ One **kilopascal** (kPa) = 1000 pascals (10^3 Pa)

❖ One **megapascal** (MPa)= 1,000,000 pascals (10^6Pa)

❖ One **gigapascal** (GPa) =1000,000,000 pascals (10^9Pa)

2.4. Calculation of stress (some exercises)

- There is **no direct way to measure the stresses in a body,** but they **may be calculated if the external forces are known.**

- If a body is compressed or stretched, **the stress is referred to a plane perpendicular to the direction in which the external forces are acting.**

- Thus, if a **vertical square column** 10 inches on a side supports a load of 5000 pounds, every horizontal plane in the column is subjected to a compressive force of 5000 pounds if we neglect the weight of the column itself.

- Each square inch of these horizontal planes supports a load of 50 pounds per **square inch**.

- Thus, The compressive stress is said to be 50 pounds **per square inch** (Billings, p.17).

- If a vertical rod with a cross-sectional area of 10 square inches carries a weight of 5,000 pounds at its lower end, every horizontal plane in the rod is subjected a pull of 500 pounds per square inch.

- The tensile stress is said to be 500 pounds per square inch.

- Imagine a **large block of granite** resting on a marble column.
- The **weight of the granite block constitutes a gravitational load** that **imposes a stress** on the marble column.
- The **magnitude of the stress** is found by dividing the force created by the load of the granite block by the cross sectional area of the top of the marble column.

Granite Load

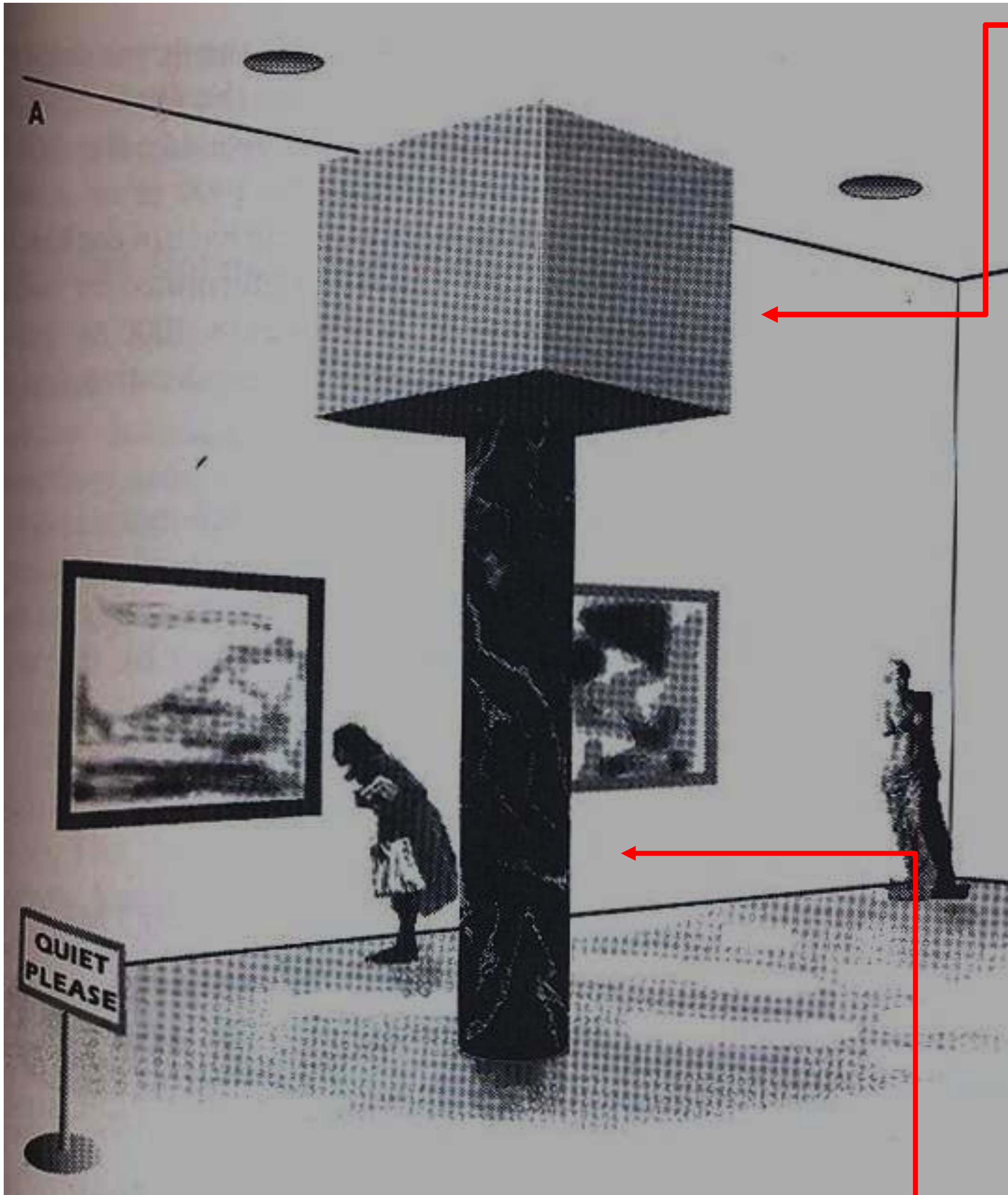


Fig. 3.7. (A)

This piece of sculpture of granite on marble creates an opportunity for calculating the force exerted by the granite block, and the stress generated by it on the top of the marble column.

Marble column

- The **force created by the granite block** is the product of **mass (*m*)** times **acceleration (*g*)** due to gravity.

Or, $F = mg$

Mass is volume **(*V*)** times density **(*p*)**

Or, $(m = V \times p)$

(because $m/V = p$)

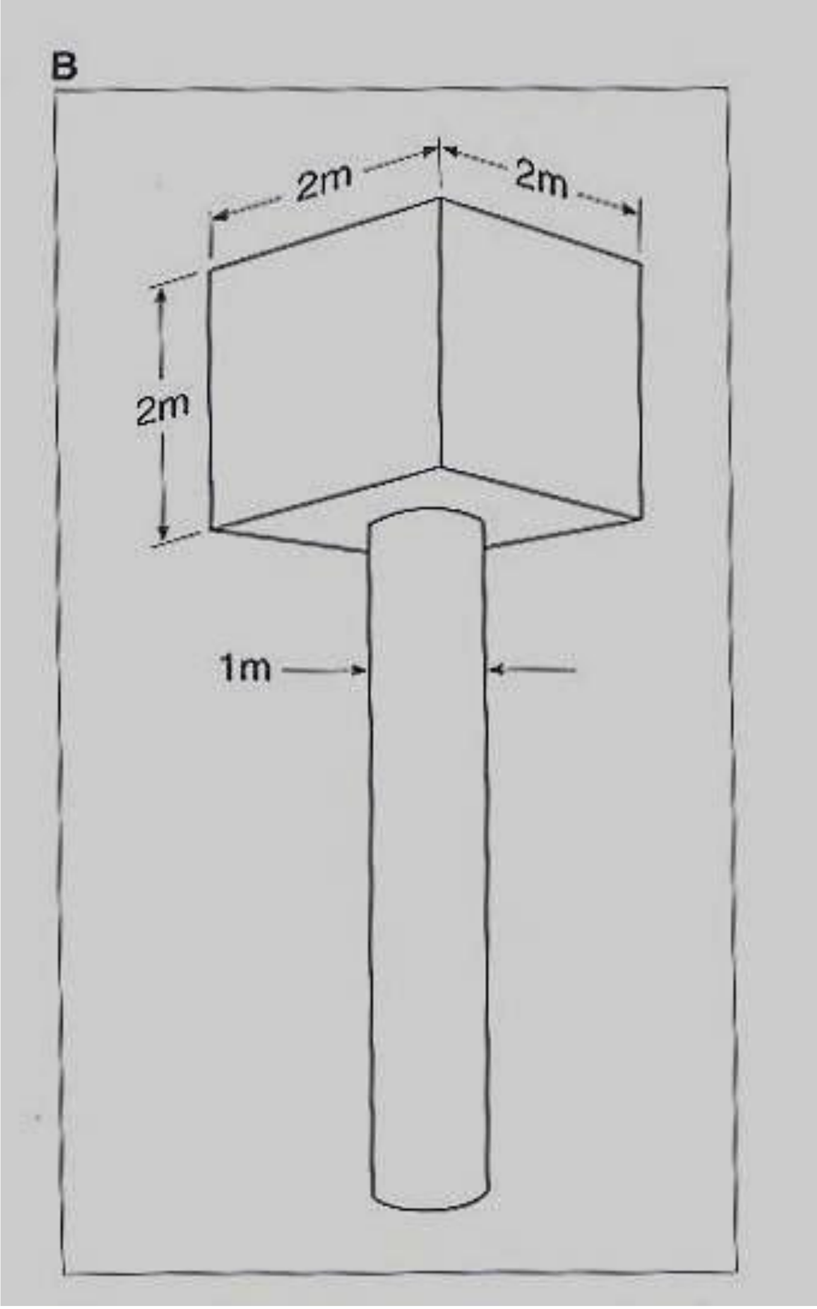


Fig. 3.7 (B).

- Force is determined by measuring the volume of the granite block and multiplying it by its density and by the acceleration due to gravity.
- The stress is determined by diving the force by the cross-sectional area of the marble column.

In this case:

Volume (***V***) = Width (***W***) x (Breadth (***B***) x (Height (***H***)
= 2 m x 2 m x 2 m

Therefore, $V = 8\ m^3$

Density (***ρ***) 2.7 g/cm^3 = 2700 kg/m^3

Mass (***m***) = ***V ρ*** = $8\ m^3$ x 2700 kg/m^3 = 21,600 *kg*

And **force**, as just stated, is compounded by multiplying mass (***m***) times acceleration due to **gravity (*g*)**

Or, **(*m x g*)**

Force (***F***) = mass (***m***) x acceleration (***g***) = ***mg***

F = 21,600 kg x 9.8 m/s^2 = 211,680 N

The **stress**, represented by the Greek letter **sigma (σ),** created by the load of the granite block on the marble column is force dived by area:

$\sigma = F/A$

Circular Area $(A) = Пr^2$ = 3.14 x (0.5) m^2 = 0.79 m^2

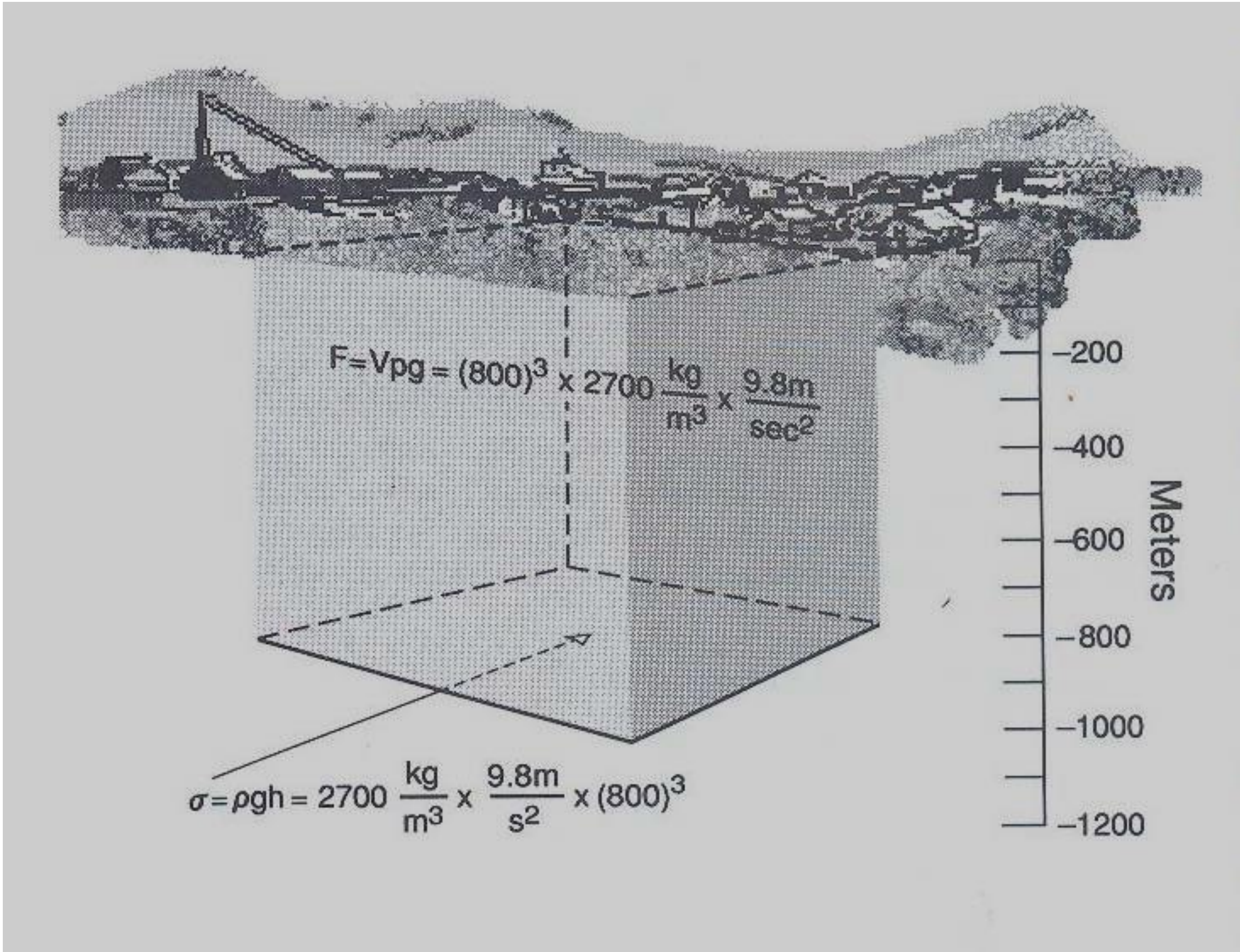
$\sigma = F/A$ = 211,680 N / 0.79 m^2 = 267,949 N / m^2

= **267,949 Pa = 268 *kPa***

2.4. Calculating Stress cont'd

Calculating stress underground:

- In a manner quite similar to the museum-piece example, we can calculate the stress created by the weight of a very large cube of granite in the upper crust.
- Let us picture a region of the Earth where the upper several kilometers of the crust are entirely composed of granite.
- Then let us calculate, for a given depth level, the traction $\mathbf{T}^{\mathbf{down}}$ **(stress)** created by the load of the granite.



We can apply our museum-piece calculation to a huge block of granite nestled in the crust of the Earth.

- We can choose -1000 m (-1 km) as the depth level of interest to us.
- To set up the calculation, it is helpful to visualize the – 1000 m depth level as overlain by a giant cube of granite 1000 m on a side.
- Our goal is to compute the magnitude of the **traction** at the base of the block**.**

The force generated by the weight of the block:

$$F = mg$$

$$p = \frac{m}{V}, \quad \text{and therefore, } m = pV$$

So, $F = Vpg$

It is determined by multiplying the volume of the block (V = 1000 m x 1000 m x 1000 m) times the density of the granite ($p = 2700\ kg/m^3$) times the acceleration due to gravity ($g = 9.8\ m/s^2$).

F = ***V*** (1000m x 1000m x 1000 m) x ***p*** (2700 kg/m^3) x **g**(9.8 m/s^2)

- The Traction **(T^{down})** and stress **(σ)** created by the weight of the block of granite acting on the base of the block is determined by dividing the force **(*F*)** by the area (*A* = 1000m x 1000 m):

$$(T^{down})\ \boldsymbol{\sigma} = \frac{F}{A} = \frac{1000\text{ m} \times 1000\text{ m} \times 1000\text{ m} \times 2700\text{ kg/m}^3 \times 9.8\text{ m/s}^2}{1000\text{m} \times 1000\text{ m}}$$

$$(T^{down})\ \boldsymbol{\sigma} = 1000\text{ m} \times 2700\text{ kg/m}^3 \times 9.8\text{ m/s}^2$$

$$= 26{,}460{,}000\text{ Pa} = 26.5\text{ MPa}$$

- This tells us the ***lithostatic stress gradient*** at depth.
- Lithostatic stress increases **26.5 MPa/km**, which is equivalent to **265 bars or 0.264 kbar/km**.
- In other words, for each **3.8 km** depth, lithostatic stress increases by **1 kbar** or **100 MPa**.

There is s shortcut to determine the stress at any given depth in the granite:

Stress (σ) is simply the product of **density (*p*)** times **gravity (*g*)** times **depth (*h*).**

$\sigma = pgh = 2700\ kg/m^3 \times 9.8\ m/s^2 \times 1000\ m = 26.5\ MPa$

The **calculated stress** level of **26.5 MPa** is very similar to direct ‘*in situ*’ measurements of stress in deep mines at depth levels in the range of **1000 m**.

02/04/2024

End of Introduction to Stress